{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2019(5),1, 1-8

Received: 2019-01-30
Accepted: 2019-06-18
Online published: 2019-10-15
DOI: https://doi.org/10.15414/meraa.2019.05.01.1-8

Original Paper

Nonparametric distribution of the daylight factor

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ABSTRACT

Kernel density estimation (KDE) approximates the distribution of statistical data similar to the histogram. The histogram of data is a special kind of the Kernel density. In the reconstructed building of stall in Oponice (Slovakia), we measured the values of daylight factor. The obtained data proved a bimodal distribution, so it was not appropriate to use some of the usual parametric distributions. This paper describes how Kernel density can be applied to measured results. We find out the values of the cumulative distribution function of such density, by probability procedures, that serves us comparison with the prescribed values of the daylight factor in the standard, on the one hand for animals (1.0%) and on the other hand for the people (1.5%) who care for animals. The results obtained from the measurements and the same ones approximated by KDE are in good agreement.

KEYWORDS: Kernel density estimation, bandwidth, daylight factor

JEL CLASSIFICATION: C13, C16

INTRODUCTION

Nonparametric probability density estimation is an important tool in statistical data analysis [2]. It allows capture multimodality, skewness or other irregular structure of obtained measurements. Compared to the classical parametric estimation of distributions, it has greater flexibility and efficiency. With the parametric approach [6], it is necessary to predict some distribution model, whose small number of parameters is estimated by the likelihood principle. A nonparametric approach does not require an a priori assumption of probability density distribution [4]. The probability density distribution is created directly from the examined data. The most popular nonparametric approach to estimate probability density is Kernel density estimation (KDE). The data, that are possible to model by KDE, can come from very complicated distributions that mathematically do not even have to be described exactly. In this paper, we fit KDE to the distribution of the daylight factor for cattle.

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MATERIAL AND METHODS

Kernel density estimation

The histogram is the simplest nonparametric estimate of the probability density and concurrently it belongs to the most used. To create a histogram, we need two parameters: the starting position of the first bin and the bin width. However, the histogram has several drawbacks. Density estimate depends on the starting point of histogram. The discontinuities of the estimate are not created by underlying density, but only by the bin width of the histogram. The histogram is only suitable for one- or two-dimensional problems, because with a larger number of dimensions it becomes unclear.

A more convenient tool for nonparametric modelling of the distribution density is Kernel density estimation. Its advantage over the histogram lies in that it is smooth and continuous [8]. KDE assembles the measurement of values and creates so called kernels of the bandwidth in these values, which in the sum form the Kernel density estimation. If we have a set of data x_i, i = 1, ..., n, the KDE is expressed according to [12] by the relation

$$\mathrm{KDE}(x) = \frac{1}{n.bw}\sum_{i=1}^{n} K\left(\frac{x - x_i}{bw}\right), \tag{1}$$

where $K\left(\frac{x - x_i}{bw}\right)$ represents the kernel and symbol bw means bandwidth. Figure 1 illustrates a method of producing KDE from individual kernels; on the x axis there are selected these points {14, 16, 20, 30, 33, 35}.

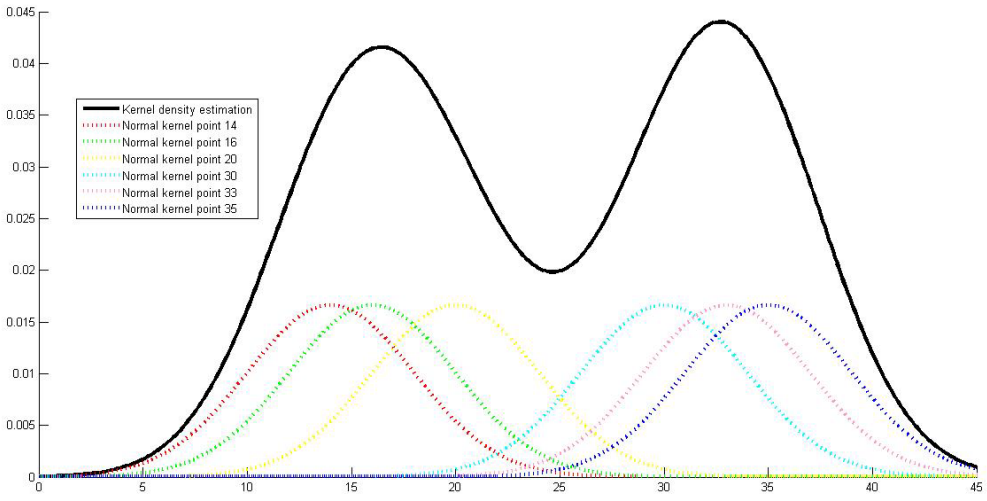

Figure 1 Example of the sum of individual kernels to the resulting KDE, probability density functions

The most used four types of kernels are (in parametric form)

$$\text{Normal:} \qquad K(t) = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} \tag{2}$$

$$\text{Box:} \qquad K(t)=0.5 \tag{3}$$

$$\text{Triangular:} \qquad K(t)=1-|t| \tag{4}$$

$$\text{Epanechnikov:} \qquad K(t)=\frac{3}{4}\left(1-t^2\right) \tag{5}$$

As Breheny stated in [3], each kernel must satisfy three conditions:

$$\text{symmetric to } 0; \quad \int K(u)\,du=1; \quad \int u^2 K(u)\,du>0 \tag{6}$$

In the Figure 2 the various types of kernels (2) - (5) are drawn and the property (6) can be demonstrated.

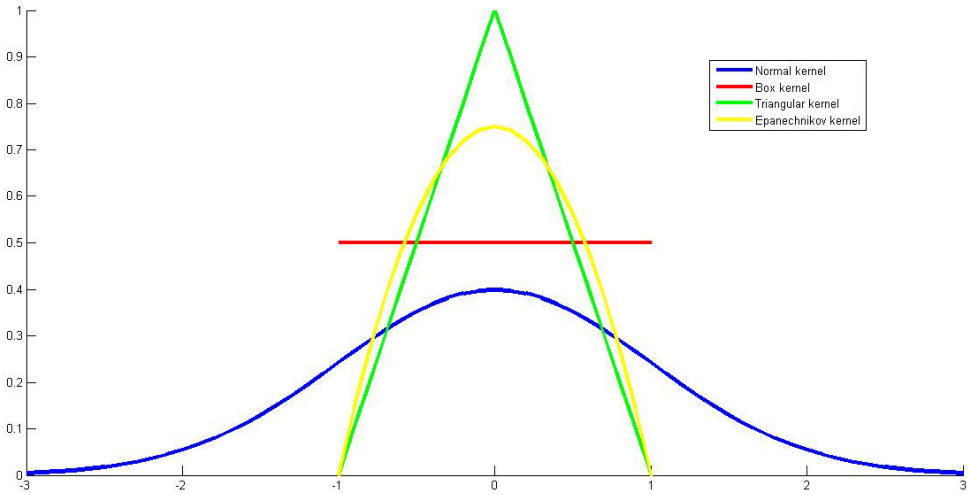

Figure 2 Types of kernels (2) - (5), probability density functions

It turns out that selection of the kernel is not as substantial as choice of the bandwidth bw. By choosing an incorrect value of bw, we can oversmooth the distribution by overvaluing it and undersmooth by undervaluing. The bandwidth can be optimized, for example, for the normal kernel [8].

Another important parameter for representation of KDE is the number of evenly spaced points on the x axis, in which the values of KDE are computed. Obviously, with an increasing number of points, the performance of the density will be more accurate.

Realization of the experiment

Daylighting was measured in the stall for cattle in Oponice (Slovakia). The stall has undergone extensive reconstruction to improve the conditions for animals. Satisfactory lighting is the part of a successful breeding. A detailed description of the building can be found in [1] and [5]. The Figure 3 shows ground plan and cross-section of the studied building also with the position of the measured points.

We measured the internal illumination E and at the same time the external comparison of light E_h in the depicted points. As it is determined in [11], the basic condition for performance of the measurements is the evenly cloudy sky with a brightness distribution for dark landscape. The measurements were made in the precision class 3, because the cattle conditions are degraded in comparison with residential homes [9]. We measured with two identical luxmeters Testo 545 [7]. We calculated the daylight factor by the formula

$$D = \frac{E}{E_h}.100, \tag{7}$$

where D is daylight factor [%], E is internal illuminance [lx], E_h is external comparison illuminance [lx].

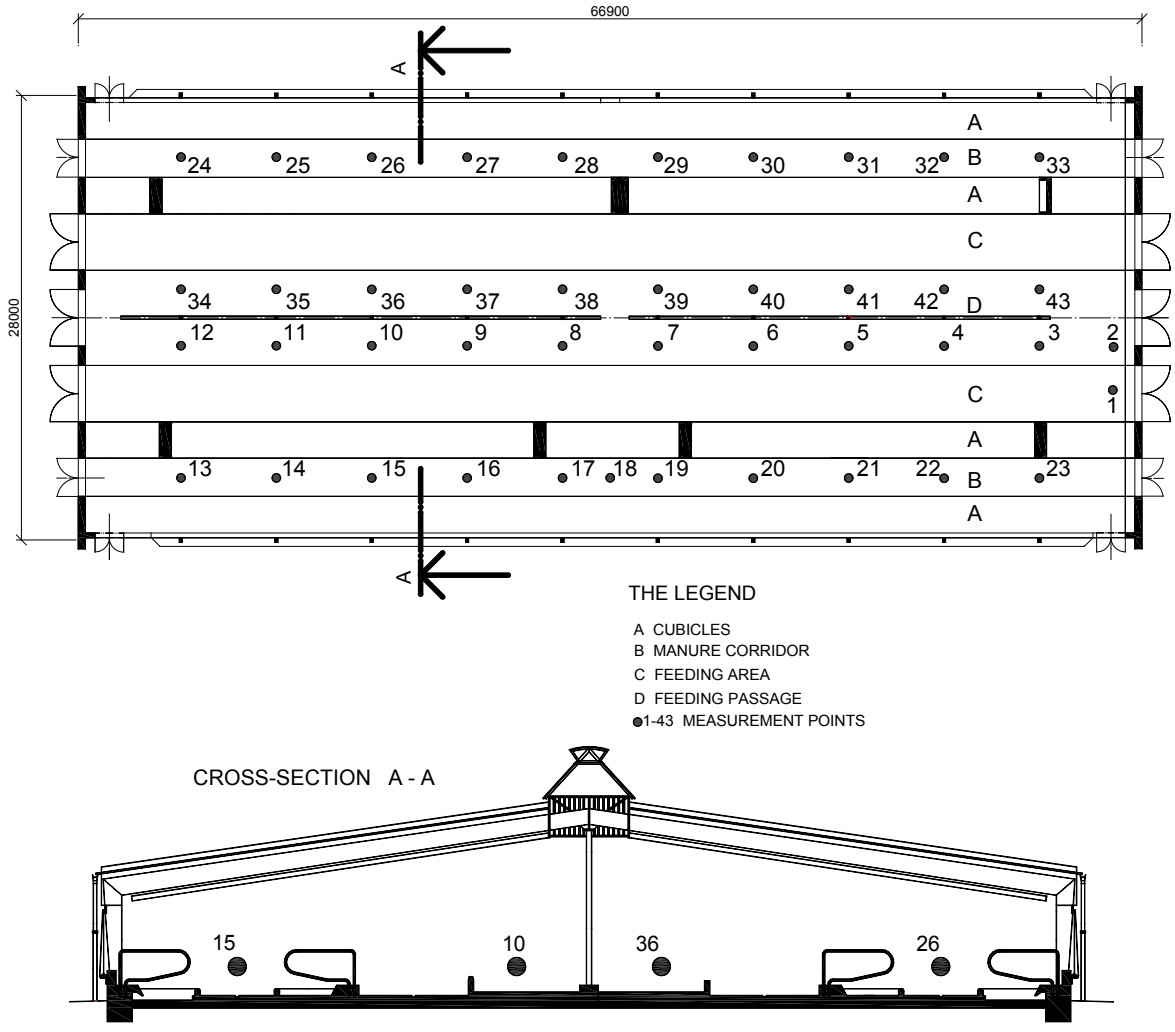

Figure 3 Ground plans and the cross-section of the measured building

The standard values of the daylight factor are listed in [10]. For a stall with dairy cows in free housing, the minimum D value should be 1.0 %. The value of D = 1.5% is needed for cattle workers.

RESULTS AND DISCUSSION

The resulting data in the measurement points transformed by the relation (7) are presented in the Table 1. At each point in the Figure 3 illuminance was measured five times, giving a total number of 215 measurements. As can be seen from the histograms of these data (Figures 4–6) the values have a bimodal probability density distribution. Such distribution shows the heterogeneity of data, probability feature is composed of two phenomena and it is worth considering to divide the data into two more homogeneous sets [13]. To fit on these values the classical parametric distribution is practically impossible. Kernel density estimation, however, without difficulty creates a probability density distribution over such data.

Table 1 Values of the daylight factor organized in ascending order, 215 values, obtained from measurements at 43 points in Figure 3

0.2195	0.6925	1.9011	2.3853	2.8451	3.8000	6.3425	6.8849	7.0987
0.2493	0.6938	1.9162	2.4482	2.8966	3.8211	6.3441	6.8872	7.0991
0.2524	0.6968	1.9336	2.4553	2.9219	3.8349	6.3457	6.8902	7.1041
0.2527	0.7005	1.9645	2.4593	2.9433	3.8356	6.4251	6.9383	7.1443
0.2534	0.7036	2.0274	2.4814	2.9801	3.8410	6.4270	6.9814	7.1684
0.2609	0.7167	2.0410	2.4861	2.9814	4.2793	6.4671	6.9870	7.1768
0.2624	0.7767	2.0644	2.5052	2.9952	4.4198	6.4889	6.9946	7.1811
0.2667	0.8051	2.1371	2.5095	3.0132	4.4201	6.4989	6.9977	7.1885
0.2696	0.8124	2.1492	2.5223	3.0212	4.4337	6.4996	7.0027	7.2394
0.2779	0.8266	2.1753	2.5279	3.0267	4.4873	6.5003	7.0059	7.2926
0.4505	0.8368	2.1765	2.5434	3.0332	5.9975	6.5095	7.0177	7.3122
0.4613	1.1022	2.2032	2.5502	3.0752	6.1256	6.5562	7.0191	7.3755
0.4679	1.1198	2.2202	2.5510	3.1553	6.1597	6.6477	7.0287	7.3833
0.4810	1.1486	2.2453	2.5770	3.2882	6.1828	6.7038	7.0414	7.4780
0.4941	1.3133	2.2494	2.5777	3.3212	6.1876	6.7145	7.0531	7.4851
0.6006	1.3657	2.2527	2.5878	3.3369	6.2071	6.7248	7.0543	7.5118
0.6197	1.6054	2.2544	2.6134	3.3614	6.2113	6.7422	7.0566	7.6676
0.6312	1.6172	2.2661	2.6240	3.3724	6.2427	6.7524	7.0585	7.7039
0.6323	1.6989	2.2688	2.6246	3.3827	6.2604	6.7616	7.0585	8.1081
0.6346	1.7200	2.2828	2.6415	3.6708	6.2733	6.7879	7.0689	8.1120
0.6718	1.7480	2.3068	2.6581	3.7432	6.2939	6.8052	7.0853	8.1128
0.6752	1.7847	2.3118	2.7064	3.7685	6.2959	6.8380	7.0856	8.2334
0.6792	1.8516	2.3205	2.7462	3.7798	6.3017	6.8768	7.0863	8.2486
0.6808	1.8562	2.3423	2.8032	3.7955	6.3370	6.8775	7.0873	

On the Figure 4 KDE is created using data with the optimized bandwidth bw = 1.2853 [8]. Obviously, by reducing the bw to 0.5, the distribution gets closer to the columns of the histogram and the number of its peaks increases, the distribution is undersmoothed. On the contrary, by increasing the value of bw to 2, the distribution is more aligned, the number of peaks decreases, the distribution is oversmoothed [12].

The Figure 5 presents the effect of choosing a different kernel on the form of the probability density. It turns out that the box kernel is quite jump, and also the triangular and

Epanechnikov kernels are not quite ideal. The smoothest shape has the normal kernel. But it is confirmed that the shape of KDE is not too sensitive to the kernel type, because the functions are not different.

The Figure 6 illustrates the effect of the number of evenly distributed points in which kernels are counted. Obviously, the highest value of 100 points gives smoother shape than the lower values 10 and 20 points. With a further increase in points, over 100, there is no more smoother function, so we do not show the higher number of points in the figure.

For KDE, normal kernel and optimized bandwidth bw = 1.2853 (Figures 4 - 6 depicted in blue) we calculated the cumulative distribution function values for daylight factor 1.0 % and 1.5 % as mentioned above [10]. In the Table 2, we compare them with the cumulative frequency calculated directly from the data. The value 1.0 %, acceptable for the animals, is approximated by KDE almost exactly and the value 1.5 %, ideal for the people, who care of animals, differs slightly.

Table 2 Comparison of the cumulative distribution function calculation from data and from Kernel density estimation

	1.0 %	1.5 %
Fractile from data	16.51 %	18.84 %
Fractile from KDE	16.28 %	22.30 %

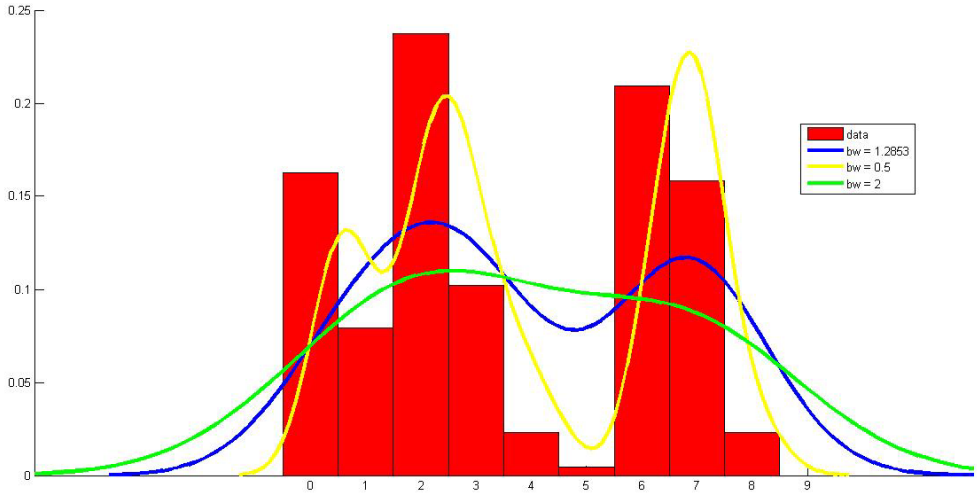

Figure 4 Histogram of the daylight factor together with Kernel distribution estimation for different bandwidth values

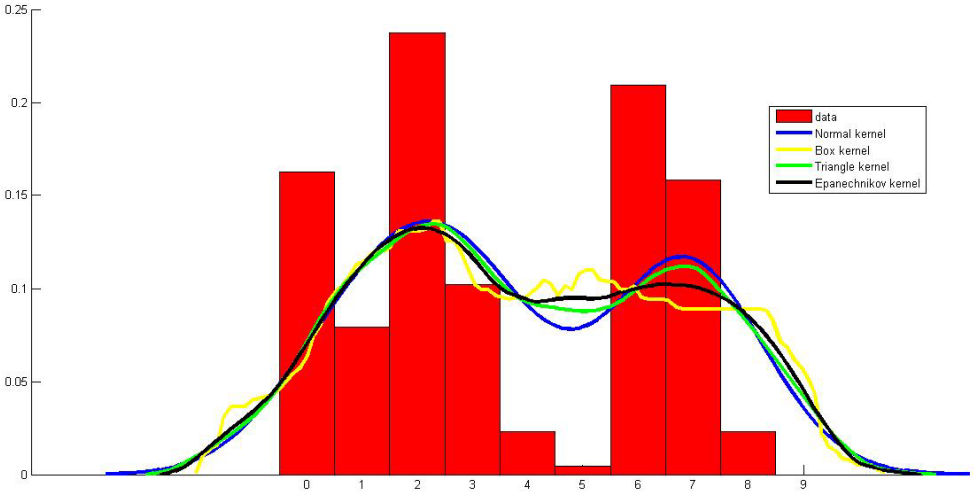

Figure 5 Histogram of the daylight factor together with Kernel distribution estimation for different types of kernels

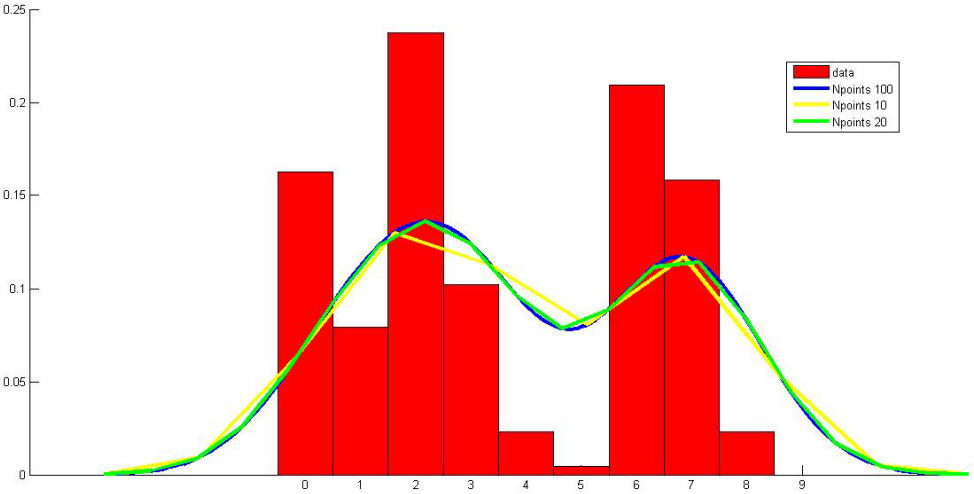

Figure 6 Histogram of the daylight factor together with Kernel distribution estimation for different numbers of approximate points

CONCLUSIONS

We applied the Kernel density estimation for the data obtained by measuring the daylight factor. The effect of bandwidth, kernel and number of points was examined on the shape of the density. It was particularly important to adjust the bandwidth, Figure 4, which the most influences the resulting shape of the distribution. By computing fractiles of the density, we found that roughly 16 % of the values measured in the examined building did not meet the light requirements for the animals and about 20 % of the values did not meet the requirements of the staff. The fractiles calculated from the Kernel density estimation are only slightly different from the fractiles calculated directly from the measured data and can be considered as a suitable approximation.

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{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2019(5), 1, 9-15

Received: 2019-04-11
Accepted: 2019-06-18
Online published: 2019-10-15
Published under
DOI: https://doi.org/10.15414/meraa.2019.05.01.9-15

Original Paper

The sum of the series of reciprocals of the cubic polynomials with one zero and double non-zero integer root

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ABSTRACT

This contribution is a follow-up to author's previous published papers and deals with the sum of the series of reciprocals of the cubic polynomials with one zero and double non-zero integer root. We derive the formula for the sum of these series and verify it by some examples using the basic programming language of the computer algebra system Maple 16. This contribution can be an inspiration for teachers who are teaching the topic Infinite series or as a subject matter for work with talented students.

KEYWORDS: sum of the series, telescoping series, generalized harmonic number, digamma function, Euler's constant, computer algebra system Maple

JEL CLASSIFICATION: I30

INTRODUCTION

This scientific paper is following published contributions [2], [3] and [4], and deals with the sum of the special series. Let us recall some basic terms. The series

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots$$

converges to a limit s if and only if the sequence of partial sums $\{s_n\} = \{a_1 + a_2 + \cdots + a_n\}$ converges to s, i.e. $\lim_{n\to\infty} s_n = s$. We say that the series has a *sum* s and write $\sum_{k=1}^{\infty} a_k = s$. The *sum of the reciprocals* of some positive integers is generally the sum of unit fractions. The nth *harmonic number* is the sum of the reciprocals of the first n natural numbers:

$$H(n) = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n},$$

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Math Educ Res Appl, 2019(5), 1, 9-15

where $H(0)$ is being defined as 0. The *generalized harmonic number* of order n in power r is the sum

$$H(n,r)=\sum_{k=1}^{n}\frac{1}{k^r},$$

where $H(n,1)=H(n)$ are harmonic numbers. Every generalized harmonic number of order n in power 2 can be written as a function of harmonic numbers using formula (see [5])

$$H(n,2)=\sum_{k=1}^{n-1}\frac{H(k)}{k(k+1)}+\frac{H(n)}{n}.$$

Basic and as well interesting information about harmonic numbers can be found in [1].

The *telescoping series* is any series where nearly every term cancels with a preceding or following term, so its partial sums eventually only have a fixed number of terms after cancellation. For example, the series

$$\sum_{k=1}^{\infty}\frac{1}{k(k-3)},$$

where obviously the summation index $k\neq 3$, has the general kth term, after partial fraction decomposition, in a form

$$a_k=\frac{1}{k(k-3)}=\frac{1}{3}\left(\frac{1}{k-3}-\frac{1}{k}\right).$$

After that we arrange the terms of the nth partial sum $s_n=a_1+a_2+a_4+\cdots+a_n$ in a form where can be seen what is cancelling. Then we find the limit $\lim_{n\to\infty}s_n$ of the sequence of the partial sums s_n in order to find the sum s of the infinite telescoping series. In our case we get

$$s_n=\frac{1}{3}\left[\left(-\frac{1}{2}-\frac{1}{1}\right)+\left(-\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\cdots\right.$$
$$\left.\cdots+\left(\frac{1}{n-6}-\frac{1}{n-3}\right)+\left(\frac{1}{n-5}-\frac{1}{n-2}\right)\right]+\left(\frac{1}{n-4}-\frac{1}{n-1}\right)+\left(\frac{1}{n-3}-\frac{1}{n}\right)\Bigg]=$$
$$=\frac{1}{3}\left(-3+1+\frac{1}{2}+\frac{1}{3}-\frac{1}{n-2}-\frac{1}{n-1}-\frac{1}{n}\right).$$

So we have

$$s=\lim_{n\to\infty}\frac{1}{3}\left[-3+H(3)-\frac{1}{n-2}-\frac{1}{n-1}-\frac{1}{n}\right]=\frac{1}{3}\left(-3+\frac{11}{6}\right)=\frac{-7}{18}=-0{,}3\overline{8}.$$

THE SUM OF THE SERIES OF RECIPROCALS OF THE CUBIC POLYNOMIALS WITH ONE ZERO AND DOUBLE NON-ZERO INTEGER ROOT

Let us consider the series of reciprocals of the normalized cubic polynomials with one zero and double non-zero integer root a, i.e. the series

$$\sum_{\substack{k=1\\k\neq a}}^{\infty}\frac{1}{k(k-a)^2},\qquad(1)$$

and let us determine its sum $s(0,a_2)$. We differentiate two cases – a case of negative integer root a and a case of positive integer root a.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2019(5), 1, 9-15

i) First, let us assume the case of a negative integer root a and denote the sum of the series (1) by $s(0,a_2^-)$. Then, after partial fraction decomposition, we get the general kth term in a form

$$a_k = \frac{1}{k(k-a)^2} = \frac{A}{k} + \frac{B}{(k-a)^2} + \frac{C}{k-a},$$

so $1 = A(k-a)^2 + Bk + Ck(k-a)$. For k = 0 we have $A = 1/a^2$, for $k = a$ is $B = 1/a$ and by comparing coefficients in second powers is $0 = A + C$, whence $C = -1/a^2$, so

$$a_k = \frac{1}{k(k-a)^2} = \frac{1}{a^2}\cdot\frac{1}{k} + \frac{1}{a}\cdot\frac{1}{(k-a)^2} - \frac{1}{a^2}\cdot\frac{1}{k-a} = \frac{1}{a^2}\left(\frac{1}{k} - \frac{1}{k-a}\right) + \frac{1}{a}\cdot\frac{1}{(k-a)^2}.$$

Example 1 Using nth partial sum calculate the sum $s(0,-3_2)$ of the series

$$\sum_{k=1}^{\infty} \frac{1}{k(k+3)^2}.$$

Because

$$a_k = \frac{1}{k(k+3)^2} = \frac{1}{9}\left(\frac{1}{k} - \frac{1}{k+3}\right) - \frac{1}{3}\cdot\frac{1}{(k+3)^2},$$

then the nth partial sum $s_n(0,-3_2) = a_1 + a_2 + \cdots + a_n$ is

$$s_n(0,-3_2) = -\frac{1}{3}\left[\frac{1}{4^2} + \frac{1}{5^2} + \cdots + \frac{1}{(n+3)^2}\right] + \frac{1}{9}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right) + \right.$$

$$\left. + \left(\frac{1}{4} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{n-3} - \frac{1}{n}\right) + \left(\frac{1}{n-2} - \frac{1}{n+1}\right) + \left(\frac{1}{n-1} - \frac{1}{n+2}\right) + \left(\frac{1}{n} - \frac{1}{n+3}\right)\right] =$$

$$= -\frac{1}{3}\left[\frac{1}{4^2} + \frac{1}{5^2} + \cdots + \frac{1}{(n+3)^2}\right] + \frac{1}{9}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right).$$

Considering the facts that for any positive integer c is $\lim_{n\to\infty}(1/c) = 1/c$ and $\lim_{n\to\infty}[1/(n+c)] = 0$ and that

$$\lim_{n\to\infty} H(n,2) = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k^2} = \sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6},$$

whence

$$\lim_{n\to\infty}\left[\frac{1}{4^2} + \frac{1}{5^2} + \cdots + \frac{1}{(n+3)^2}\right] = \lim_{n\to\infty}[H(n+3,2) - H(3,2)] = \frac{\pi^2}{6} - \frac{49}{36},$$

we get

$$s(0,-3_2) = \lim_{n\to\infty} s_n(0,-3_2) = -\frac{1}{3}\left[\frac{\pi^2}{6} - H(3,2)\right] + \frac{H(3)}{9} = \frac{49-6\pi^2}{108} + \frac{11}{54} = \frac{71-6\pi^2}{108}$$

$$\doteq 0.1091.$$

Clearly, for arbitrary negative integer a is $A = -a$ positive integer, so $k - a = k + A$ are also positive integers for all $k = 1,2,\ldots,n$. After cancellation all of the inner terms we get the nth partial sum $s_n = a_1 + a_2 + \cdots + a_n$ in a form

$$s_n(0,a_2^-) = \sum_{k=1}^{n}\left[\frac{1}{A^2}\left(\frac{1}{k} - \frac{1}{k+A}\right) - \frac{1}{A}\cdot\frac{1}{(k+A)^2}\right] = -\frac{1}{A}\left[\frac{1}{(1+A)^2} + \frac{1}{(2+A)^2} + \cdots\right.$$

$$\cdots+\frac{1}{(n+A)^2}\Big]+\frac{1}{A^2}\Big[\Big(\frac{1}{1}-\frac{1}{1+A}\Big)+\Big(\frac{1}{2}-\frac{1}{2+A}\Big)+\cdots+\Big(\frac{1}{A}-\frac{1}{2A}\Big)+$$
$$+\Big(\frac{1}{1+A}-\frac{1}{1+2A}\Big)+\cdots+\Big(\frac{1}{n-A}-\frac{1}{n}\Big)+\Big(\frac{1}{n-A+1}-\frac{1}{n+1}\Big)+\cdots$$
$$\cdots+\Big(\frac{1}{n-1}-\frac{1}{n+A-1}\Big)+\Big(\frac{1}{n}-\frac{1}{n+A}\Big)\Big]=$$
$$=-\frac{1}{A}\Big[\frac{\pi^2}{6}-H(A,2)\Big]+\frac{1}{A^2}\Big(1+\frac{1}{2}+\cdots+\frac{1}{A}-\frac{1}{n+1}-\frac{1}{n+2}-\cdots-\frac{1}{n+A}\Big).$$

Considering the three facts above we have

$$s(0,a_2^-)=\lim_{n\to\infty}s_n(0,a_2^-)$$
$$=-\frac{1}{A}\lim_{n\to\infty}\Big[\frac{\pi^2}{6}-H(A,2)\Big]+\frac{1}{A^2}\lim_{n\to\infty}\Big[H(A)-\frac{1}{n+1}-\frac{1}{n+2}-\cdots-\frac{1}{n+A}\Big].$$

Altogether, we derived this statement:

Theorem 1 The series

$$\sum_{k=1}^{\infty}\frac{1}{k(k-a)^2}$$

has for a negative integer a the sum in the form

$$s(0,a_2^-)=\frac{\pi^2}{6a}-\frac{H(-a,2)}{a}+\frac{H(-a)}{a^2}, \qquad (2)$$

where $H(n)$ is the harmonic number and $H(n,2)$ is the generalized harmonic number.

Let us note, for example, that $s(0,-1_2)=2-\pi^2/6$ and $s(0,-2_2)=1-\pi^2/12$.

ii) Now, let us assume the case of a positive integer root a and denote the sum of the series (1) by $s(0,a_2^+)$. Then, after partial fraction decomposition, we also get, as in the case **i)**, that

$$a_k=\frac{1}{a^2}\Big(\frac{1}{k}-\frac{1}{k-a}\Big)+\frac{1}{a}\cdot\frac{1}{(k-a)^2}.$$

Let us note that the summation index k must be different from a to avoid division by zero.

Example 2 Using nth partial sum calculate the sum $s(0,3_2)$ of the series

$$\sum_{\substack{k=1\\k\neq3}}^{\infty}\frac{1}{k(k-3)^2}.$$

Because

$$a_k=\frac{1}{k(k-3)^2}=\frac{1}{9}\Big(\frac{1}{k}-\frac{1}{k-3}\Big)+\frac{1}{3}\cdot\frac{1}{(k-3)^2},$$

then the nth partial sum $s_n(0,3_2)=a_1+a_2+a_4+\cdots+a_n$ is

$$s_n(0,3_2)=\frac{1}{3}\Big[\frac{1}{2^2}+\frac{1}{1^2}+\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{(n-3)^2}\Big]+$$

$$+\frac{1}{9}\left[\left(\frac{1}{1}+\frac{1}{2}\right)+\left(\frac{1}{2}+\frac{1}{1}\right)+\left(\frac{1}{4}-\frac{1}{1}\right)+\left(\frac{1}{5}-\frac{1}{2}\right)+\cdots\right.$$
$$\left.\cdots+\left(\frac{1}{n-3}-\frac{1}{n-6}\right)+\left(\frac{1}{n-2}-\frac{1}{n-5}\right)+\left(\frac{1}{n-1}-\frac{1}{n-4}\right)+\left(\frac{1}{n}-\frac{1}{n-3}\right)\right]=$$
$$=\frac{1}{3}\left[\frac{5}{4}+\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{(n-3)^2}\right]+\frac{1}{9}\left(3-\frac{1}{1}-\frac{1}{2}-\frac{1}{3}+\frac{1}{n-2}+\frac{1}{n-1}+\frac{1}{n}\right).$$

Analogously, as in the case **i)**, we get

$$s(0,3_2)=\lim_{n\to\infty}s_n(0,3_2)=\frac{1}{3}\left(\frac{5}{4}+\frac{\pi^2}{6}\right)+\frac{3-H(3)}{9}=\frac{15+2\pi^2}{36}+\frac{7}{54}=\frac{6\pi^2+59}{108}$$
$$\doteq 1.0946.$$

For arbitrary positive integer a, after cancellation all of the inner terms, we get the nth partial sum $s_n=a_1+a_2+\cdots+a_{a-1}+a_{a+1}+a_{a+2}+\cdots+a_n$ in a form

$$s_n(0,a_2^+)=\sum_{\substack{k=1\\k\neq a}}^{n}\left[\frac{1}{a^2}\left(\frac{1}{k}-\frac{1}{k-a}\right)+\frac{1}{a}\cdot\frac{1}{(k-a)^2}\right]=\frac{1}{a}\left[\frac{1}{(a-1)^2}+\frac{1}{(a-2)^2}+\cdots\right.$$
$$\left.\cdots+\frac{1}{1^2}+\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{(n-a)^2}\right]+\frac{1}{a^2}\left[\left(\frac{1}{1}+\frac{1}{a-1}\right)+\left(\frac{1}{2}+\frac{1}{a-2}\right)+\cdots+\left(\frac{1}{a-1}+\frac{1}{1}\right)\right.$$
$$+\left(\frac{1}{a+1}-\frac{1}{1}\right)+\left(\frac{1}{a+2}-\frac{1}{2}\right)+\cdots+\left(\frac{1}{2a-1}-\frac{1}{a-1}\right)+\left(\frac{1}{2a}-\frac{1}{a-2}\right)+\cdots$$
$$\left.+\left(\frac{1}{n-1}-\frac{1}{n-a-1}\right)+\left(\frac{1}{n}-\frac{1}{n-a}\right)\right]=$$
$$=\frac{1}{a}[H(a-1,2)+H(n-a,2)]+\frac{1}{a^2}\left[H(n)-\frac{1}{a}-H(n-a)+H(a-1)\right].$$

Therefore we have

$$s(0,a_2^+)=\lim_{n\to\infty}s_n(0,a_2^+)=\frac{1}{a}\left[H(a-1,2)+\frac{\pi^2}{6}\right]+\frac{1}{a^2}\left[H(a-1)-\frac{1}{a}\right].$$

Theorem 2 The series

$$\sum_{\substack{k=1\\k\neq a}}^{\infty}\frac{1}{k(k-a)^2}$$

has for a positive integer a the sum in the form

$$s(0,a_2^+)=\frac{\pi^2}{6a}+\frac{H(a-1,2)}{a}+\frac{H(a-1)}{a^2}-\frac{1}{a^3}, \qquad (3)$$

where $H(n)$ is the harmonic number and $H(n,2)$ is the generalized harmonic number.

For example, $s(0,1_2)=\pi^2/6-1$, $s(0,2_2)=\pi^2/12+1/2+1/4-1/8=\pi^2/12+5/8$. Because formulas (2) and (3) are very similar, they can be obviously expressed by the only common formula given in the following statement.

Corollary 1 The series

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
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$$\sum_{\substack{k=1 \\ k\neq a}}^{\infty} \frac{1}{k(k-a)^2}$$

has for a non-zero integer a the sum in the form

$$s(0,a_2) = \frac{\pi^2}{6a} + \frac{H\left(|a| - \frac{\mathrm{sgn}(a)+1}{2}, 2\right)}{|a|} + \frac{H\left(|a| - \frac{\mathrm{sgn}(a)+1}{2}\right)}{a^2} - \frac{\mathrm{sgn}(a)+1}{2a^3}, \qquad (4)$$

where $H(n)$ is the harmonic number and $H(n,2)$ is the generalized harmonic number.

NUMERICAL VERIFICATION

We solve the problem to determine the values of the sum $s(0,a_2)$ for $a = -10,-9,\ldots,-1$ and for $a = 1,2,\ldots,10$. We use on the one hand an approximative direct evaluation of the sum

$$s(0,a_2,t) = \sum_{\substack{k=1 \\ k\neq a}}^{t} \frac{1}{k(k-a)^2},$$

where $t = 500000$, using the basic programming language of the computer algebra system Maple 16, and on the other hand the formula (4) for evaluation the sum $s(0,a_2)$. We compare 20 pairs of these ways obtained sums $s(0,a_2,500000)$ and $s(0,a_2)$ to verify the formula (4). We use following simple procedure `rp30aa` and following two `for` statements:

```
> rp30aa:=proc(a,t)
      local k,s0a2t,s0a2; s0a2t:=0;
      for k from 1 to t do
          if k<>a then s0a2t:=s0a2t+1/(k*(k-a)*(k-a))
                  else s0a2t:=s0a2t+0;
          end if;
      end do;
      s0a2:=Pi*Pi/(6*a)+harmonic(abs(a)-(signum(a)+1)/2,2)/abs(a) +harmonic(abs(a)-
      (signum(a)+1)/2)/(a*a)-(signum(a)+1)/(2*a*a*a);
      print("s(",a,")=",evalf[10](s0a2));
      print("s(",a,t,"=",evalf[10](s0a2t));
      print("diff=",evalf[10](abs(s0a2t-s0a2)));
  end proc:

> for i from -10 to -1 do
      rp30aa(i,500000);
  end do;

> for i from 1 to 10 do
      rp30aa(i,500000);
  end do;
```

Table 1 The approximate values of the sums $s(0,a_2)$

a	-10	-9	-8	-7	-6
$s(0,a_2)$	0.0197730489	0.0232403856	0.0338958782	0.0338958782	0.0424646925
a	-5	-4	-3	-2	-1
$s(0,a_2)$	0.0550687422	0.0748775942	0.1090960516	0.1775329664	0.3550659331
a	1	2	3	4	5
$s(0,a_2)$	0.6449340668	1.4474670340	1.0946076520	0.8504696278	0.6890423689
a	6	7	8	9	10
$s(0,a_2)$	0.5768871594	0.4951306848	0.4331516578	0.3846660630	0.3457598625

Source: own modelling in Maple 16

The approximate values of the sums $s(0,a_2)$, for $a=-10,-9,\ldots,-1,1,2,\ldots 9,10$, obtained by means of the formula (4) are written in Table 1; values of the sums $s(0,a_2)$ are rounded to 10 decimals.

Computation of 20 pairs of the sums $s(0,a_2)$ and $s(0,a_2,500000)$ took 10 hours and 35 minutes. The absolute errors, i.e. the differences $|s(0,a_2)-s(0,a_2,500000)|$, are all only between $2\cdot 10^{-10}$ and 10^{-9}.

CONCLUSIONS

We dealt with the sum of the series of reciprocals of the cubic polynomials with one zero and double non-zero integer root a, i.e. with the series

$$\sum_{\substack{k=1\\k\neq a}}^{\infty}\frac{1}{k(k-a)^2}.$$

We derived that the sum of this series is given by formula

$$s(0,a_2)=\frac{\pi^2}{6a}+\frac{H(|a|-\sigma(a),2)}{|a|}+\frac{H(|a|-\sigma(a))}{a^2}-\frac{\sigma(a)}{a^3},$$

where $\sigma(a)=[\mathrm{sgn}(a)+1]/2$, $H(n)$ is the harmonic number and $H(n,2)$ is the generalized harmonic number. We verified this main result by computing 20 sums by using the computer algebra system Maple 16. The series above so belong to special types of the series, such as geometric and telescoping ones, which sums are given analytically by means of a formula.

ACKNOWLEDGEMENTS

The work presented in this paper has been supported by the project "Rozvoj oblastí základního a aplikovaného výzkumu dlouhodobě rozvíjených na katedrách teoretického a aplikovaného základu FVT (K215, K217)".

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{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2019(5), 1, 16-29

Received: 2019-08-30
Accepted: 2019-09-30
Online published: 2019-10-15
DOI: https://doi.org/10.15414/meraa.2019.05.01.16-29

Original Paper

Growing role of CSR activities in the area of human resources management and its evaluation using quantitative methods

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ABSTRACT

The long-term interest of an enterprise should be to keep employees as long as possible, while at the same time they should deliver sustainable added value. The main goal of human resources management is to achieve success and competitive advantage through the strategic provision and deployment of capable and committed employees. The aim of the scientific paper is to point out the role of CSR activities in human resources management. We analyzed the topic in 331 food companies in Slovakia. The tool for analyze was the questionnaire based on the 5-degree Likert scale and controlled interviews. Data processing was performed using statistical program SAS Enterprise Guide 7.1. The statistical relations and correlations between variables were performed using Cronbach alpha, Spearman's test, Kruskal-Wallis´ test. Based on the results of our research and the verification of six established research presumptions, we can conclude, that we see important to ensure the balance between the working and private lives of all employees, not only managers, as well as provide the suitable space for their mental hygiene. The companies should deal with different opportunities within corporate philanthropy, which are closely related to the help and protection of various disadvantaged employees. It is also necessary to protect the intellectual property of employees and respect their rights and freedoms and systematically combat bureaucracy and corruption.

KEYWORDS: human resources management, corporate social responsibility, employee, food companies

JEL CLASSIFICATION: M12, M14

INTRODUCTION

The absolute factor in providing competitive business potential is the formation of personnel with certain quantitative, structural and qualitative characteristics and its effective use both in the current period and in the future. Therefore, the development of personnel is an important component of strategic enterprise management.

One of the fundamental approaches to studying Corporate Social Responsibility (CSR) and its relationship with the community is a functionalistic approach, which demonstrates the

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company's level of interconnection with the ethical, social, environmental and economic spheres [5].

However, it should be noted that most of the works created in the field of CSR are focused on meeting the needs of external stakeholders. Companies should be aware that employees are also an important stake, which has a vast influence on the enterprise [9].

In the conditions of development of the socially oriented economy, enterprises need to develop new approaches in the system of human resource management (HRM). The core of such innovations should be the concept of Corporate Social Responsibility since on its basis it is possible to develop measures in the system of HRM that will be effective both from a social and economic point of view. Therefore, senior management of an enterprise has to develop and implement a socially responsible policy, which helps to satisfy all desires of each entity [14]. This approach involves studying the behaviour of the employee at the enterprise, identifying the laws in the field of labour and personality development, evolving and implementing socially responsible actions that can the best way to cater the needs of each employee.

The human resources management is not only an important bearer of social responsibility to employees, but also plays an irreplaceable role in building a business.

The integration of corporate social responsibility and human resource management represents a shift from the established focus of day-to-day human resource management to the business with an emphasis on the effective use of employee work to achieve business goals towards the long-term impact of personal work [2].

The interconnection of the personal and social responsibility of the company is as follows. The term social function of a company defines as the development of those activities that aim not only economic growth, but also the improvement of people's living and working environment in general and their personal and professional development for them themselves. This is reflected both in the external and internal environment of the company. The intersection of the external and internal dimensions is in the existence of strong links between the activities by which an enterprise seeks to valorise its human capital and the activities by which an enterprise develops in order to fulfill its social function. The reason for linking these two areas of business is not only the links that exist between them, but also the potential for enrichment and interaction [18].

In the context of corporate social responsibility and human resources management and interaction with each other, the business faces two tasks: performing personally and sustainably human resources (creating a mutually beneficial relationship between employees and business), and promoting corporate social responsibility and sustainability [2].

In such a modernized system of relations between management and hired workers, the role of Corporate Social Responsibility will be as follows [7]:

- CSR as a special approach to management provides an opportunity to rethink the list of priority goals and objectives;
- CSR allows developing a code of managerial behaviour principles, which the company can steadfastly use for making decisions of any level;
- CSR is a management system that functions through the use of tools that will coordinate the progress of social responsibility in the human resources management system and control its results.

It should be noted that enterprises of economically developed countries use Corporate Social Responsibility as a reliable tool aimed at increasing the efficiency of HRM, motivation and loyalty of employees, forming a positive image of the company and the brand on the market. The results of the international research prove that the company receives a lot of benefits (including financial) from the implementation of the CSR activities which are connected with the workers [14].

Porter and Miles believe that the connection between CSR and HRM are extremely promising, but the process of measuring the impact of CSR actions in relation to the company's efficiency and reputation is long-term and often difficult to fix [17]. Voegtlin and Greenwood [21] also argue that the impact of links between CSR and HRM practically impossible to inquire. The authors explain that there is a reciprocal link between these two concepts, that is, CSR can affect the practice of personnel management as well as the practice of human management can affect the company's actions in the context of CSR.

The essence of CSR principles in labour relations is the following [7]:

- observance of the legislation of country and international norms;
- avoidance of reduction of existing social rights and guarantees;
- workers' participation in decision-making concerning their interests;
- permanence - socially responsible behaviour is a constant feature of the social partners' activities;
- transparency - openness in relation to own activity, provision of reliable information in an accessible and full extent;
- accountability - the willingness to report on the impact of its activities;
- efficiency - achievement of the maximum result at minimum expenses;
- the principle of feedback - the current result should be compared with the goal set at the beginning, the discrepancy should be promptly detected and correct by managerial decisions;
- scientific feasibility - methods and tools of management should be scientifically substantiated and tested in practice.

Jamali, El Dirani, and Harwood [8] believe that CSR does not act as a traditional function of HRM, since HRM is concentrated on the organization's employees, and CSR can often be focused on other stakeholders. Today, human resource (HR) is often asked to play an increasingly important role in the development and implementation of CSR initiatives. If HR continues in this role, HR managers should strive to make CSR initiatives productive both for business performance and for HR practices [19].

Present-day confirms that modern companies are concentrated on informing employees about the company's actions in CSR. The growing role of HRM in CSR pushes enterprises to create events that can promote the value of Corporate Social Responsibility among workers. There is a practice of implementation CSR training as a promotion of skills for employees.

The challenges of creating a breakthrough in the concept of human resources management and social responsibility are the need to change human resource management based on the demands of this understanding, as well as the need to take over the role of coordinator for the implementation of the concept of social responsibility. Socially responsible human resources management must lead from a boom in the company's impact on the worker to a boom in the worker's impact on the business. Corporate social responsibility is the transformation of a company into an institution that takes very active responsibility for the impact of its own

activities on employees and their family members, thus contributing to their job satisfaction in a significant way. Satisfied employees are more responsible in the workplace towards suppliers, customers, towards the community and the environment where the business is developing [10].

The implementation of CSR principles depends on management's expectations. However, the CSR program is implemented by people employed in the company, specifically by its employees and human resources. Therefore, HRM, as one of the components of governance, should play an important role in the implementation of the CSR policy [12].

Oliynyk assures that activities connected with CSR of enterprises should pay attention to the training and qualification of staff. The main steps in this area are [15]:

- determining the training needs of employees in accordance with current and future plans, business development strategies, market conditions;
- the selection of methods, forms and training programs that are adequate to the needs;
- creation of equal opportunities for training and development of workers, stimulation of their education;
- retraining of their employees in case of reorganization and restructuring of the company in accordance with new production requirements and tasks;
- development and creation of conditions for the implementation of individual programs of professional and career development, especially valuable and prospective employees;
- encouraging employees to study in the workplace through the purchase of professional literature, assignment of creative, more complex and responsible tasks, involvement in decision-making processes that go beyond standard works;
- developing an effective system of a job promotion, ensuring equal opportunities for the increase of the position of all employees, depending on their competence, results of work, etc.

It should be noted that a sustainable effect (economic, organizational, social) on the use of Corporate Social Responsibility in the field of HRM is possible if the implementation of CSR principles is related to the daily economic activities of the organization and the constant interaction of social partners. Therefore, the actual scientific tasks are the development of conceptual foundations for the implementation of CSR in the area of employees' development, aimed at creating conditions for the harmonious development of the person.

At the same time, one of the most important elements of the conceptual framework is the choice of the forms of CSR implementation in the field of HRM. The development, approval and implementation of the internal socially responsible system should become part of the general management programs.

Kurinna and others are convinced that for the implementation of effective labour practices, it is necessary to continuously maintain a dialogue with employees and receive feedback. To do this, the company can use different tools [11]:

- to conduct questionnaires, interviews;
- to form committees (groups) of employees for solving various issues;
- organize meetings and sessions;
- create a hotline or corporate chat;
- create a mailbox for complaints and suggestions, etc.

Besides, authors assume that employees should be involved in the solving of important decisions for the organization, for example, the formulation of values, business development strategies, etc. Indeed, an effective system of the human resource management involves not only the engagement of employees in the adoption of strategic decisions but also the use of their creative and intellectual potential for the accumulation of innovative ideas.

Corporate Social Responsibility begins with its transformation into an institution that actively assumes responsibility for the impact of its activities on employees and their families, what greatly contributes to their job satisfaction. Satisfaction further leads to strengthening personal responsibility at the workplace, towards customers, suppliers, the environment, and the community where the company is developing its activities [3].

MATERIAL AND METHODS

The aim of the scientific paper is to point out the role of CSR activities in the area of human resources management and to evaluate it by using quantitative methods. Our task was to analyse (using questionnaire) the relationship of social area of CSR and human resources in analysed food companies within Slovakia.

Our research was realized from May to September 2018. It was based on a sample of 331 respondents, which holds a different position in the asked companies (such as owner, manager, the employee of the economic, logistics, production or sales department and so on). The tool for analyse was the questionnaire based on the 5-degree Likert scale, where the respondents expressed the degree of their agreement or the disagreement with the particular statement. To complete information we realized also controlled interview through face to face meetings with selected respondents from food companies from Slovakia.

Data processing was performed using statistical program SAS Enterprise Guide 7.1. The statistical relations and correlations between variables were performed using Cronbach alpha, Spearman's test, Kruskal-Walis´ test. In general like in each research, the reliability and accuracy of measurement are important. In conducted survey, we used the Cronbach coefficient of alpha as part of the statistical evaluation, where: the higher the homogeneity of the elements, the higher the reliability. We applied mathematical and statistical methods such as non-parametric Kruskal-Wallis test and correlation analysis using Spearman test for statistical hypothesis testing.

The Spearman correlation coefficient is a non-parametric measure of association. The values of this coefficient are from 1 to –1. Values approaching 1 represent strong positive correlation. Values approaching 0 represent none correlation and values approaching –1 represent strong negative correlation. The Spearman correlation coefficient provides better estimate than the Pearson correlation coefficient [1].

The most widely used method for assessing the reliability of scales is Cronbach Coefficient Alpha. It represents the degree of internal consistency of the scale. The values of Cronbach Coefficient Alpha 0.7 and higher mean sufficient internal consistency of the scale [1].

We have tested first three research presumptions using nonparametric one-way ANOVA by Kruskal-Wallis test. The last three research presumptions where testing for relationship using Spearman correlation coefficient.

Interpretation of Spearman and Cronbach correlation coefficient values was based on the following table 1 [4].

Table 1 Interpretation of correlation coefficient values

Correlation value	Dependence interpretation
0.01 – 0.09	Trivial or none
0.10 – 0.29	Low to medium
0.30 – 0.49	Medium to essential
0.50 – 0.69	Essential to very strong
0.70 – 0.89	Very strong
0.90 – 0.99	Almost perfect

Source: De Vaus, 2002

We determined and verified in our research following 6 research presumptions (hypotheses):

H1: We assume that there is statistically significant difference between the respondent's position in the company and providing space for mental hygiene (for example rest, nutrition or regeneration area)

H2: We assume a positive correlation between corporate philanthropy and protection of specific groups of employees (as disabled…).

H3: We expect a positive correlation between protection of an intellectual property and respect for human rights and freedoms.

RESULTS AND DISCUSSION

Slovak Republic has faced significant changes in the social, political and environmental spheres during the last decades. The concept of CSR for this region is comparatively new. However, it should be noted that the practice of using CSR develops rather rapidly within the framework of integration of this region with the European Union, as well as under the effect of the increase of foreign investors and the development of international companies. Therefore, the assessment of CSR state in this region is becoming more interesting not only for the scientists but also for the companies themselves. In our opinion, an important part of the analysis conducted in the study was the assessment of the level of CSR in human resource management, as labour is considered as one of the most valuable resources of each enterprise.

The socio-political changes in these countries, as well as the transformation of their economies from centrally planned to market form, could have caused entrepreneurs' lack of interest in the requirements of workers, as companies could think that needs of other stakeholders are more important to satisfy. Thus, the purpose of this study is to determine the level of CSR quality measures in the relationship between the firm and employees.

The first research presumption was formulated in the part "Methods" and based on it we set these zero and alternative hypothesis:

H0: There is no statistically significant difference between the respondent's position in the company and providing space for mental hygiene (for example rest, nutrition or regeneration area).

H1: There is statistically significant difference between the respondent's position in the company and providing space for mental hygiene (for example rest, nutrition or regeneration area)

The hypotheses were tested using nonparametric one-way ANOVA by Kruskal-Wallis test. The test results are shown in the Table 2.

Table 2 Kruskal-Wallis test for the third research presumption

Wilcoxon Scores (Rank Sums) for Variable providing space for mental hygie Classified by Variable Your position in the firm					
Your position in the firm	**N**	**Sum of Scores**	**Expected Under H0**	**Std Dev Under H0**	**Mean Score**
owner	18	1287.00	2988.0	350.120734	71.500000
manager	36	6678.00	5976.0	480.697283	185.500000
the employee of the economic department	27	6034.50	4482.0	422.598629	223.500000
the employee of the logistics department	27	6925.50	4482.0	422.598629	256.500000
the employee of the production department	198	30865.50	32868.0	756.951101	155.886364
the employee of the sales department	25	3155.50	4150.0	407.981177	126.220000
Average scores were used for ties.					

Kruskal-Wallis Test	
Chi-Square	75.6263
DF	5
Pr > Chi-Square	<.0001

Source: own research

As we can see in the Table 6, the obtained p-value is lower than 0.0001. If we compare it with determined Alfa = 0.05, we see that p-value is lower. At the same time p-value is lower also than Alfa = 0.01. It means that we reject the zero hypothesis and accept the alternative hypothesis at the level of significance of 99%. In accordance of Kruskal-Wallis test we can say, that there is highly statistically significant difference between the respondent's position in the company and providing space for mental hygiene (for example rest, nutrition or regeneration area) at the level of significance of 99%.

Afterwards we have continued with further testing and we have realized multiple range test. We have decided to realize Fisher´s least significant difference (LSD) test, which results are shown in Table 3 below:

Table 3 Fisher´s least significant difference test for the third research presumption

Comparisons significant at the 0.05 level are indicated by *.**			
Your position in the firm Comparison	**Difference Between Means**	**95% Confidence Limits**	
the employee of the logistics department - the employee of the economic department	0.33333	0.03103	0.63564 ***
the employee of the logistics department - manager	0.58333	0.30056	0.86611 ***
the employee of the logistics department - the employee of the production department	0.78788	0.56001	1.01575 ***
the employee of the logistics department - the employee of the sales department	0.97333	0.66504	1.28162 ***

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ISSN 2453-6881
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Comparisons significant at the 0.05 level are indicated by ***.			
Your position in the firm Comparison	Difference Between Means	95% Confidence Limits	
the employee of the logistics department - owner	1.33333	0.995351.67132	***
the employee of the economic department - manager	0.25000	-0.032780.53278	
the employee of the economic department - the employee of the production department	0.45455	0.226680.68241	***
the employee of the economic department - the employee of the sales department	0.64000	0.331710.94829	***
the employee of the economic department - owner	1.00000	0.662021.33798	***
manager - the employee of the production department	0.20455	0.003300.40579	***
manager - the employee of the sales department	0.39000	0.100830.67917	***
manager - owner	0.75000	0.429361.07064	***
the employee of the production department - the employee of the sales department	0.18545	-0.050300.42121	
the employee of the production department - owner	0.54545	0.272010.81890	***
the employee of the sales department - owner	0.36000	0.016650.70335	***

Source: own research

The Fisher´s LSD test confirms realized Kruskal-Wallis test, so that there are highly statistically significant differences between the respondent´s position in the firm and providing space for mental hygiene. We have found out that there are significant differences between groups of respondents (according to the position in the company), which are marked in Table 3 with "***".

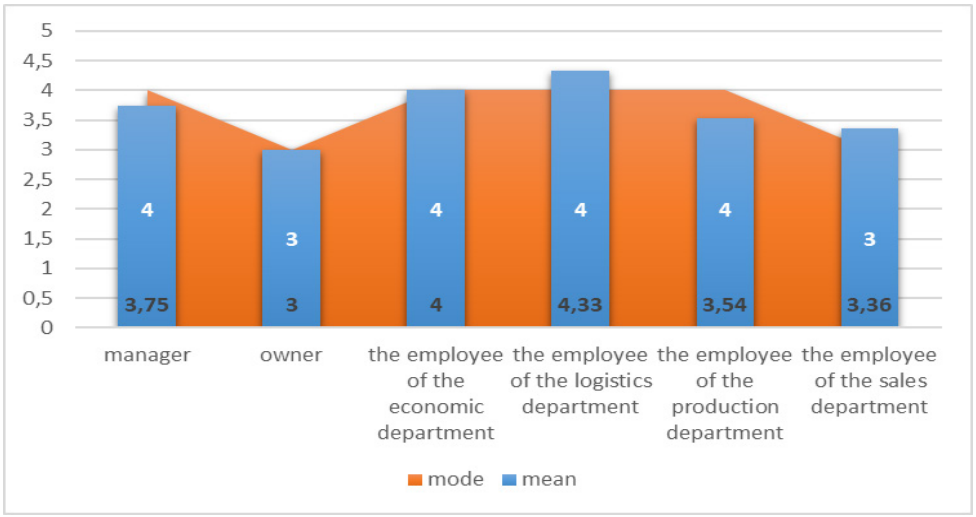

Figure 1 Characteristics of the location of the evaluation criteria providing space for mental hygiene (according to the position in the company). Source: own research

There is a great need to improve the understanding of the effects of the built environment on the incidence, severity and persistence of various diseases, as well as factors that promote mental health and human performance. Since we spend most of our time in homes, schools, offices, and other areas, it is important to investigate the consequences of mental health related to how we design, operate and maintain buildings [6].

The last two research presumptions deal with correlations. As the first we have determined Cronbach coefficient alpha to evaluate reliability of the realized research. As we can see in the Table 4, the value of this Cronbach Coefficient Alpha is 0.892629. It represents very high file reliability based on interpretations of correlation according to De Vaus [4].

Table 4 Cronbach Coefficient Alpha

Cronbach Coefficient Alpha	
Variables	**Alpha**
Raw	0.881915
Standardized	0.892629

Source: own research

In the following tables 5 and 6 there are shown all values of Spearman correlation coefficient for relationship of all questions between each other. Values marked with "*" mean, that they are statistically significant at the level of significance of 95% (Alfa = 0.05). Values marked with "**" mean, that they are highly statistically significant at the level of significance 99% (Alfa = 0.01). Values marked with "pale green" mean, that they represent essential to very strong correlation and values marked with "dark green" mean, that they represent very strong correlation between two variables based on interpretations of correlation according to De Vaus (2002) [4].

The second research presumption, which was formulated in the part "Methods", was about positive correlation between corporate philanthropy and protection of specific groups of employees. On what bases we set a zero and alternative hypothesis as following:

H0: There is no positive correlation between corporate philanthropy and protection of specific groups of employees (as disabled…) (files are not dependent).

H1: There is positive correlation between corporate philanthropy and protection of specific groups of employees (as disabled…) (files are dependent).

Also for testing the fifth hypothesis we used the Spearman correlation coefficient to evaluate the correlation relationship between statements about corporate philanthropy and protection of specific groups of employees (as disabled…).

In the table 5 we can see, that Spearman correlation coefficient for this relationship has value 0.54101. This value represents essential to very strong correlation between corporate philanthropy and protection of specific groups of employees (as disabled…) at the level of significance of 99% (Alfa = 0.01).

The obtained p-value in this case is also lower than 0.0001. This p-value is lower than the determined value Alfa = 0.05. So that means, that in this case we also reject the zero hypothesis and we accept the alternative hypothesis. At the same time the obtained p-value is also lower than Alfa = 0.01. We have confirmed that there is essential to very strong positive correlation between corporate philanthropy and protection of specific groups of employees (as disabled…) at the level of significance of 99%, so these files are dependent. Due to this positive correlation, we can claim, that the more the company deals with philanthropy the more it tries to protect in its organization specific groups of employees for example disabled.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2019(5), 1, 16-29

Table 5 Spearman Correlation Coefficients, correlations of Q1 – Q9 with Q10 – Q18

Spearman Correlation Coefficients, N = 331									
	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**
Q10	0.19195**	0.19334**	0.54928**	0.56997**	0.32883**	0.28844**	0.29235**	0.13073*	0.78238**
Q11	-0.12036*	0.38941**	0.54855**	0.52268**	0.20603*	0.25446*	0.17242**	-0.15612**	0.53917**
Q12	0.21077**	0.48557**	0.63067**	0.53943**	0.41209**	0.52184**	0.49213**	0.15095**	0.49084**
Q13	0.37989**	0.38402**	0.37859**	0.42896**	0.54101**	0.63305**	0.52888**	0.36061**	0.21730**
Q14	0.10778	0.07239	0.04672	0.18068**	0.17077**	0.16646**	0.37787**	0.24058**	0.09680
Q15	-0.30639**	0.26582**	0.32390**	0.16718**	-0.13230*	0.17082	-0.06307	-0.35418**	0.29047**
Q16	0.00297	0.48079**	0.56053**	0.46280**	0.33943**	0.42112**	0.11013*	-0.10741	0.47639**
Q17	0.14586**	0.09001	0.49642**	0.48015**	0.25838**	0.34318**	0.29024**	-0.06361	0.35930**
Q18	0.64127**	0.22210**	0.32488**	0.47737**	0.66869**	0.53181**	0.50763**	0.53403**	0.35239**

Source: own research

Q1 – our company is a socially responsible company
Q2 – awareness of employees about important matters relating to the company
Q3 – communication with the employees
Q4 – compliance with labour standards
Q5 – corporate philanthropy
Q6 – development of qualification, skills, and long-lasting career of its employees
Q7 – early preparation of the staff to changes
Q8 – employee loyalty to the company
Q9 – fighting against corruption and bribery
Q10 – gender mainstreaming and the minimization of discrimination
Q11 – helping redundant workers and their families
Q12 – protection of an intellectual property
Q13 – protection of specific groups of employees (as disabled…)
Q14 – providing employment benefits to the employees (as working from home…)
Q15 – providing space for mental hygiene (as rest, nutrition, regeneration area)
Q16 – respect for human rights and freedoms
Q17 – safety at work
Q18 – work-life balance of employees

Corporate philanthropy as the highest form of corporate social responsibility (CSR) is an essential means of enhancing reputation and strengthening relationships between organizations and their stakeholders [23]. There are many reasons why companies incorporate philanthropic activities into their strategy. For example, good relationships with the region where they exist, public relations (a good tool to create a corporate identity), marketing (causal marketing, raising company awareness), human resources (better internal communication, better employee loyalty), new employee experience - volunteering), tax benefits [20]. Managers are generally satisfied with the work done by disabled employees. Managers' satisfaction grows when these employees perform their duties correctly, work on schedule, teamwork integrates collective adherence to organizational values, contributes to achieving goals, and applies their knowledge in practice [13].

The last one - third research presumption from the part "Methods" was about positive correlation between protection of an intellectual property and respect for human rights and freedoms. Based on this, we set following zero and alternative hypothesis:

H0: There is no positive correlation between protection of an intellectual property and respect for human rights and freedoms (files are not dependent).

H1: There is positive correlation between protection of an intellectual property and respect for human rights and freedoms (files are dependent).

We used the Spearman correlation coefficient to evaluate the correlation between protection of an intellectual property and respect for human rights and freedoms.

In the Table 6 we can see, that Spearman correlation coefficient for this relationship has value 0.51654. This value represents essential to very strong correlation protection of an intellectual property and respect for human rights and freedoms at the level of significance of 99% (Alfa = 0.01).

The obtained p-value for this correlation is lower than 0.0001. It is lower than the determined value Alfa = 0.05. So it means, that we reject the zero hypothesis and we accept the alternative hypothesis. At the same time the p-value is lower than Alfa = 0.01. Therefore, we have confirmed that there is essential to very strong positive correlation between protection of an intellectual property and respect for human rights and freedoms at the level of significance of 99%. So files are dependent. Based on this result, we can say, that the more the company try to protect an intellectual property the more this company care about respect for human rights and freedoms.

Table 6 Spearman Correlation Coefficients, correlations of Q10 – Q18 with each other

Spearman Correlation Coefficients, N = 331									
	Q10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**	**Q16**	**Q17**	**Q18**
Q10	1.00000	0.53917**	0.47361**	0.21730**	0.09680	0.19289**	0.31976**	0.24710**	0.44926**
Q11	0.53917**	1.00000	0.38769**	0.22402**	-0.10709	0.35487**	0.49778**	0.28601**	0.08776
Q12	0.47361**	0.38769**	1.00000	0.49995**	0.25425**	0.46632**	0.51654**	0.40757**	0.31727**
Q13	0.21730**	0.22402**	0.49995**	1.00000	0.38309**	0.16015**	0.33053**	0.30285**	0.57174**
Q14	0.09680	-0.10709	0.25425**	0.38309**	1.00000	0.15636**	0.19410**	0.15203**	0.21699**
Q15	0.19289**	0.35487**	0.46632**	0.16015**	0.15636**	1.00000	0.48301**	0.06387	-0.17445**
Q16	0.31976**	0.49778**	0.51654**	0.33053**	0.19410**	0.48301**	1.00000	0.48385**	0.10511
Q17	0.24710**	0.28601**	0.40757**	0.30285**	0.15203**	0.06387	0.48385**	1.00000	0.08210
Q18	0.44926**	0.08776**	0.31727**	0.57174**	0.21699**	-0.17445**	0.10511	0.08210	1.00000

Source: own research

Q10 – gender mainstreaming and the minimization of discrimination
Q11 – helping redundant workers and their families
Q12 – protection of an intellectual property
Q13 – protection of specific groups of employees (as disabled…)
Q14 – providing employment benefits to the employees (as working from home…)
Q15 – providing space for mental hygiene (as rest, nutrition, regeneration area)
Q16 – respect for human rights and freedoms
Q17 – safety at work
Q18 – work-life balance of employees

Human rights are defined as inalienable fundamental rights to which a person is naturally entitled simply because he is a human being; they cover political, civil and socio-economic and cultural rights as defined in the UN Universal Declaration of Human Rights and, more generally, the International Human Rights Act. Human rights are most often discussed in connection with abuse by criminal or violent parties. Their link to legitimate business activities has hardly been explored, in particular in the context of the management and research of international companies. This is despite growing evidence of the participation of the business sector in human rights disputes, including child labour, trafficking in human beings, engaging in rogue regimes and violations of the right to life and health due to environmental degradation [22].

Corporate social responsibility for respect for human rights means that companies must avoid, mitigate and remedy any negative impact on human rights caused or related to their activities or the activities of their business partners. The challenge for multinationals is how they can achieve respect for human rights in practice [16].

CONCLUSIONS

It should be noted that a sustainable effect (economic, organizational, social) on the use of Corporate Social Responsibility in the field of HRM is possible if the implementation of CSR principles is related to the daily economic activities of the organization and the constant interaction of social partners. Present-day shows, modern companies are concentrated on informing employees about the company's actions in CSR. The growing role of HRM in CSR pushes enterprises to create events that can promote the value of Corporate Social Responsibility among workers. There is a practice of implementation CSR training as a promotion of skills for employees.

The company's success is based on employees who can increase productivity, care for customers and bring innovative ideas. The claim that people are our most important capital can not only be taken as an exhausted phrase but as an effective tool for achieving competitiveness.

Based on the results of our research and the verification of six established research presumptions, our conclusions are:

- There are highly statistically significant differences between the respondent´s position in the firm and the opportunity to use the space reserved for mental hygiene; the employees of the economic and logistic department have responded on average value 4 and 4.33, which represents attitude “I rather agree”. In average, the managers, owners, employees of the production and sales department have mean values 3.75, 3, 3.54 and 3.36, which represent indecisive answer;
- There is essential to very strong positive correlation between corporate philanthropy and protection of specific groups of employees; the more the company deals with philanthropy the more it tries to protect specific groups of employees;
- There is essential to very strong positive correlation between protection of an intellectual property and respect of human rights and freedoms; the more the companies try to protect an intellectual property the more these companies respect human rights and freedoms of the employees.

Based on our research and its results, our recommendations for practice of companies in Slovakia are as follows:

- To ensure (by the available means) the balance between the working and private lives of all employees, not only managers, as well as provide the suitable space for their mental hygiene; the good idea should be to communicate with employees very often;
- Companies should deal with different opportunities within corporate philanthropy, which are closely related to the help and protection of various disadvantaged employees; this is donation of companies that seek to solve the problems of society by supporting education, innovation and skills development to enable them to help themselves;
- Ensure that companies unconditionally protect the intellectual property of their employees and respect their rights and freedoms and systematically combat bureaucracy and corruption.

Compliance and strengthening the above, companies can significantly contribute to the development of socially responsible business activities, ensure a sufficient quality and loyal employees and targeted towards sustainable development.

ACKNOWLEDGEMENTS

This paper was created within the KEGA project *Theory and practice of human resources management and managerial work.* Project´s registration number: 041SPU-4/2018.

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{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2019(5), 1, 30-37

Received: 2019-08-26
Accepted: 2019-10-30
Online published: 2019-10-30
DOI: https://doi.org/10.15414/meraa.2019.05.01.30-37

Original Paper

Differentiating under integral sign in Castigliano's theorem

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ABSTRACT

In this contribution we are dealing with application of differentiating of function defined with determined parametrical integral. The mathematical problem was analyzed form point of view of mechanics of materials. The mathematical model of loaded beam was created. Applying the cross section method we defined the exact function of bending moment. Additional properties of cantilever joint were neglected. We showed the derivation of modified Castigliano's theorem via Leibnitz rule of differentiating under integration sign. Appling the modified Castigliano's theorem we got the exact solution of the deflection of the beam. The exact solution of beam deflection was finished in PTC Mathcad Prime software (PTC - Parametric Technology). The numerical integration of the bending moment dataset and defined deflection function was done in program written in Microsoft Visual C# 2010. The Dormand-Prince numerical integration method was used for the numerical integration. Comparing the exact and numerical solution we got the error of numerical integration solution.

KEYWORDS: differentiating methods, beam deflection, parametric integral, numerical integration

JEL CLASSIFICATION: C02

INTRODUCTION

The application of modified Castigliano's theorem is the basic knowledge for the structural and civil engineering. It's have been published a many articles which deals with the application this method. But, unfortunately the some major parts are still missing in the published articles. The goals of this paper are focused on the summarization of the way how it was created and how does it work the modified Castigliano's theorem in the application in mechanical engineering. The application of generalized form of Castigliano's theorem was published by [11]. Of course, the method were replaced by the finite element method (FEM) applied in the continuum mechanic of elastic bodies. But many technical applications are the technical functions which are defined by the integral. These functions must be differentiated manually or numerically. The mathematic procedure including the differentiating the function defined by integral was analyzed by [12]. The method of differentiating the

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functions under integral sign was analyzed by [1, 5, 8, 14]. Many mathematical examples of differentiating under integral sign were published by [13] Explaining the visualization of Leibnitz rule was done by [6]. The deflection of machines parts applying the Castigliano's theorem was realized by [7, 17].

MATERIAL AND METHODS

The least energy method

The energetic method of determination of beam deflection y is defined in [3]. The main idea of the method is the determination of the difference dA of the strain energy function A of more variables where $A = f(F_1,..,F_i,..,F_n;M_1,...,M_i,..,M_n)$ by using the total derivate in the differentiating form as follows:

$$dA = \frac{\partial A}{\partial F_1}dF_1 + ... + \frac{\partial A}{\partial F_i}dF_i + ... + \frac{\partial A}{\partial F_n}dF_n + ... + \frac{\partial A}{\partial M_1}dM_1 + ... + \frac{\partial A}{\partial M_i}dM_i + ... + \frac{\partial A}{\partial M_n}dM_n \quad (1)$$

where $1 \le n \le +\infty$. The energy in deformable body is separated to two parts as described in [3]. The significant and not neglected part of strain energy has the form $dA_i = dF_i.y_i$.

Selection of $i-th$ member from equation (1) and compare them we got $dF_i.y_i = \frac{\partial A}{\partial F_i}dF_i$ and finally we got the equation for solving the deflection $y_i = \frac{\partial A}{\partial F_i}$. The general form of this equation is follows:

$$y = \frac{\partial A}{\partial F}. \quad (2)$$

Differentiating under integral sign

Let is defined the function of two variables by definite integral with boundaries $a(x), b(x)$ of the continuous function where as defined in [9]:

$$I(x,t) = \int_{a(x)}^{b(x)} f(x,t). \quad (3)$$

Differentiating function $I(x,t)$ with using the limits we got:

$$\frac{\partial I(x,t)}{\partial t} = \lim_{\Delta t \to 0} \frac{I(x,t+\Delta t) - I(x,t)}{\Delta t}.$$

After the some equations arrangements published in [2] we got the final form:

$$\frac{\partial I(x,t)}{\partial t} = f[b(x),t].\frac{d}{dt}b(x) - f[a(x),t].\frac{d}{dt}a(x) + \int_{a(x)}^{b(x)} \frac{\partial f(x,t)}{\partial t}dx. \quad (4)$$

Mathematical model

We were self-created the model of isotropic beam loaded with continuous triangular loading q_2. The self-mass of beam is modelled with uniform loading q_1. The beam section profile is square and its dimensions are in table 1.The model is depicted on the Figure 1. The model properties are in Table 1.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2019(5), 1, 30-37

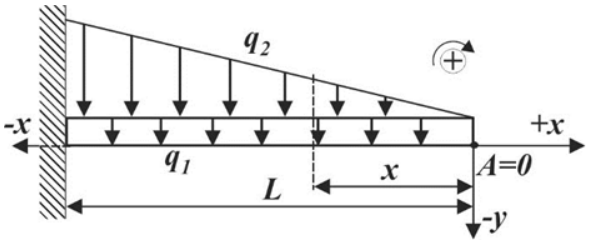

Figure 1 Model of loaded beam

Table1 Beam properties

Parameter	Value	Unit
Beam length L	1	m
Beam profile $a \times a \times t$	0,05x0,05x0,003	m
Beam mass m_B	4,16	kg
Uniform loading q_1	40,81	$N.m^{-1}$
Uniform loading q_2	4081	$N.m^{-1}$
Moment of inertia J_z	$2{,}08492.10^{-7}$	m^4
Modulus of elasticity E	$2{,}1.10^{11}$	Pa

For any position x of the cross-section the bending moment has the form in equation (5):

$$M_x = -\left(\frac{q_1.x^2}{2} + \frac{q_2.x^3}{6.L}\right) \tag{5}$$

The bending moment of the loaded beam is depicted in the Figure 2. The cantilever joint position of the beam is located in the $L = 1m$ dimension of the beam length (opposite coordinate system).

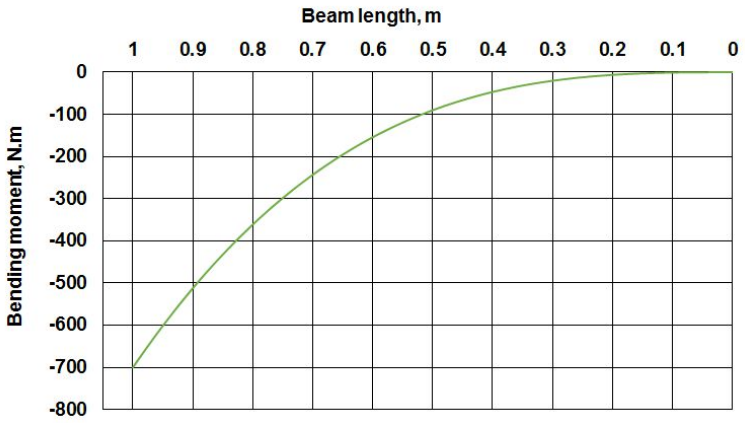

Figure 2 Bending moment function visualization

The exact solution of beam deflection on the end of beam (point A) has the form in equation (6).

$$y_A = -\left(\frac{q_1.L^4}{8.E.J_z} + \frac{q_2.L^4}{30.E.J_z}\right) \tag{6}$$

For modelling the deflection of the beam we rewrite the equation (6) to the form in equation (7).

$$y_{(i)} = -\left(\frac{q_1}{8.E.J_z} + \frac{q_2}{30.E.J_z}\right).x_{(i)}^4 \tag{7}$$

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Numerical solution of deflection

Dormand – Prince method is addressed by its authors [4]. They presented this method in the form of Butcher table, where are described coefficients of particular terms in the equations. The equations for Dormand – Prince method are as follows:

$$k_1 = h_i f\left(x_i, y_i\right)\ ,\ k_2 = h_i f\left(x_i + \frac{1}{5}h_i, y_i + \frac{1}{5}k_1\right),$$

$$k_3 = h_i f\left(x_i + \frac{3}{10}h_i, y_i + \frac{3}{40}k_1 + \frac{9}{40}k_2\right),$$

$$k_4 = h_i f\left(x_i + \frac{4}{5}h_i, y_i + \frac{44}{45}k_1 - \frac{56}{15}k_2 + \frac{32}{9}k_3\right),$$

$$k_5 = h_i f\left(x_i + \frac{8}{9}h_i, y_i + \frac{19372}{6561}k_1 - \frac{25360}{2187}k_2 + \frac{64448}{6561}k_3 - \frac{212}{729}k_4\right) \tag{8}$$

$$k_6 = h_i f\left(x_i + h_i, y_i + \frac{9017}{3168}k_1 - \frac{355}{33}k_2 - \frac{46732}{5247}k_3 + \frac{49}{176}k_4 - \frac{5103}{18656}k_5\right)$$

$$k_7 = h_i f\left(x_i + h_i, y_i + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6\right)$$

$$y_{i+1} = y_i + \frac{35}{284}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{9784}k_5 + \frac{11}{84}k6\,.$$

The numerical integration algorithm was created in the Microsoft Visual C# 2010 language. The algorithms of solving the differential equations with families of Runga-Kutta methods of higher degrees were published by [15].

RESULTS AND DISCUSSION

We define the strain energy in deformable body under the bending loading and it has the next form: $A = \frac{1}{2E.J}\int_0^L M_x^2 dx\,.$ (9)

Placing the equation (9) to equation (2) we got the form: $y = \frac{\partial}{\partial F}\left(\frac{1}{2E.J}\int_0^L M_x^2 dx\right)$. Applying the equation (4) and setting the boundaries for integral $a\left(x\right) = 0\,, b\left(x\right) = L$ we got:

$$\frac{\partial I\left(M_x, F\right)}{\partial F} = \frac{1}{2E.J_z}\left\{f[L,F].\frac{d}{dt}(L) - f[0,F].\frac{d}{dt}(0) + \int_0^L \frac{\partial f(M_x^2, F)}{\partial F}dx\right\}. \tag{10}$$

The next parts of equation (10) are zero: $f[L,F].\frac{d}{dt}(L) = 0\,,\ f[0,F].\frac{d}{dt}(0) = 0\,.$

On the next step we got for deflection:

$$y = \frac{1}{2E.J}\int_0^L \frac{\partial M_x^2}{\partial F}dx\,. \tag{11}$$

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
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The square of bending moment we should rewrite as follows: $y = \frac{1}{2E.J}\int_0^L \frac{\partial(M_x.M_x)}{\partial F}dx$.

Applying the rule for differentiating the two variables product in general form $(u.v)' = u'v + uv'$, we got: $\frac{\partial(M_x.M_x)}{\partial F} = \frac{\partial M_x}{\partial F}M_x + M_x\frac{\partial M_x}{\partial F} = 2\frac{\partial M_x}{\partial F}M_x$.

Returning to back the equation (11) we have the final form of modified Castigliano's theorem:

$$y = \frac{1}{E.J}\int_0^L \frac{\partial M_x}{\partial F}M_x dx. \tag{12}$$

If we want to define the function of bending moment for application the equation (12), we have to insert to the certain point (where we are looking for the value of deflection) the force F equal to zero. The function of bending moment for our model will be the next:

$$M_x = -\left(\frac{q_1.x^2}{2} + \frac{q_2.x^3}{6.L} + F.x\right) \tag{13}$$

Combining the equation (12) and (13) we got:

$$y = \frac{1}{E.J}\int_0^L \left\{\frac{\partial}{\partial F}\left[-\left(\frac{q_1.x^2}{2} + \frac{q_2.x^3}{6.L} + F.x\right)\right]\right\}.\left[-\left(\frac{q_1.x^2}{2} + \frac{q_2.x^3}{6.L} + F.x\right)\right]dx.$$

Result of the partial differential (in {} brackets) is the next: $(-x)$, and in the integral part (in the [] brackets) of the equation holds $F = 0$ and finally we got:

$$y = \frac{1}{E.J}\int_0^L x\left(\frac{q_1.x^2}{2} + \frac{q_2.x^3}{6.L}\right)dx. \tag{14}$$

Solving the integral (14) we got the equation (6). Rewriting the equation (6) for displaying the deflection we got the equation (7). Generating the deflection curve with step $\Delta x = 0.01m$ we got the curve depicted in the Figure 3.

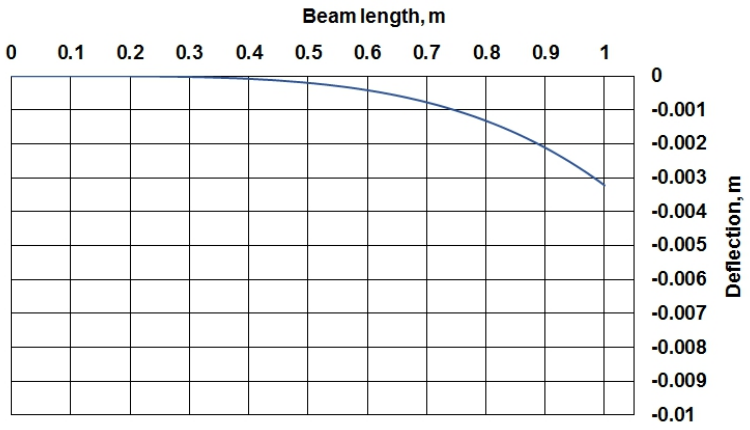

Figure 3 Deflection curve of the loaded beam form exact solution

With the exact solution of the deflection of the beam on the end of beam ($L = 1m$) we got the value $y_{ex} = -3,2234783322355.10^{-3}m$. The exact solution was realized in PTC Mathcad Prime software. For numerical solution we rewrite the equation (14) to the next form:

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$$y_{(i)} = \frac{1}{E.J}\int_0^L x_{(i)}.M_{x(i)}dx\,. \tag{15}$$

Applying the Dormand-Prince numerical method in equations (8) to solve integral equation (15), we got the data set of beam deflection curve depicted in the Figure 4. The numerical integration step was $h_i = 0{,}01$. The efficiency of the used numerical methods were declared by [10,16].

Figure 4 Deflection curve obtained by numerical integration

Finally we should compare the result of deflection obtained from exact solution with results obtained via numerical integrations. The difference (error) was solved from the equation (16).

$$E_{r(i)} = y_{ex(i)} - y_{num(i)}\ , \tag{16}$$

where: $y_{ex(i)}$ is the deflection dataset solved via exact solution, $y_{num(i)}$ is the deflection dataset solved via numerical integration, where $i\langle 0,n\rangle$. The error function is depicted in the Figure 5. The similar numerical problem was investigated and evaluated by [7].

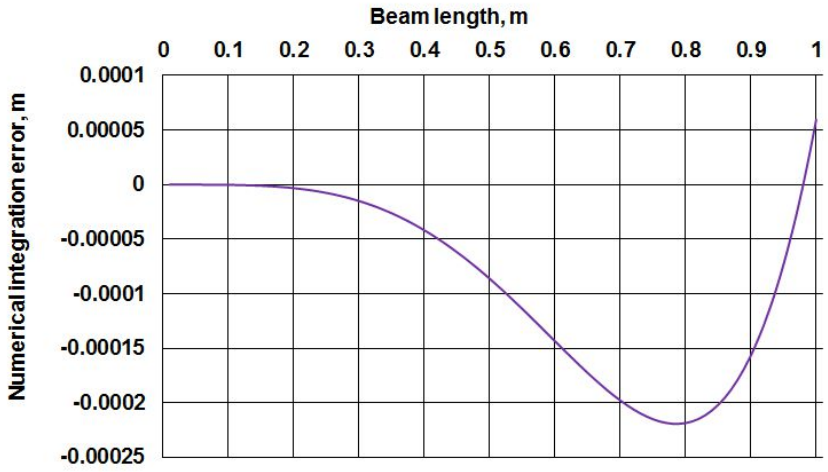

Figure 5 Error of numerical integration

The acceptable errors of numerical integration were discussed in [4].

CONCLUSIONS

In this paper we are dealing with the method of exact and numerical solving of the loaded beam deflection. For the exact solution was chosen the modified Castigliano's theorem. We showed the derivation of the modified Castigliano's theorem through the Leibnitz rule of differentiating under integral sign on defined example. We set up the mathematical model. For the exact solution we solved and visualized the function of bending moment and the beam deflection. For this purpose we used the PTC Mathcad Prime software. For the defined function of deflection we set up the algorithm of numerical integration in Microsoft Visual C# language. Used numerical method was Dormand-Prince method. The numerical integration step was chosen $h_i = 0,01$. From exact solution we got the beam deflection on the length $L = 1m$, $y_{ex} = -3,2234783322355.10^{-3} m$ and from the numerical integration $y_{num} = -3.15589088264663.10^{-3} m$. The error in the point $L = 1m$ is $E_r = 6,75874495887.10^{-5} m$.

From the realized analysis we should conclude that the presented method of differentiating under integral sign has a significant role in problems taught in mechanics of materials. Very pure explanation of the Leibnitz rule in the literature is now fixed. The presented methods are utilizable in simple engineering design process or in teaching process in mechanics of materials subject or applied mathematics. The applied numerical integration method has an acceptable accuracy $E_{r\%} = 6,75874495887.10^{-3}\%$. The percentage error was solved follows: $E_{r\%} = \left(|y_{ex}| - |y_{num}|\right) / 100$.

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{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2019(5), 1, 38-42

Received: 2019-09-30
Accepted: 2019-10-30
Online published: 2019-10-30
DOI: https://doi.org/10.15414/meraa.2019.05.01.38-42

Original Paper

Decision-making of students for their professional career in financial and insurance practice

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ABSTRACT

The Faculty of Economics and Management of the Slovak University of Agriculture in Nitra provides students within bachelor's study programs with general theoretical and practical knowledge. In the first year of bachelor study students learn the basics of higher mathematics within the subjects "Mathematics IA" and "Mathematics IB". Students acquire practical and applied knowledge in the optional subject "Financial and Insurance Mathematics" taught in the third year of bachelor's degree. The paper deals with the questionnaire survey carried out in the subject "Financial and insurance mathematics" taught at FEM SUA in Nitra. The aim of the statistical analysis is to find out whether students' attitudes towards their professional orientation in the field of financial and actuarial mathematics have changed. During the academic years 2016/17, 2017/18 and 2018/19 students were asked to fill out an identical questionnaire aimed at finding their general overview in the field of financial and actuarial mathematics, as well as opinions on their professional future in any of these areas. Methods of descriptive statistics and hypothesis testing were used to evaluate the survey results. The existence of statistically significant relationships among the obtained data was verified by the χ^2-test. In case of dependence confirmation the intensity of assessed dependence was determined. The questionnaire survey confirmed the existence of statistically significant differences between the answers of students from different types of secondary schools and academic years.

KEYWORDS: teaching of mathematics, financial and insurance mathematics, questionnaire survey, statistics

JEL CLASSIFICATION: I 21, C12

INTRODUCTION

According to Novikov and Novikova [6] throughout the early 21st century the system of education has been changing all over the world, this is caused by numerous social and political factors, situation in the labor market and technical progress. Among the reasons for

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such changes are the rise of the interest in the higher education and an increasing prestige of degrees.

Education through different stages constitutes the primordial basis for the sustainable development of countries. It also constitutes an effective tool to reduce economic and social gaps through access to a decent work [2].

The transversal competencies are an essential part of the professional and training profile of most degrees, and they have been defined as generic aspects related to knowledge, skills, and capabilities that any graduate must have achieved to improve his transition and integration in the professional life [1].

In the contemporary society the university education is the important factor for employment opportunities on the labor market [8]. According to Drábeková, Pechočiak and Matušek [3] the business leaders and educational organizations are calling for new education policies that target the development of broad, transferable skills and knowledge. Országhová [7] said that graduates of the 1st education level at the Faculty of economics and management are prepared for various economic areas of economy and agriculture. For the graduates of The Faculty of Economics and Management of the Slovak University of Agriculture in Nitra (FEM SUA in Nitra) knowledge obtained in particular study programs and language competences create wide opportunities of application at different levels of business management in agro-food resort, companies of biological and technical services, businesses of foreign trade as well as business departments of companies in agro resort and in financial institutions. Students are educated also for the requirements of institutions of public administration and self-administration, consultancy companies, research departments and education.

Financial mathematics provides appropriate applications of mathematics in the financial area and analogically the insurance mathematics provides applications of mathematics in the insurance practice. Knowledge of financial mathematics enables more effective and rational manner of its use in borrowing or investing of financial means. Knowledge and methods shall be applied within job decisions but as well as in private decisions on finance assessment. Insurance mathematics provides better orientation in insurance products, possibilities and types of insurance and insurance practice in general [4]. Methods of financial mathematics can be applied in a lot of economic branches. Accounting, financial planning and decision making is the part of many professional courses and specialized subjects [5]. According to Papcunová and Gecíková [10] the quality of human decision making is largely contingent on their qualifications and experience.

MATERIAL AND METHODS

Financial and Actuarial Mathematics is a compulsory subject for the Quantitative Methods in Economics and an optional subject for the Accounting and Business Economics study programs. The course is attended every year by several dozen bachelor students of FEM SAU in Nitra. During the school years 2016/17, 2017/18 and 2018/19 these students were given an identical questionnaire aimed at finding their general overview in the field of financial and actuarial mathematics, and also at opinions on their professional career in any of these areas. The following basic methods of descriptive statistics and hypotheses testing were utilized in the assessment of survey results. The existence of statistically significant relations between acquired assessments was verified by mean of χ^2–test. Statistically demonstrated differences

in the assessment were based on the significance of testing (p-value), presenting the error probability which is reached when the H_0 hypothesis is rejected even it is true. In case the p-value of testing characteristic is lower than 0.05 (5.00 E-02), a null hypothesis about the equality of observed features is rejected and the difference in values of a statistical feature is considered as statistically significant. In our case we dealt with the statistical samples of range *n* and analysed two statistical features – the first studied character (A) was the type of secondary school or the school year in which the survey was conducted and the second (B) was one of the options to answer each question: Yes / No / Don't know. The statistics χ^2 is used as a testing criterion and is presented by the following ratio [9].

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{r}\frac{(a_i b_j - (a_i b_j)_0)^2}{(a_i b_j)_0}$$

We tested the following null hypothesis H_0: There in no dependence between the observed features *A* and *B*. The alternate hypothesis H_1 as opposite: There is dependence between the observed features *A* and *B*.

RESULTS AND DISCUSSION

The questionnaire was filled out by 188 students. More than two-thirds (67.55%; 127 students) of these students were business or hotel academy graduates (Table 1). We can expect that these students will apply for jobs where knowledge of financial or actuarial mathematics is required. The structure of students from the perspective of the finished type of secondary school can be considered stable in case of grammar schools graduates and business or hotel academies graduates (Table 2). There is a statistically significant difference in case of another type of secondary school graduates when the proportion of such students has decreased.

Table 1 Structure of students according to the type of secondary school and form of study

Finished secondary school	School year			Total
	2016/17	2017/18	2018/19	
Grammar school	9.57%	9.04%	6.91%	25.53%
BA or HA	22.34%	21.28%	23.94%	67.55%
Other type of SS	2.13%	4.26%	0.53%	6.91%
Total	34.04%	34.57%	31.38%	100.00%

Source: authors' calculations

The survey also examined if students register an overlap in this subject with other subjects.

The questions, although in the whole survey formulated separately for financial and actuarial area, were answered by respondents in the same way. There were statistically significant differences between the responses in compared years (p-value 3.37 E-05 for "insurance mathematics" and 1.11 E-06 for "financial mathematics"). In school year 2016/17 positive answers prevailed, signaling the overlap of the subject with other subjects. After modification of the subjects' content due to the re-accreditation of study programs, the students' answers in

subsequent years changed, and the answers "*No*" prevailed in subsequent periods (p-value 0.11 for "insurance mathematics" and 0.95 for "financial mathematics").

Table 2 Results of mutual interactions determined in the survey

		Acquired assessment
		p-value
Have you experienced financial math tasks in practical life?	SS	6.83 E-01
	year	9.24 E-01
Have you experienced actuarial math tasks in practical life?	SS	3.18 E-01
	year	5.03 E-01
Have you applied anything from financial mathematics in practical life?	SS	6.20 E-01
	year	1.28 E-01
Have you applied anything from actuarial mathematics in practical life?	SS	7.37 E-01
	year	***0.47 E-03*****
Would you like to work in financial practice after graduation?	SS	1.27 E-01
	year	5.23 E-01
Would you like to work in insurance practice after graduation?	SS	1.13 E-01
	year	4.38 E-01

Source: authors' calculations

Table 2 shows the p-values to verify the existence of differences in respondents' answers to questionnaire questions divided by the type of secondary school (SS) and the year in which the survey was conducted. All p-values greater than 5.00 E-02 mean that there are no statistically significant differences in students' answers - graduates of different types of secondary schools. The structure of students' answers was the same in all compared time periods.

Statistically proven difference was found only in the answers to the question: "*Have you applied any actuarial math experience in practical life?*" In the analyzed time period, on average 50% of respondents chose a negative answer to this question. Less than 10% of students chose the answer "*I don't know*", except in the second investigated period, when this type of answer was more than a quarter (27.69%). This can be considered as the reason for the difference in the structure of answers.

CONCLUSIONS

The results of a repeated survey showed that Financial and Actuarial Mathematics overlapped with other courses, particularly in the academic year 2016/17. Based on these findings and also in connection with the re-accreditation of study programs, in the academic year 2017/18 an adjustment was made in the subject's curriculum, which also according to students' opinions, remedied these shortcomings in teaching. Transformation of the Basics of Actuarial Mathematics subject to the Financial and Actuarial Mathematics subject resulted in changes in the subject syllabus content, which reduced the overlap of the subject with other subjects taught within the bachelor study program at FEM SAU in Nitra.

Another important finding is that students' opinions on their professional implementation in the above mentioned areas do not change and a rather undecided or even negative attitude towards employment in areas using financial and actuarial mathematics prevails.

ACKNOWLEDGEMENTS

This contribution was created with the support of project KEGA 029SPU-4/2018 Digital educational applications in mathematics.

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{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2019(5), 1, 43-52

Received: 2019-02-19
Accepted: 2019-10-30
Online published: 2019-10-30
DOI: https://doi.org/10.15414/meraa.2019.05.01.43-52

Original Paper

Economic aspects of consumers' preferences when buying milk and selected dairy products

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ABSTRACT

The Slovak dairy industry brings to the market a wide range of standard dairy products as well as dairy specialties. Consumers' opinions and preferences need to be known to the manufacturers of these products, because through this information they can better understand and meet their demands and requirements. The main objective of this paper is to evaluate selected economic aspects of consumers' preferences in the purchase of milk and certain dairy products. Through the questionnaire survey we obtained opinions of consumers and the existence of relationships between selected economic indicators was verified by method of the χ2-test. This test did not confirm the statistical significance of the analyzed relationship in any formulated hypothesis. As the important fact for producers of milk and dairy products we could state that participants of the questionnaire predominantly prefer quality of a product to its price.

KEYWORDS: milk, dairy products, consumer preferences

JEL CLASSIFICATION: D40, C12, M31

INTRODUCTION

In general, dairy products are well regarded for their nutritional value. Consumer perception of dairy products is influenced by many interrelated factors but healthiness remains one of the key attributes and values for consumers. Furthermore, contemporary consumers increasingly seek out dairy products with additional health benefits and, therefore, it is essential to explore which attributes are important drivers of food choices and how producers can better respond to shifting consumer values and needs in each dairy product category [6].

World milk production has been growing by approximately 3% per year in recent years. Meanwhile, the demand for milk and dairy products has been growing at an average rate of 4% per year. As a result of the increase in population and per capita consumption, the demand for milk and dairy products worldwide is expected to continue rising. Slow growth in milk

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production has been attributed to economic hardships, frequent droughts, extreme temperatures, and climate change [5].

The transformation of the Agrifood industry of developing countries in the 1980s resulted in the formalization of a greater percentage of milk and milk product suppliers. Continuous increase in the demand of milk and milk products, coupled with slow increase in milk production has, however, resulted in vast changes in the dairy industry. The emergence and growth of the informal dairy sector is being witnessed in most developing countries. Small scale businesses, middlemen and milk vendors are taking advantage of the market gap to produce and process dairy products such as fresh milk, yoghurts, cheese, and buttermilk through the informal milk marketing channels [5].

Consumption of fluid milk has steadily declined over the last few decades. Understanding the attributes of fluid milk products that are attractive to specific consumer groups may provide a sound basis for education and marketing to encourage increased dairy consumption and reverse the downward trend. Fluid milk is a product that is often treated as a dietary staple. However, fluid milk sales have decreased significantly in recent years, with a 2.6% decline observed in 2016. Several reasons for the decline have been proposed, such as the growing popularity of milk alternatives, flavor concerns, and shelf-life concerns. Although sales within the fluid milk category are generally in decline, sales of organic milk continue to climb. Dairy is an important part of this movement and is currently the second most purchased organic food category, behind only fruits and vegetables. Relatively few studies have sought to identify and profile the typical organic dairy user. The preference for organic designation in fluid milk is typically linked to increased willingness to pay a premium price. Fluid milk label claims such as pasture-raised and recombinant bovine somatotropin -free designations have been similarly associated with greater willingness to pay by certain consumer groups. With evidence of consumer interest in mind, further investigation and appropriate valuation of different features in commercially offered fluid milk products are essential for milk producers in the modern market [1].

The food industry has a long tradition in the Slovak Republic. It is based on high-quality production, which is guaranteed by strict legislation and belongs to important industries. Very high competition after the accession of the Slovak Republic to the EU, imbalances in agricultural and food aid, financially insufficient investments in innovation and the modernization of the food sector weakened its position within the national economy and agro-food complex. The overall competitiveness of the food industry has been jeopardized in particular by the weak financial flows to the sector, preparation for accession to the European Union, market opening, the emergence of trade chains, uneven prices in the sector and lack of marketing and trade experience. The dairy industry is one of the leading food industry industries. It accounts for more than 18% of the total production and sales of the food industry. The production and sales of the dairy industry are relatively stable in the long term, with the greatest impact on the economy of this sector being the development of raw cow's milk prices on the world and European markets [4].

The supply (S) always expresses the relationship between the quantity (Q) of production and the price (P) for which are seller willing to sell. Factors that affect supply:

1. The cost of inputs (production factors) - for the enterprise to produce, it needs a variety of inputs (raw materials, materials, appliances, employee work). If the cost of one or more of

these components increases, production will be less profitable and the company will offer fewer products. If input prices increase substantially, an enterprise may cease to produce;
2. Technology - is an important determinant of the quantity offered. Using newer technology increases productivity. Reducing business costs contributes to increasing the amount of production offered;
3. The price of the goods - when the price of goods is high, the sale of goods also increases, because the company offers a large quantity. Otherwise, when the price of the goods is low, the business becomes less profitable and therefore produces less. At an even lower price, an enterprise may decide to end production and its offered quantity will drop to zero. Given that the quantity offered rises and falls together with the price, there is direct dependence between the farm's price and the quantity offered. This relationship is called bid law. If the price of the farm rises, assuming that other factors are constant, the quantity offered will increase [3].

Demand (D) represents the sum of the expected purchase volume and is also determined by the amount (but in this case the demand) and the price at which consumers are willing to buy. We distinguish between these types of demand:

- Total (aggregate) - which is determined by the total volume of production that consumers want to buy and the prices at which they are willing to buy;
- Individual - is the demand of an individual. This means how the consumer spends his / her income on the purchase of different products according to its prices and benefits;
- Market - it is the demand for one product, which represents the expected costs of different consumers [2].

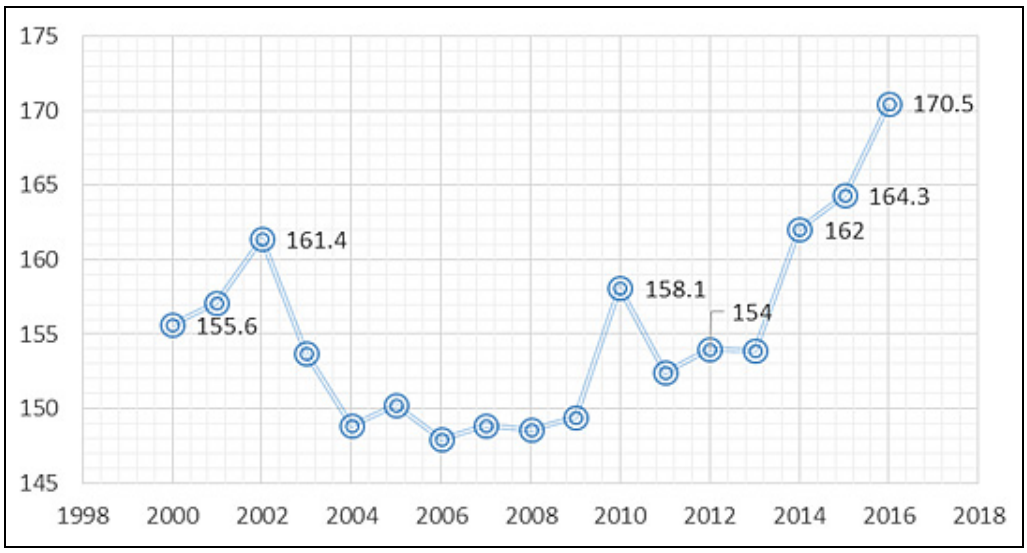


Figure 1 Development of consumption of milk and dairy products in the Slovak Republic (in liters per capita). Source: [7], own processing

Figure 1 shows that in the monitored period 2000 - 2017 the consumption of milk and dairy products in the Slovak Republic had a fluctuating tendency. At the beginning of the monitored period the consumption of milk and dairy products per capita was 155.6 liters. In 2002 it was 161.4 liters and later declined, down to 148 liters in 2006. From that moment it started to rise up to 158.1 liters in 2010. One year later, it dropped by 4 liters, but from this

year on, we recorded a steady increase in the consumption of milk and dairy products per capita, which reached 170.5 liters in 2017.

MATERIAL AND METHODS

The aim of the article was to analyze consumer preferences when buying milk and selected dairy products.

In the analysis of respondents' opinions, were set the following hypotheses:

Hypothesis 1: Education of respondents has an impact on the value of expenditures on milk and dairy products in one purchase;

Hypothesis 2: The net monthly income of the household of respondents has an impact on motivational aspects when buying milk and dairy products;

Hypothesis 3: The expenditures for milk and milk products in one purchase are affected by motivational aspects of consumers when buying milk and milk products.

The first part of the questionnaire consisted of 11 questions related to consumer preference and consumer buying behavior. The second part consisted of 9 classification questions concerning the characteristics of the respondents.

The survey was running in February and March 2018. The questionnaire was filled by 251 respondents. In this paper we used following methods and procedures:

- Method of questionnaire – it was used in written and electronic form. This method is important in view of the further analysis of data into MS Excel;
- Graphic analysis – it was used for greater clarity and better reporting ability;
- Descriptive statistics – the tools of descriptive statistics were used for sorting qualitative data.

The result of this process was the pivot table that served as the basis for graphical analysis and testing of hypotheses.

- Testing of statistical hypotheses – it was used a nonparametric test χ^2 for nominal data; criterion expression is $\chi^2 = \frac{(Eij-Tij)2}{Tij}$, where E_{ij} is empirical abundance, T_{ij} is theoretical abundance.

RESULTS AND DISCUSSION

In this part we introduce the characteristics of the respondents who participated in the questionnaire survey as well as the analysis of their opinions.

Characteristics of respondents

The first qualifying question was focused on gender. From all 251 respondents who were interviewed, 130 were men (52%) and 121 were women (48%) (Figure 2).

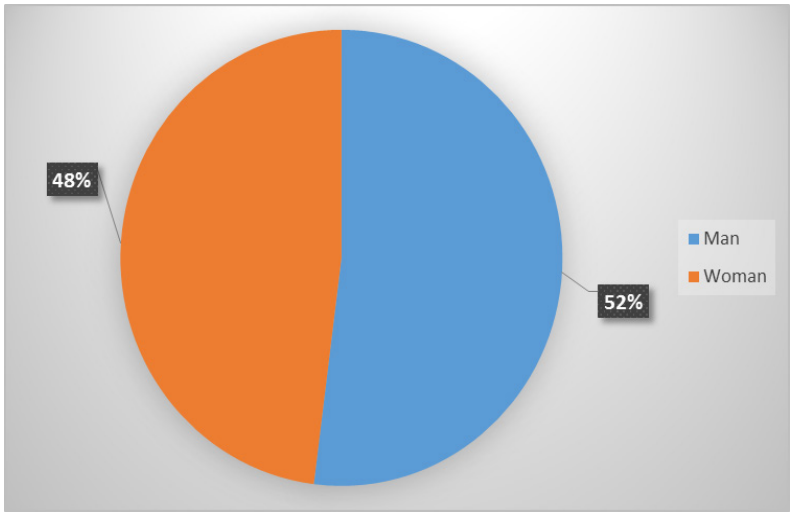


Figure 2 Structure of respondents by gender. Source: own processing

The second question in terms of the characteristics of the respondents was focused on highest educational attainment of the people who participated in the survey. Figure 3 shows that the highest number of respondents has secondary education, almost 58% of them (142 of the respondents). The college graduates represent 39% (101) respondents. The smallest group was made up of people with only basic education. This group consisted only of 8 respondents.

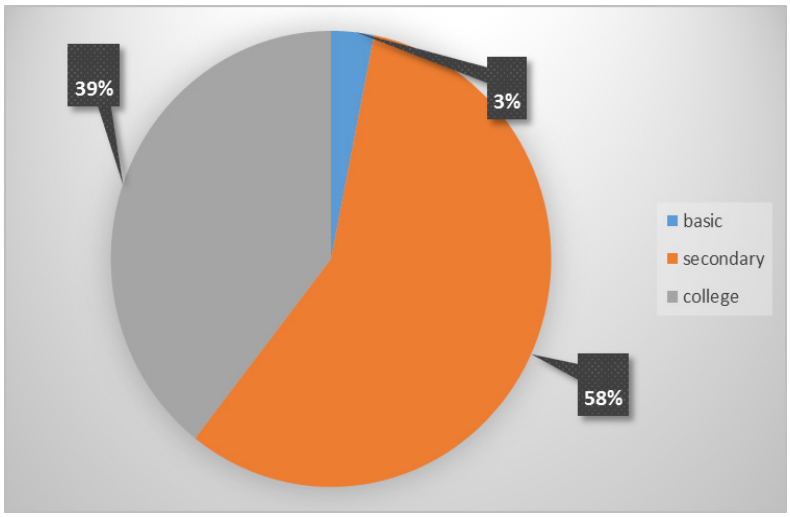


Figure 3 Structure of respondents by educational attainment. Source: own processing

Another classification question was the net monthly income of respondents' households. The most numerous were households which net monthly income exceeded 1 300 €, represented by 33% of the respondents, which in absolute terms means 84 interviewees. Figure 4 shows that the largest share, up to 40% is represent by households where the net monthly income is in the range 501 - 800 €. 50 respondents in the survey stated that the net monthly income of their household is 801 - 1 000 €. The next group consists of respondents who stated that the net monthly income of their households is between 1001 - 1 300 €, represented by 18% share (45) of interviewees. The smallest group was made up of respondents whose households had a net monthly income of only 500 €, which is in relative terms 9% represents of all respondents.

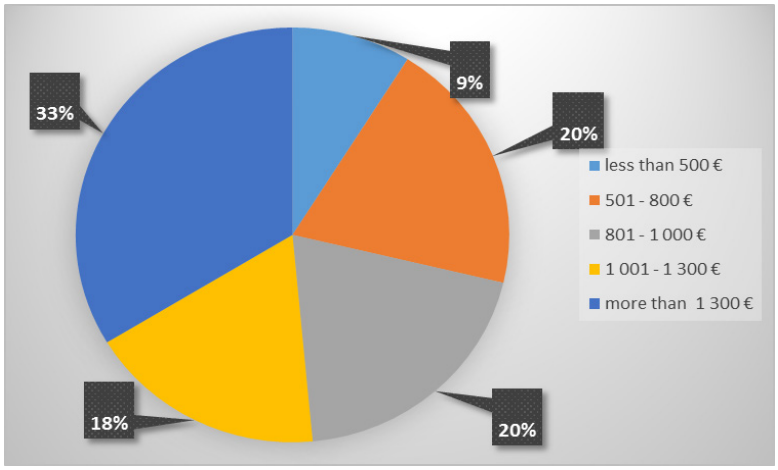


Figure 4 Structure of respondents according to the net monthly income of respondents' households. Source: own processing

Analysis of respondents' opinions

In this part of the article were analyzed the opinions of the respondents regarding to the economic aspects of consumer preferences for the purchase of milk and selected dairy products.

Question no. 1:
Estimate the monetary value of milk and milk products bought in one purchase (€)

In this issue we analyzed the relationship between the highest education attained by the respondents and the value of the expenditure on milk and dairy products. We examined the relationship using a square test of contingency. In this case we did not reject the zero hypothesis; which means that there is not statistically significant dependence between the highest achieved education of the respondents and the value of the expenditures on the purchase of milk and dairy products. The calculated value of test criterion is 6.006 and the critical value is 15.507 at chosen significance level of 5%.

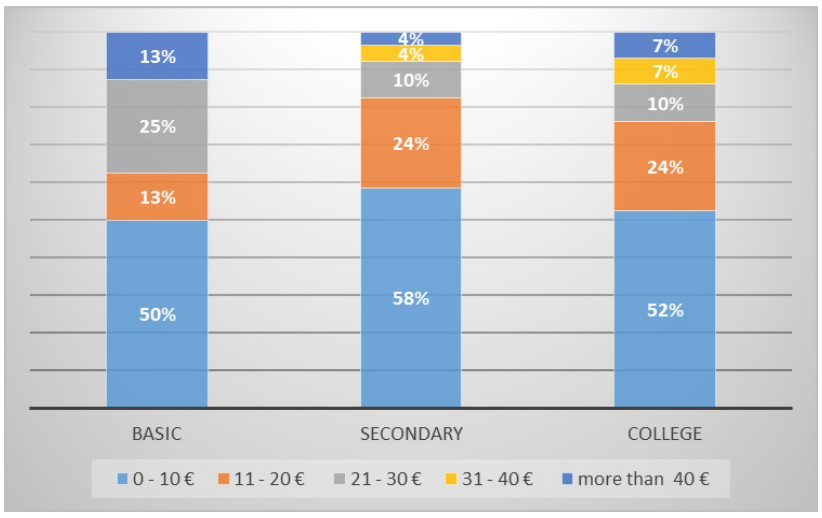


Figure 5 Structure of respondents by education and the value of expenditure on milk and dairy products. Source: own processing

Figure 5 shows that in the category up to 10 €, approximately 50% of people, was doing purchase regardless of the level of education. In the 31- 40 € category, no one with basic education shopped, which is caused by the fact that these group of respondents were involved in the survey at least.

Question no. 2:
What are aspects of your motivation when buying milk and dairy products?

In connection with this question, we found the impact of two factors. We analyzed the relationship between the net monthly income of respondents' households and the aspects of motivation that affect the purchase of milk and dairy products as well as the relationship between the aspects of motivation which influence the purchase of milk and dairy products and the expenditures on buying milk and dairy products.

In the case of the first relationship, we can conclude that based on the square test of contingency we cannot reject the zero hypothesis, meaning that there is no statistically significant dependence between the net monthly income of respondents' households and motivating aspects that affect the purchase of milk and dairy products. Value of the test criterion is 16.485 and the critical value is 21.026 at a significance level of 5%.

Figure 6 shows that people with an income higher than 1 300 € most often do shopping according to the current situation, or according to the state of household inventory, represented by 39% and 35%. Interesting is also fact, that consumers are also do shopping on the same occasion, are also purchasing consumers, whose income is between 1 001 € and 1 300 €. Equally interesting is the fact that almost ¼ of respondents who said that their net monthly income is lower than 500 € buy milk and dairy products only in sale. In addition, we could see that consumers who have net monthly income in the range of 501 - 800 € (23%) buy exclusively in discount prices.

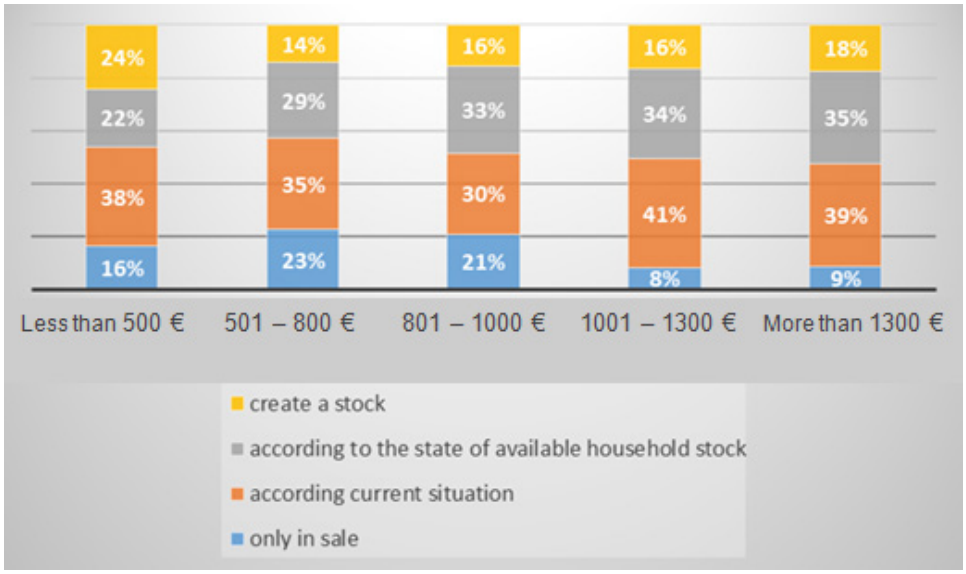


Figure 6 Motivational aspects of milk purchase according to net monthly income.
Source: own processing

In the case of the second relationship (between the motivation aspects of the purchase of milk and dairy products and the expenditures on the dairy and milk purchased) we found, using the same test, that we do not reject the null hypothesis anyway. Conclusion is that that there is no statistically significant dependence between the incentive aspects of the dairy purchase and the expenditures on the purchase of milk and milk products because the value of test criterion is 18.013 and the critical value is 24.996 at a significance level of 5%.

Figure 7 shows that people who spend more than 50 € on buying milk and dairy products do not buy in discount prices. On the other side, people spending 41 – 50 € for milk and dairy products, buy mostly in the sale (45%) and the other aspects are at the same level. Consumers spending 0 – 10 € on milk and dairy products most often buy according to their current situation.

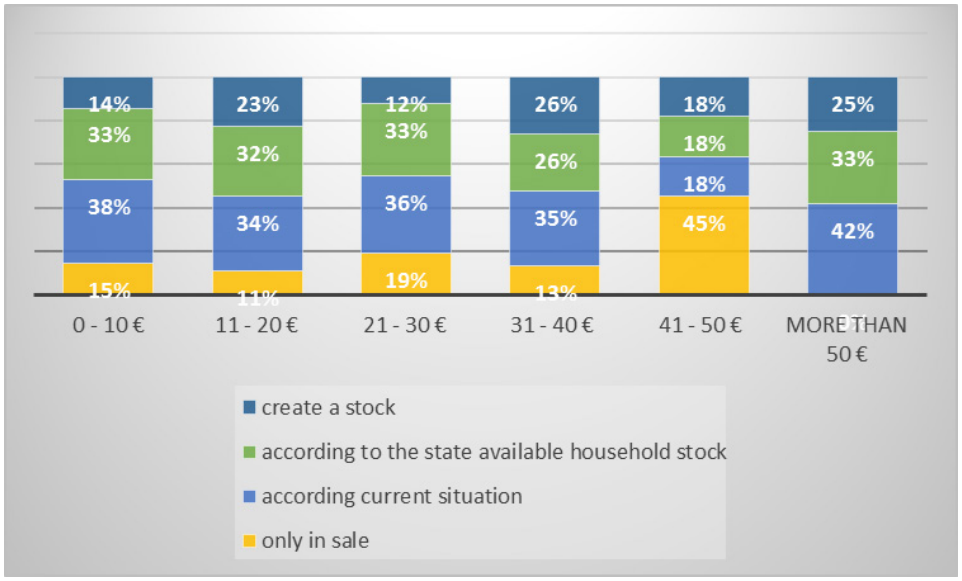
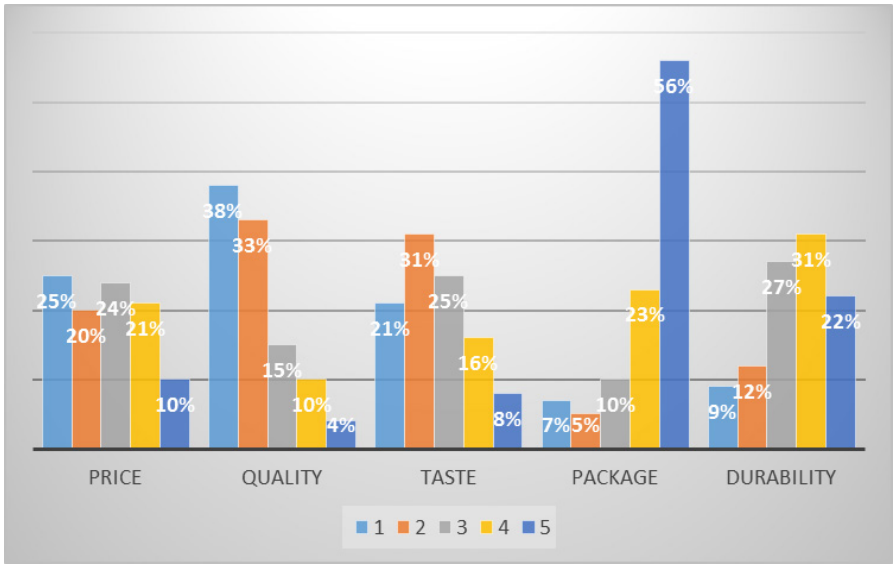


Figure 7 Motivational aspects of the buying of milk and dairy products according to the purchased value. Source: own processing

Question no. 3:
Sort the following factors according to your decision-making importance
(1 - the most important factor, 5 - the least important factor)

The respondents were asked to set the factors that influence them when purchasing this kind of food. Individual factors were price, quality, taste, packaging and durability. Each factor was assigned a point from 1 to 5 depending on the importance. An evaluation of this question can be seen in Figure 8.

Respondents in the survey decided that the most important factor was the quality because 95 consumers identified this as their first choice, representing 38% of the sample of respondents. The second most preferred criterion was the price that was rated as the most important factor by approximately ¼ respondents (64 respondents). A total of 21% of respondents ranked among the taste factors (52 consumers) and 9% durability (22 respondents). On the other hand, up to 56% of consumers chose packaging as the least important factor.



Note: 1 means the most important factor, 5means the least important factor

Figure 8 Selected factors in decision of consumers. Source: own processing

CONCLUSIONS

By analyzing selected economic aspects and consumers' preferences when buying milk and selected dairy products, we found:

1. As the first one we examined the relationship between education and the value of milk and dairy products in one purchase. Dependence has not been confirmed; the education of respondents does not have a significant impact on the value of buying milk and dairy products in one purchase.

2. As the second factor we analyzed the relationship between the net monthly income of respondents' households and the motivational aspects of respondents in the purchase of milk and dairy products. In this case, the dependence has not been confirmed, meaning that the net monthly income of the respondents does not influence aspects when buying milk and milk products.

3. As the third one we analyzed the relationship between the motivational aspects of buying milk and dairy products and the value of purchasing milk and dairy products in one purchase. Even in this case, dependence has not been confirmed.

4. Very interesting finding was the fact that participants of the questionnaire survey predominantly preferred the quality to the price as well as other factors. This finding requires manufacturers to produce quality food as they are nowadays in great demand.

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