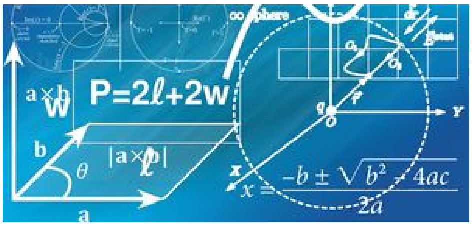
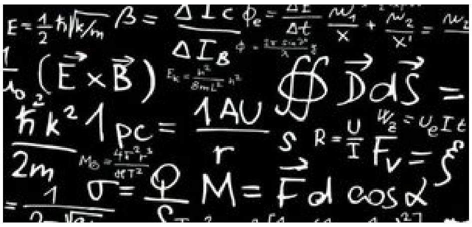
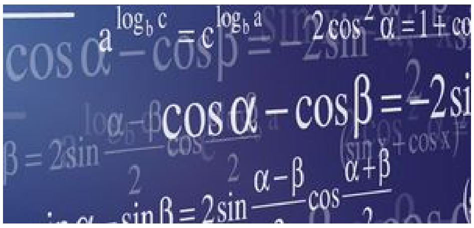

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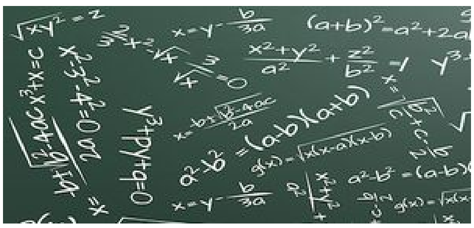
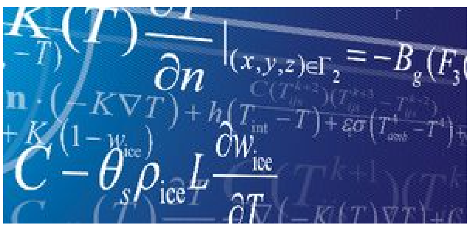
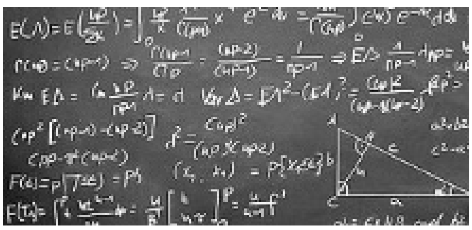

# Volume Index :: 2019(5)

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***Original Paper***

# Development and prospects of Stewart's theorem research

**Olena Melnichenko[*], Uliana Revytska**

Bila Tserkva National Agrarian University, Department of Mathematics and Physics, Ukraine

**ABSTRACT**

This paper is devoted to the study of Stewart's theorem, its consequences, development and prospects of research of the theorem under consideration. The paper focuses on the Diophantine equations and their relation to the Stewart's theorem. The problem of determination of integer solutions of the Diophantine equations was considered, and some modern researches of the Stewart's theorem are presented from the point of view of finding integer solutions of it, which are related to the first and second order Diophantine equations. The well-known integer solutions of the Stewart's theorem, and the definitions, which have been formulated in the form of a table, are presented in this paper. Some practical applications of the Stewart's theorem focused on computing the length of a segment, that connects the vertex of a triangle with its inner point, are relevant in the area of logistics, management and designing.

**KEYWORDS**: triangle geometry, Stewart's theorem, Diophantine equations, integer solutions

**JEL CLASSIFICATION:** C02, C30

## INTRODUCTION

Stewart's theorem is one of the classical problems of triangle geometry and is partially represented in the elementary geometry educational literature [7, 11]. The most well-known problem-consequences of the Stewart's theorem are formulas for calculating the lengths of the medians and bisectors of a triangle on its sides [2]. The consequence of the Stewart's theorem is the Ptolemy theorem, and the Apollonian theorem that is the partial case of Ptolemy theorem [8]. We have systematized problems-consequences and proofs of the theorem and set task to move away from the classical method of the topic presentation and to characterize modern directions of researches of the theorem and to find its relations with other sections of mathematics. The question of finding an integer solution of the Stewart equality as a search for the solution of the Diophantine equation remains interesting.

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{MERAA} Mathematics in Education, Research and Applications
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Math Educ Res Appl, 2019(5), 2, 53-60

## MATERIAL AND METHODS

The purpose of the paper is to formulate the Stewart's theorem and its development, to present modern studies of the theorem under consideration. We refer the integer solutions of the Stewart's theorem and show the relation to the Diophantine equations.

The research methods used in this paper are universally recognized methods of scientific knowledge [10]:
- Theoretical: study and analysis of relevant scientific literature, textbooks and materials of electronic publications;
- Inductive: collection, systematization and classification of existing proofs of the Stewart's theorem, its consequences and current studies of the theorem under consideration;
- Practical: application of theoretical knowledge and practical skills to create problems and their solutions.

## RESULTS AND DISCUSSION

Scottish mathematician Stewart Matthew (1717 − 1785) published a scientific paper in 1746 (Fig. 1) in which he presented his theorem with substantiation [15]. This theorem is called Stewart's theorem, and is expressed by (1) and displayed in Fig. 2.

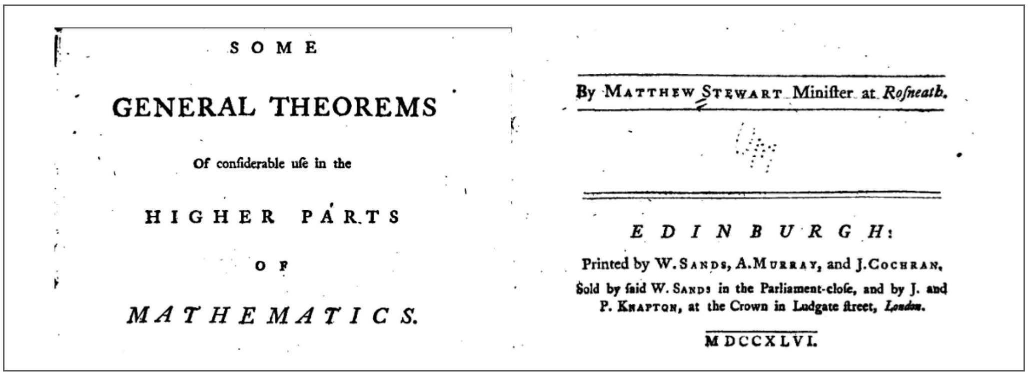

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Figure 1

**Stewart's theorem** [20]. Let ΔABC be an arbitrary triangle (Fig. 2). For any point *D* on the side of the BA the following formula holds:

$$CD^2 = CB^2 \cdot \frac{AD}{AB} + AC^2 \cdot \frac{BD}{AB} - BD \cdot AD \quad \text{or} \quad d^2 = a^2 \cdot \frac{n}{c} + b^2 \cdot \frac{m}{c} - n \cdot m \qquad (1)$$

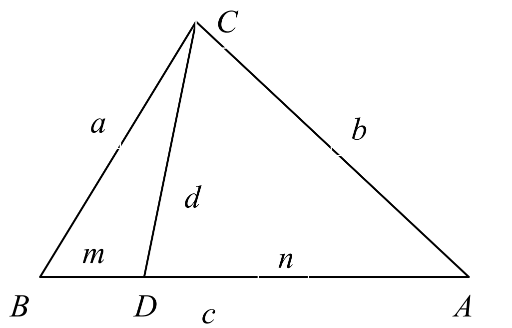


Figure 2

There are various proofs of Stewart's theorem: by Pythagorean Theorem, coordinate method, cosine theorem, vector method [1, 2, 8]. The work [2] presents the consequences of the Stewart's theorem: finding the length of a segment with ends on the sides of a triangle with possible boundary cases, and finding the distance from the vertex to the inner point of the triangle. As a development of the Stewart's theorem Willie Yong and Jim Bound in their paper Using Stewart's Theorem [20] presents the solution of the following problem:

**Task 1.** If in a triangle $\Delta ABC$ side $CB > CA$ and segment $CT$ belongs to the bisector of the angle $\angle SCA$, moreover $BS = TA$, points $S$ and $T$ belong to the $BA$ side (Fig. 3). Then:

$$CS^2 - CT^2 = (CB - CA)^2 .$$

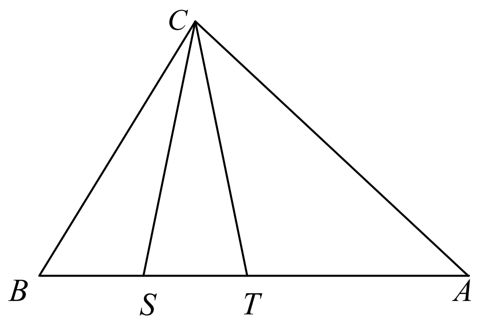

Figure 3

***Substantiation***. Known: $BT + TA = AB$ .
By the property of bisector $CT$:

$$\frac{BT}{TA} = \frac{CB}{CA}, \text{ or } \frac{BT}{BA - BT} = \frac{CB}{CA}.$$

Therefore, $$BT = \frac{BA \cdot CB}{CA + CB},\ TA = BA - BT = \frac{BA \cdot CB}{CA + CB}. \qquad (2)$$

Consider that $BS = TA$, by the Stewart's theorem we have:

$$CT^2 = CB^2 \cdot \frac{AT}{AB} + AC^2 \cdot \frac{BT}{AB} - BT \cdot AT, \quad CS^2 = CB^2 \cdot \frac{BT}{AB} + AC^2 \cdot \frac{AT}{AB} - AT \cdot BT . \qquad (3)$$

Substituting (3) into (2), we get: $CS^2 = \dfrac{CB^3 + AT}{AC + CB} - \dfrac{BA^2 \cdot CB \cdot AC}{(AC + CB)^2}$, i.e.,

$$CS^2 = CB^2 - CB \cdot CA + CA^2 - \frac{BA^2 \cdot CB \cdot AC}{(AC + CB)^2} . \qquad (4)$$

In view of (3), (4) we subtract and obtain:

$$CS^2 - CT^2 = CB^2 - 2 \cdot CB \cdot CA + CA^2 = (CB - CA)^2 .$$

One of the problems that were formulated by Willie Yong and Jim Bound is presented below [16].

**Problem 1.** Prove that in the right triangle the sum of the squares of distances from the vertex of the right angle to the three points of hypotenuse is equal to $\frac{5}{9}$ of the length of the hypotenuse squared, i.e., $CS^2 + CT^2 = \frac{5}{9} \cdot AB$ . The solution was presented in [17].

***Solution.*** In view of the Stewart's theorem the following equalities hold (Fig. 4):

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2019(5), 2, 53-60

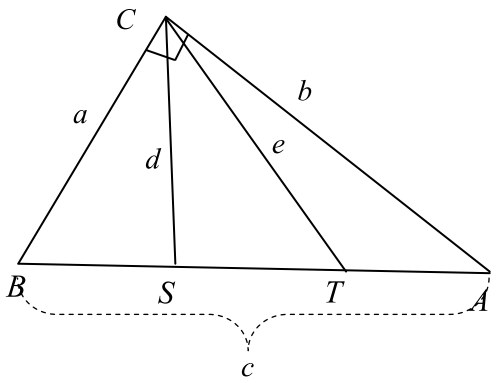

Figure 4

$$\begin{cases} c^2\cdot\left(\frac{2}{3}a\right)+b^2\cdot\left(\frac{1}{3}a\right)=a\cdot\left(d^2+\left(\frac{2}{3}a\right)\cdot\left(\frac{1}{3}a\right)\right),\\ c^2\cdot\left(\frac{2}{3}a\right)+b^2\cdot\left(\frac{2}{3}a\right)=a\cdot\left(e^2+\left(\frac{2}{3}a\right)\cdot\left(\frac{1}{3}a\right)\right).\end{cases}$$

We divide the right hand sides of the above equations with $a$ and get

$$\begin{cases} \frac{2}{3}c^2+\frac{1}{3}b^2=d^2+\left(\frac{2}{3}a\right)\cdot\left(\frac{1}{3}a\right),\\ \frac{1}{3}c^2+\frac{2}{3}b^2=e^2+\left(\frac{2}{3}a\right)\cdot\left(\frac{1}{3}a\right).\end{cases}$$

Adding the last two equations results in: $c^2+b^2=d^2+e^2+\frac{4}{9}\cdot a$.

According to the Pythagorean theorem we have: $d^2+e^2=\frac{5}{9}\cdot a$. What was to be shown.

Bretschneider's formula describes the relation among the elements of the quadrilateral $ABCD$ and is related to the Stewart's theorem.
Let us denote (Fig. 5) $AB=a$, $DC=b$, $CD=c$, $AD=d$, $AC=e$, $BD=f$ [19].

**Theorem** (Bretschneider's formula). The following formula holds

$$(e\cdot f)^2=(a\cdot c)^2+(b\cdot d)^2-2\cdot a\cdot b\cdot c\cdot d\cdot\cos(\angle A+\angle C).$$

***Proof.*** We construct two triangles $ABF$ and $ADE$, which are similar to the triangles $CAD$ and $CAB$ respectively.
The similarty of the triangles yields to
$\frac{AF}{a}=\frac{c}{e}$, $\frac{BF}{a}=\frac{d}{c}$, $\frac{AE}{b}=\frac{d}{e}$, $\frac{DE}{b}=\frac{a}{e}$.
Thus, $AF=\frac{ac}{e}$, $BF=\frac{ad}{c}$, $AF=\frac{bd}{e}$, $DE=\frac{ad}{e}$.
The sum of vertex angles $D$ and $E$ of quadrilateral $BDEF$ equals the sum of angles of $\Delta ABD$, i.e., is $180^\circ$.
Thus, the lines $BF$ and $DE$ are parallel.

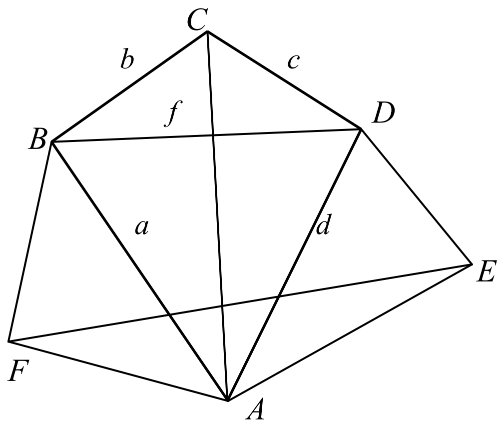

Figure 5

Since $BF = DE$, the quadrilateral $BDEF$ is a parallelogram, $FE = BD = f$.
The angle $\angle EAF$ of triangle $\Delta AEF$ equals to the sum of $\angle A$ and $\angle C$ (from construction), in other words $\angle EAF = \angle A + \angle C$. Law of cosines for $\Delta AEF$ implies that

$$(f)^2=\left(\frac{a\cdot c}{e}\right)^2+\left(\frac{b\cdot d}{e}\right)^2-\frac{2\cdot a\cdot b\cdot c\cdot d}{e}\cdot\cos(\angle A+\angle C).$$

Stewart's theorem can be considered as the consequence of Bretschneider's formula, in case of degenerate quadrilateral $ABCD$, i.e., when the point $D$ belongs to $AC$.

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2019(5), 2, 53-60

Let us consider the notion of Diophantine equations and its relation with the Stewart's theorem. A Diophantine equation is a polynomial equation, usually in two or more unknowns, such that only the integer solutions are sought or studied. It has to be mentioned that the main problems of study of Diophantine equations are existence of its solutions and also the existence of an algorithm for solving such equations [12, 13]. The problem of finding all integer solutions of Diophantine equations was quite popular among mathematicians. For example, the study of existence of integer solutions of Diophantine equations in the form $ax^2+bxy+cy^2=d$, where *a*, *b*, *c*, *d* are arbitrary integer numbers, were presented in [9, 18].

In [3] Keskin considered the Diophantine equations of the form $x^2-L_nxy+(-1)^n y^2=\pm 5^\gamma$, which under the assumptions $n>0$ and $r>1$ have the integer solutions that are Fibonacci numbers or Lucas numbers.

The authors Keskin and Yosma [6] considered the Diophantine equations of the following form: $x^2-5F_nxy+5\cdot(-1)^n y^2=\pm 5^\gamma$, with $n>1$ and $r>0$. It was also highlighted that for all $n\geq 2$ the Fibonacci-Lucas sequences are defined:

$$F_n=F_{n-1}+F_{n-2},\ F_0=0, F_1=1,\ L_n=L_{n-1}+L_{n-2},\ L_0=2, L=1.$$

Besides, the Fibonacci and Lucas numbers are defined for negative indices, for all $n\geq 1$, $F_{-n}=(-1)^{n+1}F_n$, $L_{-n}=(-1)^{n+1}L_n$ [4, 5, 14].

There is a relation between the Stewart's theorem and Diophantine equations, thus for finding integer solutions of Stewart's equation one has to apply the known algorithms for solving Diophantine equations. Consider the finding of integer solutions of the Stewart's equation of the form $\frac{b^2}{a}\cdot x+\frac{c^2}{a}\cdot y-xy=p^2$. To the right- and left-hand sides we add $-\frac{b^2}{a}\cdot\frac{c^2}{a}$.

We obtain: $\frac{b^2}{a}\cdot x+\frac{c^2}{a}\cdot y-xy-\frac{b^2}{a}\cdot\frac{c^2}{a}=p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}$.

We factorize the equation: $\frac{b^2}{a}\cdot\left(x-\frac{c^2}{a}\right)-y\cdot\left(x-\frac{c^2}{a}\right)=p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}$

and get: $\left(\frac{b^2}{a}-y\right)\cdot\left(x-\frac{c^2}{a}\right)=p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}$.

In case $p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}=0$, we have that one of the factors is zero.

In other words $\left(\frac{b^2}{a}-y\right)=0$ or $\left(x-\frac{c^2}{a}\right)=0$.

Let $\left(x-\frac{c^2}{a}\right)=0$, then we choose the value so that $c^2$ could be completely divided by *a*.

This implies $\left(x-\frac{c^2}{a}\right)=0$ and thus we compute $x=\frac{c^2}{a}$.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2019(5), 2, 53-60

We find $b^2$ such that $\frac{b^2}{a}\cdot\frac{c^2}{a}$ is a square of some integer and is equal to $p^2$, this could be rewritten as $p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}=0$.
Results of the applied selection method are presented in Table 1.

Table 1 Selection of integers

| $c^2$ | $a$ | $\frac{c^2}{a}=x$ | $b^2$ | $b^2c^2$ | $\frac{b^2}{a}\cdot\frac{c^2}{a}$ | $p^2$ | $\left(x-\frac{c^2}{a}\right)=0$ | $p^2-\frac{b^2}{a}\cdot\frac{c^2}{a}=0$ |
|---|---|---|---|---|---|---|---|---|
| 16 | 8 | 2 | 100 | 1600 | 25 | 25 | 0 | 0 |
| 36 | 12 | 3 | 64 | 2304 | 16 | 16 | 0 | 0 |
| 36 | 9 | 4 | 144 | 5184 | 64 | 64 | 0 | 0 |
| 64 | 16 | 7 | 144 | 9216 | 36 | 36 | 0 | 0 |

We find the values of y, such that $\left(\frac{b^2}{a}-y\right)$ is an integer or $b^2-ay$ can be factored by $a$.

As a result we get:

$$a=8, b^2=100\Rightarrow\left(\frac{100}{8}-y\right)=k\in Z\Rightarrow y=6,$$
$$a=12, b^2=64\Rightarrow\left(\frac{64}{12}-y\right)=k\in Z\Rightarrow y=9,$$
$$a=9, b^2=144\Rightarrow\left(\frac{144}{9}-y\right)=k\in Z\Rightarrow y=5,$$
$$a=16, b^2=144\Rightarrow\left(\frac{144}{16}-y\right)=k\in Z\Rightarrow y=9.$$

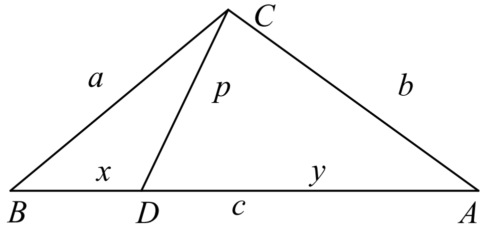

Figure 6

Thus, after calculations we have computed four different integer solutions of the Stewart's equation in a form that corresponds to the equilateral triangles in notations of Fig. 6 (Fig. 7).

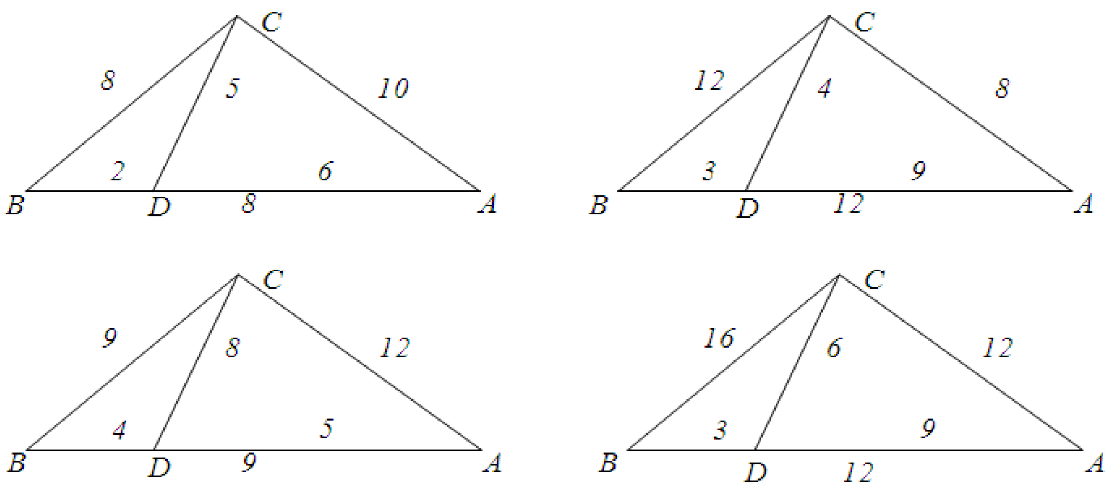

Figure 7

We consider the prospect of further research to investigate algorithms for finding integer solutions of the Stewart's equation, provided an in-depth study of the solutions of first- and second-order Diophantine equations, which have the basis of Fibonacci and Lucas numbers, and an understanding of the Fermat's little theorem and Fermat's last theorem.

## CONCLUSIONS

In this paper the Stewart's theorem was considered as one of the classical problems of geometry of a triangle and its application to different problems. We studied the problem of computing integer solutions of Diophantine equations and presented some results of studies of the Stewart's theorem in view of finding its integer solutions, that is related to first- and second-order Diophantine equations. The method for computing integer solutions of Stewart's equations was presented and four different integer solutions for equilateral triangles were computed, using the described method.

From the obtained results we can conclude that application of the Stewart's theorem is an effective tool for solving numbers of geometrical problems. The generalizations of the Stewart's theorem and its applications to computing the length of a segment, that connects the vertex of a triangle with its inner point, can be applied in logistics, management and designing. For example, to choose the right coordinates for set of three stores, which are the vertices of the triangle or to find correct place for Wi-Fi router for a few settlements (for three, for example).

## REFERENCES

[1] *Geometry*. Retrieved 2019-08-01 from http://math.cmu.edu/~cargue/arml/archive/13-14/geometry-12-15-13.pdf

[2] Kadubovska, V. M., Kadubovska, O. L., & Kadubovskij, O. A. (2012). *Navkolo Teoremi Styuarta: naslidki, uzagaljnenja ta zastosuvanja* (Навколо теореми Стюарта: наслідки, узагальнення та застосування, in Ukrainian). Retrieved 2019-08-02 from https://www.researchgate.net/publication/277141893_

[3] Keskin, R. (2010). Solutions of some quadratic Diophantine equations. *Comput. Math. Appl.*, Vol. 60, № 8, pp. 2225-2230.

[4] Keskin, R. & Bitim, B. (2011). Fibonacci and Lucas congruences and their applications. *Acta Math. Sin., Engl. Ser.*, Vol. 27, № 4, pp. 725-736.

[5] Keskin, R. & Yosma, Z. (2011). On Fibonacci and Lucas numbers of the form $cx^2$. *J. Integer Seq.*, Vol. 14, № 9, pp. 9-12.

[6] Keskin, R. & Yosma, Z. (2009). Positive integer solutions of the Diophantine equations $x^2-5F_nxy-5\cdot(-1)^n y^2=\pm 5^r$. *J. Integer Seq.*, Vol. 10, № 5, pp. 22-25.

[7] Kolmogorov, N. A. (1936). Analog teoremy i tozdestva Styuarta (Аналог теоремы и тождества Стюарта, in Russian). *Matematiceskoje prosvescenije*, Part 7, pp. 22-25.

[8] Kuschnir, I. A. (2002). Geometritchni formuli, stcho ne vyjschli do schkiljnich pidrucchnikiv: Dovidnik (Геометричні формули, що не ввійшли до шкільних підручників: Довідник, in Ukrainian). K.: Fakt, 112 p.

[9] Matthews, K. (2002). The Diophantine equation $ax^2+bxy+cy^2=N$. *J. Theor. Nombres Bordx.*, Vol. 14, № 1, pp. 257-270.

[10] Melnichenko, O. P. (2012). Naukova dijaljnist u svitli prognozuvanija, postanovka problemi, aktualjnosti, objekta i predmeta (Наукова діяльність у світлі прогнозування, постановка проблеми, актуальності, об'єкта і предмета, in Ukrainian). Naukovij visnik LNUVMB, Vol. 14, № 3(53), pp. 48-51.

[11] Merzljak, A. G., Polonskij, V. G. & Jakir, M. S. (2009). Geometrija: pidrucchnik dlja 9 klasu schkil z pogliblenim vivtchenjam matematiki (Геометрія: підручник для 9 класу шкіл з поглибленим вивченням математики, in Ukrainian). X.: Gimnazija, 272 p.

[12] Mollin, R. (2006). Quadratic Diophantine equations $x^2 - Dy^2 = c^n$. *Ir. Math. Soc. Bull.*, Vol. 58, pp. 55-68.

[13] Mordell, L. (1969). Diophantine equations. *Pure and Applied Mathematics*. London-New York: Academic Press, Vol. 30.

[14] Novosad, M. V. & Ditchka, I. A. (2009). Tchisla Ljuka (Числа Люка, in Ukrainian). *Naukovij visnik Tcherniveckogo universitety*, Vol. 446, pp. 11- 15.

[15] *Proving Stewarts Theorem*. Retrieved 2019-08-09 from https://math.stackexchange.com/questions/46616/proving-stewarts-theorem

[16] Robertson, J. *Matthews' method for solving* $ax^2 + bxy + cy^2 = N$ (manuscript). Retrieved 2019-08-18 from http://www.jpr2718.org/mtthbin.pdf

[17] *Stewart Theorem*. Retrieved 2019-08-09 from https://www.qc.edu.hk/math/Small%20problems/Stewart%20Theorem.pdf

[18] *Stewarts Theorem*. Retrieved 2019-08-18 from https://brilliant.org/wiki/stewarts-theorem/

[19] *Teorema Styuarta*. Retrieved 2019-08-12 from http://hijos.ru/2015/01/23/sootnoshenie-bretshnajdera-i-teorema-styuarta/

[20] Yong, W. & Bound, J. *Using Stewart´s Theorem*. Retrieved 2019-08-12 from https://sms.math.nus.edu.sg/smsmedley/Vol-31-Using%20Stewart's%20Theorem %20(Willie%20Yong%20and%20Jim%20Boyd)

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***Original Paper***

# The universal solution of equations of balance of the transversely isotropic plate with initial stresses with slippery strength of the flat borders

**Oksana Strigina[1*], Ivan Khoma[2]**

[1] Bila Tserkva National Agrarian University, Department of Mathematics and Physic, Ukraine
[2] National Academy of Sciences of Ukraine, Institute of Mechanics S.P. Timoshenko, Ukraine

**ABSTRACT**

The topic of this paper is concentrated on the problem of mechanics of a deformed solid. In the first part we solved equilibrium equations of a transversal isotropic plate with initial stresses under mixed conditions on planar faces where we applied the method of decomposition of the sought functions into Fourier series by Legendre polynomials. Normal displacement and tangent voltage were assumed to be zero. In the second part we proposed a method of representing the general analytical solution of the obtained equilibrium equations.

**KEY WORDS:** transversely isotropic plate, initial stresses, equations of balance, universal solution

**JEL CLASSIFICATION:** C02, C30

## INTRODUCTION

Initial stresses are widely used in solving problems of a formed solid [2, 3]. In [4, 5], a method for constructing equations of anisotropic shells and plates with initial (residual) stresses is outlined. It is based on the method of decomposition of sought functions into Fourier series by Legendre polynomials of thickness coordinate [8]. With respect to the coefficients of expansions, a system of differential equations and corresponding boundary conditions were obtained as a function of two independent variables. On this basis, in [6] a solution to the problem of the stress state of a transversal-isotropic plate with initial stresses weakened by a circular cylindrical cavity was found.

## MATERIAL AND METHODS

The cavity surface and flat faces are free of external forces, and at infinity the plate is subject to constant tensile and shear forces. In this work, by the method of decomposition of the sought functions into Fourier series by Legendre polynomials, we derive the equation of the

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elastic equilibrium of a transversal-isotropic plate with initial stresses at the sliding finishing of plane faces (with zero of normal displacement and tangential stresses). The method of representing the general analytical solution of the obtained system of differential equations is presented.

**RESULTS AND DISCUSSION**

**1 Equilibrium equations**

Assume that the plate is related to the Cartesian coordinate system $x_i$ $(i=1,2,3)$, and it is located on the median plane $S$ coinciding with the isotropy plane, and $x_3 \in [-h,h]$. The frontal boundary planes $x_3 = \pm h$ are slidably fixed, i.e.

$$u_3(x_1,x_2,\pm h)=0,\ \sigma_{3\alpha}(x_1,x_2,\pm h)=0\ (\alpha=1,2)\ , \tag{1.1}$$

and the boundary conditions on the cylindrical surface $\partial S \times [-h,h]$ are arbitrary.

For the problem, we use the method of decomposition of the components of the vector of displacements $u_j(x_1,x_2,x_3)$ into the Fourier series by Legendre polynomials $P_k(\zeta)$ of the thickness coordinate. Consider, given the boundary conditions (1.1), the components of the displacement vector in the form

$$u_\alpha = \sum_{k=0}^{N} u_\alpha^{(k)}(x)P_k(\varsigma),\ u_3 = \sum_{k=0}^{N} u_3^{(k)}(x)[P_k(\varsigma)-P_{k+2}(\varsigma)] \tag{1.2}$$

And we present the components of the stress tensor as follows

$$\sigma_{ij} = \sum_{k=0}^{N} \sigma_{ij}^{(k)}(x)P_k(\zeta), \tag{1.3}$$

where $x=(x_1,x_2)\in S,\ \varsigma = h^{-1}x_3 \in [-1,1],\ u_j^{(k)}(x),\ \sigma_{ij}^{(k)}(x)$ - coefficients of expansions, called moments (the moment number corresponds to the order of the Legendre polynomial), $N-$ is a natural number which we shall consider even $N=2n$ $(n=1,2,...\langle\infty)$. With respect to the coefficients of expansions $\sigma_{ij}^{(k)},\ u_j^{(k)}$, a system of differential equations and corresponding boundary conditions is composed as a function of two independent variables. For a transversal isotropic plate, it splits into two independent groups of equations describing, respectively, symmetric and obliquely symmetric (relative to the median plane S) deformations of the plate. In symmetric deformation, taking into account boundary conditions (1.1), it has the form [6]

$$\begin{gathered}\partial_\alpha \sigma_{\alpha\beta}^{(2k)} - (4k+1)h^{-1}\sum_{s=1}^{k}\sigma_{3\beta}^{(2s-1)} = 0 \quad (\beta=1,2;\ k=0,1,...,n),\\ \partial_\alpha \sigma_{\alpha 3}^{(2k-1)} - (4k-1)h^{-1}\sum_{s=0}^{k-1}\sigma_{33}^{(2s)} + \frac{4k-1}{2h}(\sigma_{33}^{+}+\sigma_{33}^{-}) = 0 \quad (k=1,...,n),\end{gathered} \tag{1.4}$$

where $\partial_\alpha = \partial/\partial x_\alpha,\ \sigma_{33}^{+},\ \sigma_{33}^{-}-$ is the normal stress on the planes $x_3=h,\ x_3=-h$ .

Consider a plate with a homogeneous field of initial stresses $P_{ij}^{(0)}$, and assume that $P_{ij}^{(0)} = const$ for $i=j$ and $P_{ij}^{(0)}=0$, if $i \neq j$ .

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Based on the equations [1]

$$\sigma_{ij} = c_{ijlm}\partial_\ell u_m + p_{il}\partial_\ell u_j \tag{1.5}$$

where $c_{ijlm}$ – the elastic modulus tensor, we obtain, taking into account the expansions (1.2), the relations for a transversal-isotropic plate with a homogeneous field of initial stresses, hence

$$\begin{aligned}
&\sigma_{11} = \sum_{l=0}^{n}\left[\left(c_{11} + p_{11}^{(0)}\right)\varepsilon_{11}^{(2l)} + c_{12}\varepsilon_{22}^{(2l)} - (4l+1)c_{13}h^{-1}u_3^{(2l-1)}\right]P_{2l}(\zeta),\\
&\sigma_{22} = \sum_{l=0}^{n}\left[c_{12}\varepsilon_{11}^{(2l)} + \left(c_{11}\varepsilon_{11}^{(2l)} + p_{22}^{(0)}\right)\varepsilon_{22}^{(2l)} - (4l+1)c_{13}h^{-1}u_3^{(2l-1)}\right]P_{2l}(\zeta),\\
&\sigma_{33} = \sum_{l=0}^{n}\left[c_{13}\left(\varepsilon_{11}^{(2l)} + \varepsilon_{22}^{(2l)}\right) - (4l+1)(c_{33} + p_{33}^{(0)})h^{-1}u_3^{(2l-1)}\right]P_{2l}(\zeta),\\
&\sigma_{12} = \sum_{l=0}^{n}\left[\left(c_{66} + p_{11}^{(0)}\right)\varepsilon_{12}^{(2l)} + c_{66}\varepsilon_{21}^{(2l)}\right]P_{2l}(\zeta), \qquad \sigma_{21} = \sum_{l=0}^{n}\left[c_{66}\varepsilon_{12}^{(2l)} + \left(c_{66} + p_{22}^{(0)}\right)\varepsilon_{21}^{(2l)} + c_{12}\varepsilon_{22}^{(2l)}\right]P_{2l}(\zeta),\\
&\sigma_{13} = \sum_{l=1}^{n}\left[\left(c_{44} + p_{11}^{(0)}\right)\varepsilon_{13}^{(2l-1)} + c_{44}h^{-1}\underline{u}_1^{(2l)}\right]P_{2l-1}(\zeta),\ \ \sigma_{31} = \sum_{l=1}^{n}\left[c_{44}\varepsilon_{13}^{(2l-1)} + \left(c_{44} + p_{33}^{(0)}\right)h^{-1}\underline{u}_1^{(2l)}\right]P_{2l-1}(\zeta),\\
&\sigma_{23} = \sum_{l=1}^{n}\left[\left(c_{44} + p_{22}^{(0)}\right)\varepsilon_{23}^{(2l-1)} + c_{44}h^{-1}\underline{u}_2^{(2l)}\right]P_{2l-1}(\zeta),\ \ \sigma_{32} = \sum_{l=1}^{n}\left[c_{44}\varepsilon_{23}^{(2l-1)} + \left(c_{44} + p_{33}^{(0)}\right)h^{-1}\underline{u}_2^{(2l)}\right]P_{2l-1}(\zeta).
\end{aligned} \tag{1.6}$$

Here

$$\underline{u}_2^{(2l)} = (4l-1)\sum_{s=l}^{n}u_\alpha^{(2s)}\ ,\quad \varepsilon_{\alpha\beta}^{(2l)} = \partial_\alpha u_\beta^{(2l)}\ ,\ \varepsilon_{\alpha 3}^{(2l-1)} = \partial_\alpha(u_3^{(2l-1)} - u_3^{(2l-3)}),$$

$c_{11}, c_{12},..., c_{66}$ – two-index designations of elastic constants, i.e.

$c_{11} = c_{1111},\ c_{12} = c_{1122},..., c_{66} = c_{1212}$. From relations (1.6) it follows that

$$\sigma_{33}^{+} + \sigma_{33}^{-} = 2\sum_{k=0}^{n}\left[c_{13}\left(\varepsilon_{11}^{(2k)} + \varepsilon_{22}^{(2k)}\right) - (4k+1)(c_{33} + p_{33}^{(0)})h^{-1}u_3^{(2k-1)}\right]. \tag{1.7}$$

Multiplying (1.6) by Legendre polynomials and integrating over the plate thickness, we obtain a relation connecting the moments of the stress components and the displacement vector, i.e.

$$\begin{aligned}
&\sigma_{11}^{(2k)} = \left(c_{11} + p_{11}^{(0)}\right)\varepsilon_{11}^{(2k)} + c_{12}\varepsilon_{22}^{(2k)} - (4k+1)c_{13}h^{-1}u_3^{(2k-1)},\\
&\sigma_{22}^{(2k)} = c_{12}\varepsilon_{11}^{(2k)} + \left(c_{11} + p_{22}^{(0)}\right)\varepsilon_{22}^{(2k)} - (4k+1)c_{13}h^{-1}u_3^{(2k-1)},\\
&\sigma_{33}^{(2k)} = c_{13}(\varepsilon_{11}^{(2k)} + \varepsilon_{22}^{(2k)}) - (4k+1)(c_{33} + p_{33}^{(0)})h^{-1}u_3^{(2k-1)},\\
&\sigma_{12}^{(2k)} = \left(c_{66} + p_{11}^{(0)}\right)\varepsilon_{12}^{(2k)} + c_{66}\varepsilon_{21}^{(2k)};\ \sigma_{21}^{(2k)} = c_{66}\varepsilon_{12}^{(2k)} + \left(c_{66} + p_{22}^{(0)}\right)\varepsilon_{21}^{(2k)},\\
&\sigma_{13}^{(2k-1)} = \left(c_{44} + p_{11}^{(0)}\right)\varepsilon_{13}^{(2k-1)} + c_{44}h^{-1}\underline{u}_1^{(2k)};\ \sigma_{31}^{(2k-1)} = c_{44}\varepsilon_{13}^{(2k-1)} + \left(c_{44} + p_{33}^{(0)}\right)h^{-1}\underline{u}_1^{(2k)},\\
&\sigma_{23}^{(2k-1)} = \left(c_{44} + p_{22}^{(0)}\right)\varepsilon_{23}^{(2k-1)} + c_{44}h^{-1}\underline{u}_2^{(2k)};\ \sigma_{32}^{(2k-1)} = c_{44}\varepsilon_{23}^{(2k-1)} + \left(c_{44} + p_{33}^{(0)}\right)h^{-1}\underline{u}_2^{(2k)}.
\end{aligned} \tag{1.8}$$

Substituting (1.7), (1.8) into equations (1.4), we obtain such a system of equations

$$\left(c_{66}+p_{11}^{(0)}\right)\frac{\partial^2 u_\alpha^{(2k)}}{\partial x_1^2}+\left(c_{66}+p_{22}^{(0)}\right)\frac{\partial^2 u_\alpha^{(2k)}}{\partial x_2^2}+\left(c_{12}+c_{66}\right)\frac{\partial \ell^{(2k)}}{\partial x_\alpha}-$$
$$-\frac{4k+1}{n}\left[\left(c_{13}+c_{44}\right)\frac{\partial u_3^{(2k-1)}}{\partial x_\alpha}+\frac{c_{44}+p_{33}^{(0)}}{h}\sum_{s=1}^{n}\beta_{2s}^{(k)}u_\alpha^{(2s)}\right]=0\ \left(\alpha=1,2;\ k=\overline{0,n}\right); \tag{1.9}$$

$$\left(c_{44}+p_{11}^{(0)}\right)\frac{\partial^2\left(u_3^{(2k-1)}-u_3^{(2k-3)}\right)}{\partial x_1^2}+\left(c_{44}+p_{22}^{(0)}\right)\frac{\partial^2\left(u_3^{(2k-1)}-u_3^{(2k-3)}\right)}{\partial x_2^2}+$$
$$+\frac{4k-1}{n}\left[\left(c_{13}+c_{44}\right)\sum_{s=k}^{n}\ell^{(2s)}-\frac{c_{33}+p_{33}^{(0)}}{h}\sum_{s=k}^{n}(4s+1)u_3^{(2s-1)}\right]=0\ \left(k=\overline{1,n}\right), \tag{1.10}$$

where $\ell^{(2k)}=\partial u_1^{(2k)}/\partial x_1+\partial u_2^{(2k)}/\partial x_2$, $\beta_{2s}^{(k)}-$ absolute constant.

$$\beta_{2s}^{(k)}=\begin{cases} s(2s-1)\ ,1\le s\le k; \\ k(2k-1)\ ,k\le s\le n. \end{cases} \tag{1.11}$$

$$\left(c_{44}+p_{11}^{(0)}\right)\frac{\partial^2 u_3^{(2k-1)}}{\partial x_1^2}+\left(c_{44}+p_{22}^{(0)}\right)\frac{\partial^2 u_3^{(2k-1)}}{\partial x_2^2}+\sum_{s=1}^{n}\beta_{2s}^{(k)}[\left(c_{13}+c_{44}\right)\ell^{(2s)}-$$
$$-\frac{(4s+1)(c_{33}+p_{33}^{(0)})}{h}u_3^{(2s-1)}]=0. \tag{1.12}$$

Assuming $p_{11}^{(0)}=p_{22}^{(0)}$ and introducing complex variables $z=x_1+ix_2$, $\overline{z}=x_1-ix_2$, we write equations (1.9) and (1.12) in this way

$$c_{66}^{\bullet}\Delta u_+^{(2k)}+2(c_{12}+c_{66})\partial_{\overline{z}}\ell^{2k}-(4k+1)h^{-1}[2c^{\bullet}c_{44}\partial_{\overline{z}}u_3^{(2k-1)}$$
$$+\widetilde{c}_{44}h^{-1}\sum_{s=1}^{n}\beta_{2s}^{(k)}u_+^{(2s)}]=0\quad\left(k=\overline{0,n}\right), \tag{1.13}$$

$$c_{44}^{\bullet}\Delta u_3^{(2k-1)}+h^{-1}\sum_{s=1}^{n}\beta_{2s}^{(k)}[c^{\bullet}c_{44}\ell^{(2s)}-(4s+1)\widetilde{c}_{33}u_3^{(2s-1)}]=0\quad\left(k=\overline{1,n}\right), \tag{1.14}$$

Here $\Delta=4\partial_z\partial_{\overline{z}}-$ is the Laplace operator, $2\partial_z=\partial/\partial x_1-i\partial/\partial x_2$, $2\partial_{\overline{z}}=\partial/\partial x_1+i\partial/\partial x_2$.

$$u_+^{(2k)}=u_1^{(2k)}+iu_2^{(2k)},\ \ell^{(2k)}=\partial_z u_+^{(2k)}+\partial_{\overline{z}}\overline{u}_+^{(2k)},$$
$$c_{44}^{\bullet}=c_{44}+p_{11}^{(0)},\ c_{66}^{\bullet}=c_{66}+p_{11}^{(0)},\ \widetilde{c}_{44}=c_{44}+p_{33}^{(0)},\widetilde{c}_{33}=c_{33}+p_{33}^{(0)},c^{\bullet}=\left(c_{13}+c_{44}\right)/c_{44}. \tag{1.15}$$

## 2 General analytical solution

We present a method for representing a general analytical solution to the system of equations (1.13), (1.14). We write equalities (1.14) in the form

$$\sum_{s=1}^{n}\beta_{2s}^{(k)}\ell^{(2s)}=b_k\ \left(k=\overline{1,n}\right), \tag{2.1}$$

where

$$b_k = -\frac{c_{44}^{\bullet}h}{c^{\bullet}c_{44}}\Delta u_3^{(2k-1)} + \frac{\widetilde{c}_{33}}{c^{\bullet}c_{44}h}\sum_{s=1}^{n}(4s+1)\beta_{2s}^{(k)}u_3^{(2s-1)} \tag{2.2}$$

and define the values of the functions

$$\ell^{(2k)} = \frac{1}{4k-1}(b_k - b_{k-1}) - \frac{1}{4k+3}(b_{k+1} - b_k)\ (k = 1, n-1),$$
$$\ell^{(2n)} = \frac{1}{4n-1}(b_n - b_{n-1}). \tag{2.3}$$

From here, taking into account expression (2.2), we find

$$\ell^{(2k)} = \frac{c_{44}^{\bullet}h}{c^{\bullet}c_{44}}\left[\frac{1}{4k+3}\Delta(u_3^{(2k+1)} - u_3^{(2k-1)}) - \frac{1}{4k-1}\Delta(u_3^{(2k-1)} - u_3^{(2k-3)})\right] + \frac{(4k+1)\widetilde{c}_{33}}{c^{\bullet}c_{44}h}u_3^{(2k-1)};$$
$$\ell^{(2n)} = -\frac{c_{44}^{\bullet}h}{(4n-1)c^{\bullet}c_{44}}\Delta(u_3^{(2n-1)} - u_3^{(2n-3)}) + \frac{(4n+1)\widetilde{c}_{33}}{c^{\bullet}c_{44}h}u_3^{(2n-1)}. \tag{2.4}$$

We apply the operation $\partial_z$ to equation (1.13) and in the resulting equality we consider the real part. Taking into account the formula (1.15), we obtain

$$c_{11}^{\bullet}\Delta\ell^{(0)} = 0 \quad (k = 0), \tag{2.5}$$

$$c_{11}^{\bullet}\Delta\ell^{(2k)} - (4k+1)h^{-1}[c^{\bullet}c_{44}\Delta u_3^{(2k-1)} + \widetilde{c}_{44}h^{-1}\sum_{s=1}^{n}\beta_{2s}^{(k)}\ell^{2s}] = 0 \quad (k = 1, n). \tag{2.6}$$

It follows from (2.5) that

$$\mathrm{c}_{66}\ell^{(0)} = æ_e\mathrm{u}, \tag{2.7}$$

where $u-$ arbitrary harmonic function, $æ_e^{\bullet} = 2\mathrm{c}_{66}^{\bullet}/(\mathrm{c}_{12} + \mathrm{c}_{66})$.

Equalities (2.6), taking into account the values of (2.4), are transformed to

$$\frac{1}{4k+3}\Delta\Delta u_3^{(2k+1)} - \frac{2(4k+1)}{(4k-1)(4k+3)}\Delta\Delta u_3^{(2k-1)} + \frac{1}{4k-1}\Delta\Delta u_3^{(2k-3)} + \frac{(4k+1)a\widetilde{c}_{33}}{c_{11}^{\bullet}h^2}\Delta u_3^{(2k-1)} -$$
$$-\frac{(4k+1)\widetilde{c}_{33}\widetilde{c}_{44}}{c_{11}^{\bullet}c_{44}^{\bullet}h^4}\sum_{s=1}^{n}(4s+1)\beta_{2s}^{(k)}u_3^{(2s-1)} = 0\ (k = 1,\ n-1);$$
$$\frac{1}{4n-1}\Delta\Delta(u_3^{(2n-1)} - u_3^{(2n-3)}) - \frac{(4n+1)\widetilde{c}_{33}}{c_{11}^{\bullet}h^2}\left[a\Delta u_3^{(2n-1)} - \frac{\widetilde{c}_{44}}{c_{44}^{\bullet}h^2}\sum_{s=1}^{n}(4s+1)\beta_{2s}^{(n)}u_3^{(2s-1)}\right] = 0\ (k = n). \tag{2.8}$$

Introducing the notation, $c_{66}u_3^{(2k-1)} = u_k \quad (k = 1,2,...,n)$, we represent the system (2.8) in standard form in this way

$$\sum_{k=1}^{n}L_{pk}(\Delta)u_k = 0\ (p = 1,2,...,n)\ , \tag{2.9}$$

where $L_{pk}(\Delta)$ – differential operators of the form

$$L_{pk}(\Delta) = \alpha_{pk}h^4\Delta\Delta + \beta_{pk}h^2\Delta + \gamma_{pk} \tag{2.10}$$

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Math Educ Res Appl, 2019(5), 2, 61-68

$\alpha_{pk}, \beta_{pk}, \gamma_{pk}$ – dimensionless constants whose explicit expressions are easy to write out.

To solve the system of equations (2.9), we use the operator method [7]. Consider the characteristic equation

$$\det\left\|\alpha_{pk}k^2 + \beta_{pk}k + \gamma_{pk}\right\| = 0 \tag{2.11}$$

And we will assume that it has simple, non-zero roots $k_m$ $(m = 1,2,\ldots,2n)$. Then, using the same method [6], we find

$$c_{66}u_3^{(2k-1)} = \sum_{m=1}^{2n} c_m^{(2k-1)} V_m, \tag{2.12}$$

where $V_m$ – meta-harmonic functions satisfying the equalities

$$\Delta V_m - k_m h^{-2} V_m = 0, \tag{2.13}$$

and $c_m^{(2k-1)}$ – constants defined by algebraic complements of elements of some line of the determinant

$$\left|\alpha_{pk}k_m^2 + \beta_{pk}k_m + \lambda_{pk}\right|_{n\times n}.$$

According to (2.12), the moments of deformations (2.4) take the form

$$c_{66}he^{(2k)} = \sum_{m=1}^{2n} c_m^{(2k)} V_m, \tag{2.14}$$

where

$$\begin{gathered} c_m^{(2k)} = \frac{c_{44}^{\bullet}k_m}{c^{\bullet}c_{44}}\left[\frac{1}{4k+3}c_m^{(2k+1)} - \frac{2(4k+1)}{(4k-1)(4k+3)}c_m^{(2k-1)} + \frac{1}{4k-1}c_m^{(2k-3)}\right] + \frac{(4k+1)\tilde{c}_{33}}{c^{\bullet}c_{44}}c_m^{(2k-1)}; \\ c_m^{(2n)} = -\frac{c_{44}^{\bullet}k_m}{(4n-1)c^{\bullet}c_{44}}\left(c_m^{(2n-1)} - c_m^{(2n-3)}\right) + \frac{(4n+1)\tilde{c}_{33}}{c^{\bullet}c_{44}}c_m^{(2n-1)}. \end{gathered} \tag{2.15}$$

If we take a harmonic function $u$ in the form of the real part of some harmonic function $\varphi'(z)$ (the prime denotes the derivative with respect to the variable), i.e. $u = \varphi'(z) + \overline{\varphi'(z)}$ and take into account formula (1.15), then equalities (2.7) and (2.14) can be represented in this way

$$\begin{gathered} c_{66}\left(\partial_z u_+^{(0)} + \partial_{\bar z}\overline{u}_+^{(0)}\right) = æ_e\left[\varphi'(z) + \overline{\varphi'(z)}\right], \\ c_{66}\left(\partial_z u_+^{(2k)} + \partial_{\bar z}\overline{u}_+^{(2k)}\right) = \frac{h}{2}\sum_{m=1}^{2n} a_m^{(2k)}\Delta V_m \quad (k = 1,n). \end{gathered} \tag{2.16}$$

From here we find the moments of the components of the displacement vector

$$\begin{gathered} c_{66}u_+^{(0)} = æ_{ee}^{\bullet}\varphi(z) + ih\partial_{\bar z}y_0, \\ c_{66}u_+^{(2k)} = h\sum_{m=1}^{2n} a_m^{(2k)}\partial_{\bar z}V_m + ih\partial_{\bar z}y_k \quad (k = 1,n), \end{gathered} \tag{2.17}$$

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where $a_m^{(2k)} = 2k_m^{-1} c_m^{(2k)}$, $y_k$ – arbitrary, sufficiently smooth real functions. They must be chosen so that equations (1.13) are satisfied. Therefore, if we introduce in (1.13) the values of the moments (2.7), (2.12), (2.14) and (2.17), we obtain the equalities

$$\partial_{\bar z} \Delta y_0 = 4ih^{-1}\overline{\varphi''(z)}, \tag{2.18}$$

$$\partial_{\bar z} U_k = 0 \quad (k = 1, n), \tag{2.19}$$

where $U_k = X_k + iY_k -$ complex function, the real part of which is a linear combination of metagarmonic functions $V_m$, i.e.

$$X_k = c_0 \sum_{m=1}^{2n} O_m^{(2k)} V_m \quad \left(c_0 = c_{11}^{\bullet} / 2c_{66}h\right), \tag{2.20}$$

and the imaginary part is determined by the formula

$$Y_k = \Delta y_k - \frac{(4k+1)\tilde{c}_{44}}{c_{66}h^2} \sum_{s=1}^{n} \beta_{2s}^{(2k)} y_s \ . \tag{2.21}$$

Here

$$O_m^{(2k)} = a_m^{(2k)} k_m - \frac{(4k+1)}{c_{11}^{\bullet}} \left[ 2c^{\bullet} c_{44} c_m^{(2k-1)} + \tilde{c}_{44} \sum_{s=1}^{n} \beta_{2s}^{(k)} a_m^{(2s)} \right]. \tag{2.22}$$

It is easy to verify that the constants $O_m^{(2k)}$ are identically equal to zero for $\forall k \in [1, n]$. This follows from the fulfillment of equalities (2.6), taking into account the linear independence of meta-harmonic functions $V_m$. According to (2.18), we have

$$y_0 = ih^{-1}\left[ z\overline{\varphi(z)} - \bar{z}\varphi(z) + \overline{\psi_0(z)} - \psi_0(z) \right], \tag{2.23}$$

where $\psi_0(z)$ – arbitrary holomorphic function; equation (2.19) after integration over the variable is reduced to the equality

$$U_k = f_k(z), \tag{2.24}$$

in which arbitrary analytic functions $f_k(z)$ are denoted. It follows from the last equality that the real part $U_k$ should be a harmonic function, and since it is identically equal to zero, then $\mathrm{Re}[f_k(z)] = 0$, therefore, $f_k = ic_k$. Given that the functions $y_k$ are determined up to constant terms, we can set the constants $c_k$ to zero. Thus, from the equalities $\mathrm{Y_k} = 0$ we obtain the system of equations, which we write in the standard form this way

$$\sum_{l=1}^{n} \left( q_{kl} - \delta_{kl} h^2 \Delta \right) y_l = 0 \ (k = 1, n), \tag{2.25}$$

where $\delta_{kl}$ - Kronecker symbol, $q_{kl} = (4k+1)\beta_{2l}^{(k)} \tilde{c}_{44} / c_{66}$.

Consider the characteristic equation

$$\det \left\| q_{kl} - \lambda \delta_{kl} \right\| = 0 \tag{2.26}$$

and assume that it has simple and non-zero roots $\lambda_s$. Then, by the above method, we find functions $y_k$, i.e.

$$y_k = \sum_{s=1}^{n} b_s^{(2k)} w_s \,, \tag{2.27}$$

where $w_s$ – meta-harmonic functions satisfying the equalities

$$\Delta w_s - \lambda_s h^{-2} w_s = 0, \tag{2.28}$$

constants $b_s^{(2k)}$ are determined by algebraic complements of elements of some line of the determinant

$$\left| q_{kl} - \lambda_s \delta_{kl} \right|_{n \times n}.$$

According to formulas (2.23) and (2.27), the moments of the variable $u_+^{(2n)}$ take the form

$$\begin{aligned} & c_{66} u_+^{(0)} = æ^{\bullet} \varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}; \\ & c_{66} u_+^{(2k)} = h \sum_{m=1}^{2n} a_m^{(2k)} \partial_{\bar{z}} V_m + ih \sum_{s=1}^{n} b_s^{(2k)} \partial_{\bar{z}} w_s \,, \end{aligned} \tag{2.29}$$

where $\overline{\psi(z)} = \overline{\psi'_{\bullet}(z)}, \quad æ^{\bullet} = 1 + æ_e^{\bullet}.$

Thus, the values of functions (2.12), (2.14) and (2.29), together with equalities (2.13) and (2.28), constitute the general solution of the system of equations (1.13), (1.14).

## CONCLUSIONS

We applied method of decomposition of unknown functions into Fourier series, in the Legendre exposition of polynomials we took into account equations of equilibrium elasticity of the transversely isotropic plate with initial stresses at mixed conditions on the flat borders. We supposed the normal transference and touch stresses equal to zero. We proposed method of presentation of universal analytic solution of received equations. Found solution allows describing the stress state on the surface of a transversally isotropic plate.

## REFERENCES

[1] Bolotin, V. V. (1970). Variational principles of the theory of elastic stability. In *Problems of the Mechanics of a Deformable Solid*. Leningrad, pp. 83-88 (in Russian).

[2] Brunsev, E. & Robertson, S. (1974). Mindlin type plates with initial stresses. *Rocket technology and astronautics*.

[3] Guz, A. (1996). Complex potentials in problems of the theory of elasticity for bodies with initial stresses. *Prikl. Mechanics*, 32, No. 12, Kyev.

[4] Homa, I. (1998). About the development of mathematical theory of shells with cob loads. Scientific problems of mathematics. *Materials of the International Sciences*, conference. Chernivtsi , Kiev.

[5] Khoma, I. (1996). Equations of the generalized theory of shells with initial stresses. *Int. Appl. Mech.*, 32, №11, Kyev.

[6] Lee P. C. Y., Wang Y. S. & Markenscoff, X. (1975). High-frequency vibration of crystal plates under initial stresses. *J. Acoust. Soc. Amer.,* 57, 95. Doi: https://doi.org/10.1121/1.380406

[7] Levy, E. (1940). On linear elliptic partial differential equations. *Uspekhi Mat. Sciences*, Vol. 8, Moskow.

[8] Vekua, I. N. (1965). Theory of thin shallow shells of variable thickness. *Trudy Tbilis. Mat. Univ. (AM Razmadze)*, 30.

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*Original Paper*

# Knowledge of mathematics at the Slovak University of Agriculture in Nitra in the context of history of planar curves

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**ABSTRACT**

The geometric visualization has been present in the different forms from the earliest recorded time all around us. In our paper we deal with the planar curves, which are an inseparable part of Mathematics, and they are applicable in the everyday practical life. The visualization programs require a user´s experience and skills to know also some significant properties of the planar curves. As there are many types of the planar curves, in the first part we concentrate our attention on the selected groups, which are characterized by the parametric expression. The selected curves can serve as a visual aid in the process of teaching about planar curves and their properties. The second objective of the paper is the analysis of the entrance test of Mathematics. We decided to compare the results of the entrance test in the academic year 2018/2019 and detect if the significant differences exist between the results in the particular study groups. The selection file was represented by the students of the study programs in the first year of study at the bachelor´s degree at the Faculty of Economics and Management of the Slovak University of Agriculture in Nitra. We used the methods of descriptive statistics and statistical hypotheses testing in order to analyze the empiric data.

**KEYWORDS**: mathematics, teaching, visualization, planar curves

**JEL CLASSIFICATION:** I21, C12, B16

## INTRODUCTION

In the antiquity people recognized that the hand sketches are insufficient, thus the scholars of that time began using a compass and ruler, which became the irrecoverable parts in solving geometric and design tasks. In spite of the fact that two thousand years have passed since the times of Euclid, a compass and ruler still remain the basic equipment of each geometrician. Thanks to the historical notes we can elucidate Mathematics and its essential part – geometry to any reader as a part of the everyday life of the human civilization. Along with the

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development of science and information technologies the new possibilities of displaying of geometric objects appeared, which are based on the graphic softwares, using a computer screen, graphic tablet and animations. The improvement of user interface and development of imaging algorithms encourage the origination of tools and programs of drawing curves of any accuracy. The most widespread and used are the softwares like: Mathematica, MatLab, Cabri Geometry, Maple, Geometer's Sketchpad GeoGebra, etc.

## MATERIAL AND METHODS

The planar curves, which are being analyzed in the paper, constitute a part of Mathematics and have the direct application in the practice. Their history is interesting and the university students can find remarkable facts in it. Cycling curves has application in design and art [10]. In the first part of results we introduce the survey of the selected types of planar curves which are applicable in the technical practice. They rank among the category entitled *roulettes,* in particular, there are cycloids, trochoids and spirals. In the theoretical part we indicate the survey of concepts which are associated with their expression in the parametric and polar coordinates.

In the second part of results we focused attention on the analysis of attainments of the students of the Faculty of Economics and Management (FEM) of the Slovak University of Agriculture in Nitra (SUA). Based on our teaching experience we can claim that the students are less interested in studying Mathematics, and the teachers` role is to change this students` approach. We teach the subject of Mathematics in the winter and summer term in the first year of study, entitled Mathematics IA and Mathematics IB, which include 2 lectures and 2 seminars each week in the study programs at FEM SUA. The students of the Faculty of European Studies and Regional Development and the Faculty of Biotechnology and Food Sciences the students study the subject Mathematics containing 1 lecture and 3 seminars per week. At the Technical Faculty the students acquire Mathematics for technicians of 2 lectures and 2 seminars a week. Our results indicate the evaluation of the entrance test at FEM in the academic year 2018/2019. We decided to detect if there are any significant differences between the results of the individual study groups.

## RESULTS AND DISCUSSION

### Brief history and survey of planar curves

The planar curves are most frequently expressed parametrically via the point function of one variable. Its definition follows.

**Definition 1** $I$ is a random interval on the number axis $R$. The delineation $P$ of set $I$ into the plane $E^2$ (for $P$ *holds*: $I \rightarrow E^2$) is called *the point function of one variable.* The set $I$ is *the definition area of this function,* if $P(t)$, $t \in I$, we can understand as the function value $P$in the point $t$.

The curve notation in the plane in ordinates: $P(t) = (x(t), y(t))$, $t \in I$.

The functions $x = x(t)$, $y = y(t)$ are called the coordinate functions of the function $P$, the numeric functions of the variable $t \in I$, or also *the parametric curve equations k,* where $k$ is the set of all points $P(t)$, $t \in I$. The variable $t$ is also called *the parameter* of the point $P(t)$.

Another possibility for the curve delineation in the plane are *polar coordinates*. We convey them in the oriented plane: we can choose the fixed point $O$ (it is usually the beginning of the system of coordinates (0, 0) and the initial half-line originating from this point (it is usually the positive part $x$-axis). Any point of the plane $P$, which is different from $O$, can be written in the form $(r, \theta)$, where $r > 0$ is the abscissa length $OP$ and $\theta$ is the oriented angle angled by the initial half-line and the half-line $OP$. The point, expressed by the polar coordinates $(r, \theta)$, has the Cartesian coordinates: $x = r \cos \theta$, $y = r \sin \theta$.

Vice versa, the point of the coordinates $(x, y)$ (holds $x \neq 0$, $y \neq 0$) can be represented by so called polar coordinates, where $r = \sqrt{x^2 + y^2}$, $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$, $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$.

**Definition 2** We state that the point $P_0$ is *the limit* of the point function $P$ in the point $t_0 \in I$, if for each $\varepsilon > 0$ exists $\delta > 0$ such that for all numbers $t$ holds: $|t - t_0| < \delta,\ t \neq t_0 \Rightarrow |P(t) - P_0| < \varepsilon$.
Notation: $\lim\limits_{t \to t_0} P(t) = P_0$.
The limit of function $P(t) = (x(t), y(t))$ is thus the *point* $\lim\limits_{t \to t_0} P(t) = \left( \lim\limits_{t \to t_0} x(t), \lim\limits_{t \to t_0} y(t) \right)$.

**Definition 3** We claim that the vector $P'(t_0)$ is the *derivation* of the point function $P$ in the point $t_0 \in$ I, if there exists $\lim\limits_{h \to 0} \frac{P(t_0 + h) - P(t_0)}{h} = P'(t_0)$. The derivation of function $P(t) = (x(t), y(t))$ is thus the *vector* $P'(t_0) = (x'(t_0), y'(t_0))$.

**Definition 4** The parametric expression of curve meets the requirement of *smoothness,* if the derivations of all orders and *regularity* exist, then $P'(t) \neq 0$ holds for all $t$. Thus the physically interpreted condition of regularity means that the moving point has still the nonzero speed.

The ancient Greeks were dealing with the problem of introduction of the *tangent curve*. At that time the definition of conic section tangent was known and the definition of the tangent of general curve was desired. However, this problem was not related only to geometry, its solution was required also by mechanics, elasticity, strength or optics. The solution was discovered by the introduction of the differential count at the end of the 17th century.

The definitions of the tangent line, the touch vector, and the significant points on the curve are given in the next section.

**Definition 5** Let the curve $k$ is determined by parametrization $P = P(t)$, $t \in I$. Let´s select the point $P(t_0)$ on the curve. Then the vector $P'(t_0) = (x'(t_0), y'(t_0))$ is called *the touch vector of curve k in the point $P(t_0)$* and the straight line, which is determined by the point $P(t_0)$ and directional vector $P'(t_0)$, is called *the tangent of curve k in point $P(t_0)$.*

In other words, the tangent of curve in the point $P(t_0)$ is the limit position of the secant connecting the points $P(t_0)$ and $P(t_0 + h)$, therefore the tangent is closest to the curve out of all straight lines intersecting the given point of the curve.

**Definition 6** *The curve peak* is no inflectional point $P(t)$, where holds $k(t) \neq 0$ and $k'(t) = 0$.

**Definition 7** The point $P(t_0)$ of curve $P = P(t)$, $t \in I$, is called *the singular point,* if $P'(t_0) = 0$.

**Definition 8** The curve *m,* which intersects upright all tangents of the planar curve *k,* is called the *curve evolvent.* The planar curve *k* is called *evolute of the curve m,* for which the planar curve *m* is evolvent. It holds that the tangent of an evolute is the normal line of evolvent.

Let´s imagine the plane where two curves are given. Rolling of one curve on the other one causes the smooth motion in the plane. We call *roulette* the trajectory of the firmly selected point in the plane of the rolling curve. In other words, it is the trajectory of the point which is placed on the curve or outside the curve, rolling without sliding along the second curve, and this point is firmly connected with the rolling curve. The basis is represented by two curves, one is static and the second one is rolling along the first one. The form of the final curve is influenced by the form of the given curves and the position of the monitored point with the regard of the moving curve.

The Table 1 indicates the best known roulettes and requirements for parameters $a$, $b$, $c$.

**Table 1** Best known roulettes

| static curve | rolling curve | relation between $a, b, c$ | roulette |
|---|---|---|---|
| straight line | circular line with radius $a$ | $c < a$ | shortened cycloid |
| straight line | circular line with radius $a$ | $c = a$ | cycloid |
| straight line | circular line with radius $a$ | $c > a$ | elongated cycloid |
| circular line | inner circular line with radius $b$ | $c < b$ | shortened hypocycloid |
| circular line | inner circular line with radius $b$ | $c = b$ | hypocycloid |
| circular line | inner circular line with radius $b$ | $c > b$ | elongated hypotrochoid |
| circular line | inner circular line with radius $b$ | $c = a - b$ | rose |
| circular line | outer circular line with radius $b$ | $c < b$ | shortened epitrochoid |
| circular line | outer circular line with radius $b$ | $c = b$ | epicycloid |
| circular line | outer circular line with radius $b$ | $c > b$ | elongated epitrochoid |

Source: own processing

**Definition 9** In general, we define *the cycloid* as the trajectory of the firmly selected point in the plane of circular line which is rolling along the straight line. The parametric expression is following: $x = at - c \sin t$, $y = a - c \cos t$, $t \in (-\infty, +\infty)$. We can divide the general cycloids into three groups: cycloid if $c = a$, shortened cycloid if $c < a$, elongated cycloid if $c > a$. In the case of cycloid, the drawing point is situated on the rolling circular line, thus holds $c = a$.

The parametric equations are the following: $x = a(t - \sin t)$, $y = a(1 - \cos t)$, for $t \in (-\infty, +\infty)$.

The cycloid "upside down" is the brachistochron curve (from Greek *brachistos* means *the shortest* and *chronos* means *time*). The length of cycloid arc is the quadruple of diameter of the rolling circular line. Thus, this lenght is independent on the number $\pi$ . We know from the history that the ancient Greeks were familiar with a cycloid. In the medieval times Mikuláš Kuzánsky and Marin Mersenne studied it as the first scientists. However, only Galileo gave it its name in 1599. Desargues suggested the usage of cycloids in the production of cogged wheels in 1630. In 1634 G. P. de Roberval calculated the area under the cycloid arc. In 1658 Sir Christopher Wren calculated the length of the cycloid arc.

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The different situation occurs when the circles touch from outside, i.e. they are situated in the opposite half-plane limited by the tangent in the point of tangency. In this case, the point which is placed in the plane of the circle rolling outside the other circle (regardless which radius is longer), forms *epitrochoid.* This curve was probably studied by Dürer for the first time and he described the method of its drawing in the publication *Underweysung der Messung* (1525). It was reinvented by Étienne Pascal, the father of Blaise Pascal, This curve was named by Gilles-Personne Roberval in 1653 as "limaçon", originated from the Latin *limax*, which means “snail“ (limacon) [1].

**Definition 10** The cardioid is a special example of epicycloid as well as Pascal limacon. The static and rolling circles have the same radius, $a = b$, thus also $c = a = b$. In this way the cardioid is formed as a classic epitrochoid.

Thanks to the property of the double formation, the cardioid can originate also as a hypocycloid: the monitored point is situated on the circle with radius $2a$, which is rolling “inside“ the static circle with radius $a$.
The parametric expression of the cardioid:
$x = a(2\cos t - \cos(2t)),\ \ y = a(2\sin t - \sin(2t)),\ t \in \langle 0, 2\pi \rangle.$
The notation of polar coordinates: $r = 2a(1+\cos(\theta))$
The equations of cardioid: $(x^2 + y^2 - 2ax)^2 = 4a^2(x^2 + y^2)$
The evolute of cardioid (Figure 1) is the cardioid similar to the original cardioid [2].
In the Figure 2 Archimede`s spiral is delineated.

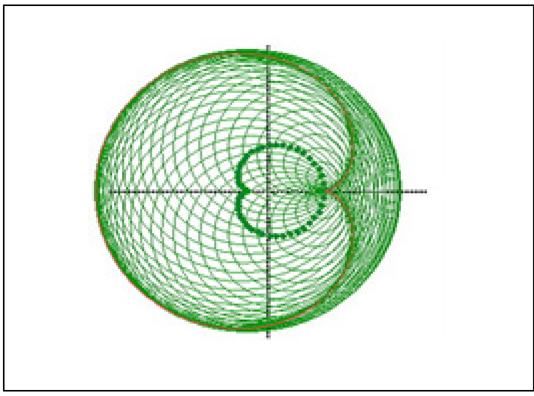

Figure 1 Evolute of cardioid
Source: own

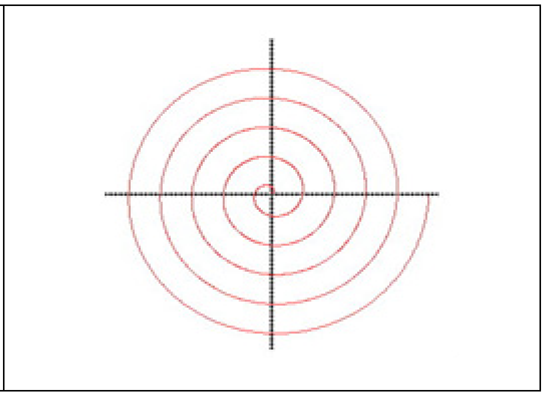

Figure 2 Archimede`s spiral
Source: own

Roemer (1674) studied a cardioid during his research of the most suitable forms of cogged wheels. The name cardioid, which means “in the shape of a heart“ (derived from Greek *cardi* – heart), was used for the first time by de Castillon in the publication *Philosophical Transactions of the Royal Society* in 1741. However, La Hire assumed the right of the cardioid discovery and he calculated its length in 1708. Nevertheless, other mathematicians had probably studied it earlier.

In the historical specialized source-books there is still the space for the completion and further enlargement of the database of visualized curves along with their properties by the modern graphic softwares. A lot of interesting information about planar curves and their properties can be found in traditional literature and electronic sources (e.g. [3], [4], [9], [11]).

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**Analysis of entrance test**

If we want to analyze the mathematical preparedness of the first year students, who are enrolled at SUA, we have to assume that the students have a good command of the secondary school Mathematics. However, in the study groups there are also those students who adapt more slowly to the pedagogic effort of a teacher. The students, who did not have to take the school leaving exam in Mathematics, studied this subject only in the second or third year of study. The students, who passed the school leaving exam in Mathematics, are prepared better for the study which is closely associated with their increased interest in this subject. Therefore, a teacher has to concentrate the pedagogic activity on better students` understanding of the principles of Mathematics ([5], [8]); then s/he can explain new teaching materials. It is necessary to focus special attention predominantly on the group of the average students, to use visualization and the personal approach in the process of explanation the particular subject units, to introduce applications in the individual fields (economy, technics).

In the academic year 2018/2019 we pursued the pedagogic survey which was oriented at the evaluation of the entrance attainments of Mathematics at FEM SUA in Nitra. The Table 2 indicates the questions and tasks of the entrance test carried out in three study groups of the following study programs: Business Management, Accounting and Environmental Economics and Management. 90 students of the first year of study participated in this survey. The tasks were related to the properties and graphs of the elementary functions with one real variable, i.e. the study material covered at secondary schools. The particular tasks (task 1 – task 5) were evaluated by 4 point for the correct solution, in total the maximum number of points was 20 for all correct tasks.

Table 2 Entrance test in Mathematics

| **Name and surname** | | **Date/study group** | |
|---|---|---|---|
| 1. How many years did you study Mathematics at secondary school? | | | |
| 2. Which mark did you achieve in Mathematics at secondary school? | | Average mark: 2.65 | |
| 3. Did you take school leaving exam in Mathematics? If yes, state the mark. | | Number of secondary school leaving students: 3 out of 90 | |
| | Group A | Group B | Group C |
| Task 1: Sketch the graph of a linear function | $y = 3x + 6$ | $y = 2x - 4$ | $y = 4x - 8$ |
| Task 2: Write the properties of this function: odd, even, limitations, $D(f); H(f)$ | $y = 3x + 6$ | $y = 2x - 4$ | $y = 4x - 8$ |
| Task 3: Find out for which numbers the given fraction has meaning | $\frac{3x+2}{x^2-9}$ | $\frac{x+2}{x^2-4}$ | $\frac{5x-2}{x^2-1}$ |
| Task 4: Sketch the graph of a quadratic function | $y = -x^2 + 1$ | $y = x^2 - 1$ | $y = -x^2 + x$ |
| Task 5: Describe the properties of the given function | $y = e^x$ | $y = 10^x$ | $y = lnx$ |

Source: own processing

The Figure 3 indicates the summary of results of the particular groups A, B, C. The students were assigned the sequence number and we calculated the average number of the achieved points for the individual tasks and also the average number of points of the whole study group. It is apparent that the group C received the best evaluation in the tasks and the point average is best (7.65).

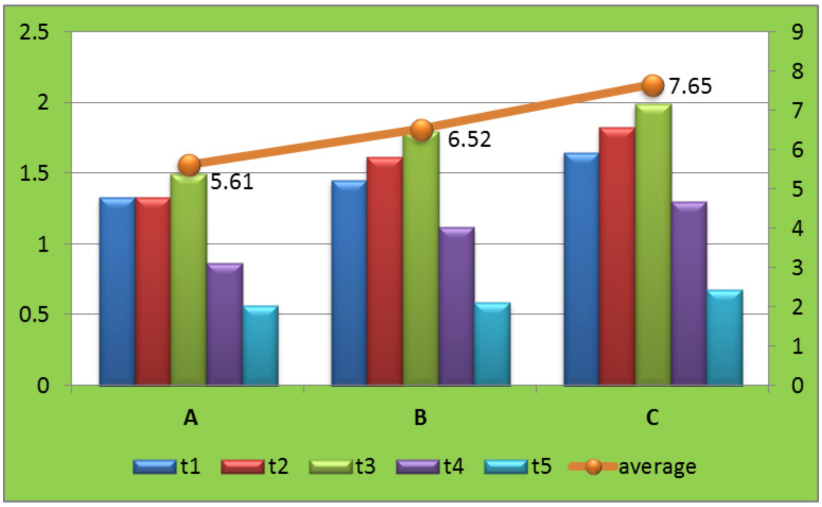


Figure 3 Entrance test in Mathematics – average number of points achieved for task solving
Source: pedagogic survey, own processing

In the course of the winter term the students sit for the progress test which is a part of the final evaluation at the exam in Mathematics IA. We were interested in the comparison of the point evaluation for the same type of tasks of the entrance test and progress test. We chose the task related to the graph of linear function, which is labelled as t1 in the entrance test and t12 in the progress test. Similarly, the results of the task related to the graph of quadratic function (in the entrance test t4) were compared with the results of the progress test (t42). The graphic delineation in the Fig. 4 shows the improvement in both tasks in all study groups A, B, C.

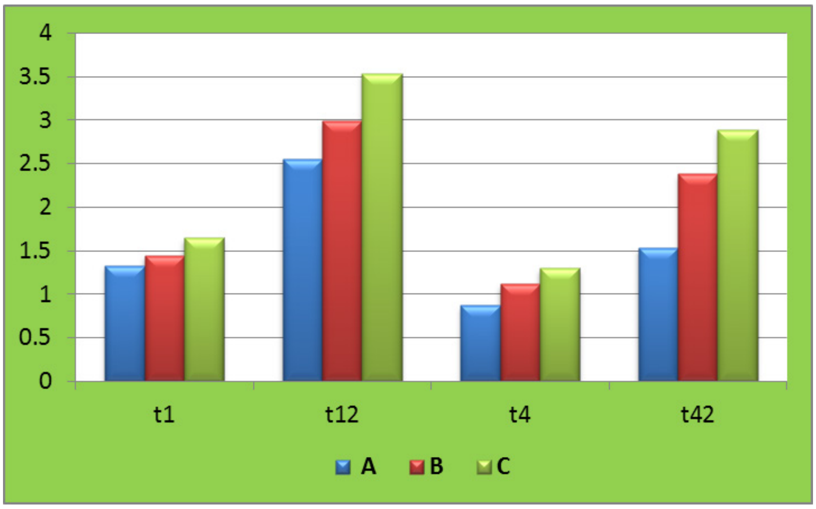


Figure 4 Comparison of points of selected tasks: entrance test and progress test
Source: pedagogic survey, own processing

The theme of the planar curves is being taught to a limited extent in the present study programs. The students study the studying material within the function with one variable about a linear and quadratic function ([6], [7]). These functions are expressed explicitly in the tasks and the students´ role is to depict the graph of the given function in the plane and describe its properties.

Table 3 Results of t-test

| Task | t1A | t1B | t1A | t1C | t1B | t1C |
|---|---|---|---|---|---|---|
| Observations | 30 | 30 | 30 | 30 | 30 | 30 |
| df | 58 | | 58 | | 58 | |
| t Stat | -1.85 | | -3.23 | | -1.4 | |
| P(T<=t) one-tail | 0.035 | | 0.001 | | 0.083 | |
| t Critical one-tail | 1.67 | | 1.67 | | 1.67 | |
| P(T<=t) two-tail | 0.07 | | 0.002* | | 0.166 | |
| t Critical two-tail | 2.001 | | 2.001 | | 2.001 | |

Source: own calculations

We evaluated the success of the students in solving the problem of the graph of the linear function using a two-sample t-test. We determined the significance of differences between study groups A, B, C at the selected significance level alpha = 0.05. The calculations were made using MS Excel tools. The results in Table 3 show that the differences between Group A and Group B are not significant; for tasks t1A and t1B, $p = 0.07$. In finding the differences between tasks t1A and t1C, we obtained the result $p = 0.002$, which indicates significant differences. Therefore, at the chosen level of significance, we reject the null hypothesis and it is true that the differences in task solving are significant between groups A and C. Finally, when testing the tasks t1B and t1C, we found that there were no significant differences between group B and C, $p = 0.166$.

The special plane curves, which involve also the roulettes, allow to create the geometric configurations with the aesthetic effect, like: windows of stained glasses, mosaics, pavements, decorative motives in stripes with repetition or changes, patterns on the ceramic surfaces, sewn or printed patterns on the different textile surfaces and others [10].

## CONCLUSION

The historical context of the mathematic and geometric themes is interesting for the students and it also increases their motivation to study Mathematics and Geometry. The present graphic softwares provide the users with the opportunities to delineate fast the planar curves. These types of software were not available for mathematicians and geometricians in the past.

The objective of teaching Mathematics at SUA in Nitra is:
- acquisition of mathematic knowledge of higher mathematics,
- command of the procedures of tasks solving in the mathematic and applied forms,
- development of the students´ abilities to formulate mathematically and solve the problem tasks by using the precise and clear mathematic stylization,
- improvement and support of the students´ divergent thinking by selecting the appropriate tasks.

The evaluation of the students´ attainments via the mathematic test is the source of the knowledge about the level of acquisition of the taught problems. The students should demonstrate the skills of the acquired mathematic curriculum by the independent solving of the testing tasks. Based on the comparison of results of the entrance and final tests, we detected the positive improvement in the point evaluation of the selected tasks.

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**REFERENCES**

[1] Božek, M. (2003 - 2004). *Učebné texty k predmetom Geometria 1, 2*. Nepublikované, 2003 – 2004 (in Slovak).

[2] Hejný, M., Zaťko, V. & Kršňák, P. (1985). *Geometria 1*. 1. vyd. Bratislava: Slovenské pedagogické nakladateľstvo, 1985 (in Slovak).

[3] Kobayashi, I., Sato, K., et al. (1996 – 2003). *Math Education and Technology* [online]. International Education Software, 1996 – 2003. Retrieved 2007-03-02 from http://www.ies.co.jp/math/java/calc/cycle_ang/cycle_ang.html.

[4] O'Connor, J.J. & Robertson, E.F. (2007). *The MacTutor History of Mathematics archive* [online]. Scotland: University of St Andrews. Retrieved 2006-10-11 from http://www-groups.dcs.st-and.ac.uk/~history/index.html

[5] Országhová, D. (2018). E-learning approach in mathematical training of future economists. In *E-learning and Smart Learning Environment for the Preparation of New Generation Specialists*. Vol. 10, 664 p. (pp. 427-442).

[6] Országhová, D. (2019). Future managers knowledge and competences in the context of university mathematics education. In *Modern technologies in education*. 1. ed. 495 p. Opole: Wyższa Szkoła Zarządzania i Administracji w Opolu, pp. 84-105.

[7] Országhová, D. (2017). The application of computational tools of it in mathematical tasks. In *Effective Development of Teachers' Skills in the Area of ICT*. 1. ed. 498 p. Katowice: University of Silesia, pp. 438-448.

[8] Országhová, D., Hornyák Gregáňová, R. & Kecskés, N. (2018). Information technology tools in mathematical topics. In *Edukacja - technika - informatyka w budowaniu lepszej przyszłości*. Radom: Wydawnictwo Uniwersytetu Technologiczno-Humanistycznego w Radomiu, pp. 51-58.

[9] Proctor, R. A. (1878). *The geometry of cycloids* [online]. London: Longmans, Green, and co. Retrieved 2007-15-01 from http://historical.library.cornell.edu/cgibin/cul.math/docviewer?did=02260001&view=75&frames=0&seq=9

[10] Raicu, L. & Rugescu, A.-M. M. (2015). Cycling curves and their applications. *Journal of Industrial Design and Engineering Graphics*, 10(2), 55-58.

[11] Weisstein, E.W. (1997). *Mathworld* – A Wolfram Web Resource [online]. Wolfram Research, Inc. Retrieved 2006-10-12 from http://mathworld.wolfram.com

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***Original Paper***

# Comparison of exam results in Mathematics at Faculty of Economics and Management, Slovak University of Agriculture in Nitra

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**ABSTRACT**

Mathematical education at the Slovak University of Agriculture in Nitra plays a very important role in university education because the subject of mathematics is a part of other subjects taught at the bachelor and master study programs. The content of Mathematical subjects differs from faculty to faculty of the Slovak University of Agriculture in Nitra because of their different orientation. At the Faculty of Economics and Management, Slovak University of Agriculture in Nitra, the basic course of higher mathematics together with example of their applications in economics is taught in the first year of the bachelor study. Subjects aimed at Economics and Finances which also comprise mathematical apparatus, are taught in the higher years of study. The aim of this paper is to analyze and evaluate the students' results in the subjects of Mathematics IA and Mathematics IB at the Slovak University of Agriculture in Nitra from 2015 to 2019. Feedback is an important part of an educational process because the analysis of the study outcomes enables teachers to evaluate its quality. The analysis of study results in selected mathematical subjects at the Faculty of Economics and Management, Slovak University of Agriculture in Nitra is carried out by selected methods of mathematical statistics. We found a long-term decline in the average mark in the research, confirming the decreasing level of knowledge and statistically significant difference in the values compared between the observed value and the long-term average. The hypothesis was confirmed by testing.

**KEYWORDS**: mathematics, study outcomes, mathematical education, mathematical subjects

**JEL CLASSIFICATION:** I 21, C12

## INTRODUCTION

The teaching of Mathematics has a long tradition at the Faculty of Economics and Management (FEM), Slovak University of Agriculture (SUA) in Nitra. Mandatory subjects of Mathematics provide the apparatus and methods applied in the scientific activities in various areas. As noted by Horská et al. [4], universities search for possibilities to attract students, to offer them an education of a high quality and to bring value added and differentiation to the university education. Snellman [10] claims, that "institutions of higher education are facing increasing pressures owing to the accelerating competition brought about by the globalization trend of higher education". The quality of higher education and the increasing

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competitiveness of universities are subject to continuous updating of the academic content as a result of interaction with the requirements for university graduates in the labour market [6]. University study programs reflect the requirements of the labour market that requires creative, flexible and quick minded people [8].

Carmona-García et al. says, that Mathematics as a knowledge field of the exact sciences, use specialized language, laws, properties, theorems, axioms, methods, and the results of the operations there of can be expressed in numerical, functional and graphical forms, which are commonly used in the academic field from arithmetic used in elementary education to higher mathematics used in Academic Programs related to engineering [1].

Many authors deal with the evaluation of study results of different subject taught at FEM SUA in Nitre. Hornyák Gregáňová, Pietriková [2] dealt with the evaluation of the study results of mandatory subjects of Mathematics I, A, Mathematics II, B and Statistics. The level of knowledge has been decreasing in recent years, the curriculum of mathematical subjects. We I suppose that the average mark has worsened in the compared subjects.

Országhová, Horváthová [7] dealt with the evaluation of the study results of subjects of Mathematics and English. Országhová et al. says, that university study programs reflect the requirements of the labour market that requires creative, flexible and quick minded people [8].

The subjects Mathematics IA and Mathematics IB are the core of the bachelor's degree at FEM SUA in Nitra. These are subjects of which the knowledge is further used in vocational subjects taught at FEM SUA in Nitra as part of bachelor and engineering studies. For these reasons, we consider it necessary to analyze the results of the examinations of these subjects to give an idea of how the students have mastered these subjects [3].

## MATERIAL AND METHODS

All first year students studying at the Faculty of Economics and Management (Slovak University of Agriculture in Nitra, later SUA) of the selected accredited study programs (7 study programs together) at the bachelor’s degree of study were included into the statistical sample (Table 1). The subjects Mathematics IA and Mathematics IB form the basis of all selected study programs studied at the bachelor’s degree at FEM SUA in Nitra. The subject content of Mathematics IA is: Function of one real variable, Derivative of a function of one real variable and Function of two real variables. Mathematics IB covers topics: Indefinite integral, Definite integral, Linear algebra and Probability theory. Basic knowledge and skills obtained from the compared subjects are further developed in the subsequently taught specialized subjects at FEM SUA in Nitra within both, the bachelor and engineering degree of study. The exam results of earlier mentioned subjects reflect the success of students in the given subjects. Those data were drawn from University information system (UIS) in the academic years from 2015/16 to 2018/19 and processed by MS Excel.

The main task of our research was to find out whether there are statistically significant differences achieved in students' assessment in the mentioned subjects depending on the study program (SP). The theoretical sources for our paper were the publications and professional academic papers which deal with the educational research and with the use of the statistical methods in this research [5] and [9].

Table 1 List of offered Saps at FEM SUA and their determination

| | | | |
|---|---|---|---|
| **EKP** | Company economics | **MAP** | Company management |
| **EMA** | Economics and management of agro sector | **OBP** | Commercial entrepreneurship |
| **MPA** | International business with agrarian commodities | **UCT** | Accounting |
| **IBA** | International business with agrarian commodities (the study program in the English language) | | |

Source: authors

Another source of the material was the experience and knowledge from teaching of subjects Mathematics IA and Mathematics IB in the first year of the undergraduate study at SUA in Nitra. Study results of mandatory subjects of Mathematics IA, Mathematics IB were assessed using the standard statistical methods.

We have verified the assumptions and used the test characteristic $T = \dfrac{\overline{X} - \mu_0}{\dfrac{\tilde{S}}{\sqrt{n}}}$

with a critical value $W_\alpha = \{t; t \geq t_{1-\alpha}\}$. The program Microsoft Excel 2017 was used for realization of calculations and determination of critical values.

## RESULTS AND DISCUSSION

The article compares the exam results from the subject Mathematics IA (Math IA) and Mathematics IB (Math IB) aimed at finding out whether are statistically significant differences in achieved students' assessment in individual academic years at FEM SUA. We used the exam results from the subjects of Mathematics IA and Mathematics IB in the academic years 2015/16 to 2018/19. Exam results from individual subjects were assessed by a standard scale ECTS A(1), B(1,5), C(2), D(2,5), E(3), FX(4). Table 2 presents the assessment of student's study results in individual subjects in research in academic years 2015/16 to 2018/19.

Table 2 Overall students' evaluation results

| | **2015/16 Math IA** | **2015/16 Math IB** | **2016/17 Math IA** | **2016/17 Math IB** | **2017/18 Math IA** | **2017/18 Math IB** | **2018/19 Math IA** | **2018/19 Math IB** |
|---|---|---|---|---|---|---|---|---|
| **A** | 36 | 31 | 34 | 28 | 37 | 37 | 32 | 36 |
| **B** | 51 | 39 | 45 | 37 | 35 | 18 | 30 | 31 |
| **C** | 92 | 70 | 61 | 57 | 56 | 48 | 54 | 55 |
| **D** | 80 | 73 | 105 | 84 | 75 | 54 | 54 | 58 |
| **E** | 162 | 194 | 178 | 171 | 109 | 133 | 112 | 123 |
| **FX** | 11 | 10 | 10 | 4 | 13 | 6 | 20 | 21 |
| **average** | **2.38** | **2.48** | **2.36** | **2.46** | **2.36** | **2.43** | **2.44** | **2.44** |

Source: authors

Table 2 shows the overall student success rate in individual study programs of the compared subjects. The average of grades varies over the years studied. We would like to draw your

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Math Educ Res Appl, 2019(5), 2, 78-83

attention to the fact that when comparing students' learning outcomes in the last three semesters of our research we found that the average of their grades was almost the same.

Overall, the highest average was reached by students in the subject Mathematics IB in the academic year 2015/16 (2.48), but it was not the highest success rate in the subject in terms of passing the exam.

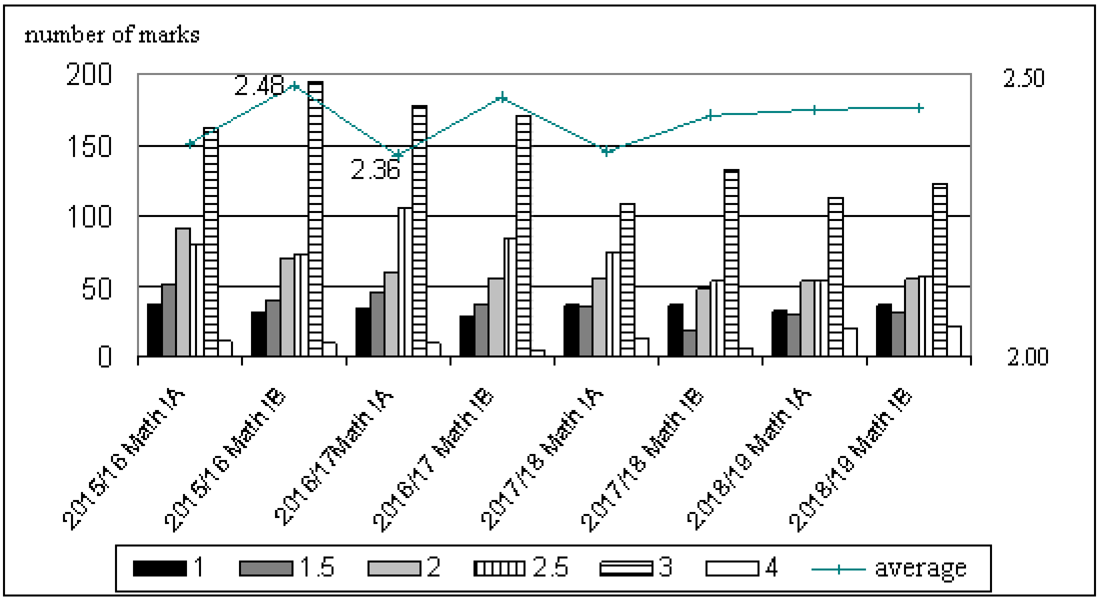


Figure 1 Comparison of students grades averages in selected years in terms of mean and median assessment
Source: authors

If we assess the students success rate from the point of the average mark (Table 3, Figure1) we have found out that the average mark has worsened in the compared subjects. There are big differences between compared subjects concerning the students' success rate.

Table 3 shows that the best average does not mean the highest success rate in the subject. The study results deteriorated last year and the number of students with the "FX" mark increased significantly (6.62% and 6.48%). In terms of students' success rate in individual subjects, the best result were obtained by students in the subject Math IB in the academic year 2015-16 and in the subject Math IA they were only slightly worse. We take into account that the majority of these students are graduates of technical secondary schools where mathematics is not considered the main subject. The median assessment reached the value of 2.5 and modus assessment reached the value 3 in all academic years. In the academic year 2018/19 the average of marks was the same in subjects Math IA and Math IB.

We calculated the value of p for all comparisons, which makes it possible to decide not to reject the null hypothesis for different levels of significance (Table 4).
Based on the before mentioned we suppose the existence of differences in achieved assessment of compared subjects between students. Examination was therefore focused on the difference determination in knowledge assessment in a regular exam term arising between students of individual academic years.

Table 3 Comparison of students' success rate in different subjects

| | 2015/16 Math IA | 2015/16 Math IB | 2016/17 Math IA | 2016/17 Math IB | 2017/18 Math IA | 2017/18 Math IB | 2018/19 Math IA | 2018/19 Math IB |
|---|---|---|---|---|---|---|---|---|
| A | 8.33% | 7.43% | 7.85% | 7.35% | 11.38% | 12.50% | 10.60% | 11.11% |
| B | 11.81% | 9.35% | 10.39% | 9.71% | 10.77% | 6.08% | 9.93% | 9.57% |
| C | 2130% | 16.79% | 14.09% | 14.96% | 17.23% | 16.22% | 17.88% | 16.98% |
| D | 18.52% | 17.51% | 24.25% | 22.05% | 23.08% | 18.24% | 17.88% | 17.90% |
| E | 37.50% | 46.52% | 41.11% | 44.88% | 33.54% | 44.93% | 37.09% | 37.96% |
| FX | 2.55% | 2.40% | 2.31% | 1.05% | 4.00% | 2.03% | 6.62% | 6.48% |
| Total | 432 | 417 | 433 | 381 | 325 | 296 | 302 | 324 |
| average | 2.38 | 2.48 | 2.36 | 2.46 | 2.36 | 2.43 | 2.44 | 2.44 |
| students' success rate | 97.45 | 97.60 | 97.69 | 98.95 | 96.00 | 97.97 | 93.38 | 93.52 |

Source: authors

Table 4 Values of testing statistics

| | 2015/16 Math IA | 2015/16 Math IB | 2016/17 Math IA | 2016/17 Math IB | 2017/18 Math IA | 2017/18 Math IB | 2018/19 Math IA | 2018/19 Math IB |
|---|---|---|---|---|---|---|---|---|
| Value of testing statistics | 1.985 * | 4.,747 ** | 4.207 ** | 4.117 ** | 1.300 | 2.527* | 2.818 ** | 2.764 ** |
| p-value | 2.36 E-02 | 1.03 E-06 | 1.30 E-05 | 1.92 E-05 | 9.68 E-02 | 5.75 E-03 | 2.42 E-03 | 2.85 E-03 |
| | * statistically significant difference | | | | ** statistically highly significant difference | | | |

Source: authors

## CONCLUSIONS

The comparison of students' results at the examination in Mathematics IA and IB at FEM SUA in Nitra confirmed the differences in exam results. Surprisingly, the best students′ success rate were achieved by students in the subject of Mathematics IB in the academic year 2016/17 and the best average of marks in the academic year 2015/16 in the subject Mathematics IB. Regarding the character of compared subjects where the majority of students were assessed by the mark “E”, but the grade “A” was achieved only in 7.43% and 7.35% of students respectively. In the following years, the average of the grades was worse, but the grade “A” was more common than in previous years. In case of arranging exam results from the poorest to the best, the order of subjects is as follows: Mathematics IB, Mathematics IA. We found a statistically significant difference in the values compared between the observed value and the long-term average.

In the last two academic years, the average of exam grades were balanced, teachers tried to change students' attitudes towards mathematics by introducing new methods in teaching terms:
- illustrating the introduction of new concepts by illustrative examples,
- elaborating new concepts,
- determining the relationship between concepts by theoretical and practical solutions tasks,
- drawing students attention to the correct solutions,
- using the knowledge of mathematical logic in working with concepts, vetoes and evidence.

By using appropriate teaching methods, the learning process can be improved and the students' level of knowledge improved, too.

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## REFERENCES

[1] Carmona-García, J.A., Garcia-Serrano, L.A., Zavala-Martínez, D. & Gutiérrez-Yurrita, P. J. (2017). Mathematical playful with MAPLE and MATHLAB in solving differential equations to determine pollutant emission profiles. *ICERI 2017 - proceedings from conference ICERI 2017.* Valencia: IATED, pp. 6692-6698.

[2] Hornyák Gregáňová, R. & Pietriková, M. (2016). Comparison of study results in selected subjects depending on study program in Faculty of economics and management, Slovak University of Agriculture in Nitra. *Mathematics in education, research and applications,* 2(1), 22-28. doi: http://dx.doi.org/10.15414/meraa.2016.02.01.22-28

[3] Hornyák Gregáňová, R., Pietriková, M. & Országhová, D. (2015). Analýza hodnotenia výsledkov skúšok z matematiky a štatistiky na FEM SPU v Nitre (The analysis of exams outcomes of mathematics and statistics at FEM SUA in Nitra). *Forum statisticum Slovacum,* 11, 26-30.

[4] Horská, E., Ubrežiová, I. & Palková, Z. (2015). Quality and value related aspects of the higher education: a case of the Slovak University of Agriculture in Nitra. *Development of Public Accreditation of Agricultural programs in Russia (PACAgro)* (543902-TEMPUS-1-2013-1-SK-TEMPUS-SMGR). Retrieved October 10, 2019 from http://spbgau.ru/files/nid/3515/sbornik_tempus_2015.pdf

[5] Matejková, E., Pietriková, M. & Poláková, Z. (2013). Praktikum zo Štatistiky A. Nitra: SPU v Nitre.

[6] Matušek, V., Drábeková, J., Országhová, D. & Farkašová, M. (2016). Mathematical competences as a part of educational objectives in economical and technical study programs. *The agri-food value chain: challenges for natural resources management and society : proceedings from conference International scientific days 2016.* Nitra : Slovak University of Agriculture. 628-633. doi: http://dx.doi.org/10.15414/isd2016.s8.08

[7] Országhová, D. & Horváthová, J. (2015). Pedagogical and statistical analysis of students´ mathematical and language competences. *Mathematics in education, research and applications.* 1(1), 28-35. doi: http://dx.doi.org/10.15414/meraa.2015.01.01.28-35

[8] Országhová, D., Kozelová, D., Hornyák Gregáňová, R., Baraníková, H. & Vollmannová, A. (2016). Mathematics study outcomes as an indicator of the quality of the university education. *The agri-food value chain: challenges for natural resources management and society: proceedings from conference International scientific days 2016.* Nitra: Slovak University of Agriculture, 652-658. doi: http://dx.doi.org/10.15414/isd2016.s8.11

[9] Pacáková, V. (2009). *Štatistická indukcia pre ekonómov a manažérov*. Bratislava: Iura Edition.

[10] Snellman, L. (2017). Globalization of education: role of education export. *ICERI 2017 - proceedings from conference ICERI 2017.* Valencia: IATED, pp. 6099-6110.

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***Original Paper***

# Economics of Facility for seniors in Gabčíkovo

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**ABSTRACT**

Current issues of social care include conditions and options of social services in facilities for seniors in the context of financial and economic situation in the Slovak Republic. The main objective of this paper is the analysis of providing social services in the Facility for seniors in Gabčíkovo. First part of the research data were obtained via questionnaire survey as well as from interview with the facility director; second data source was created from internal documents, annual reports and financial statements of the facility. Then we analyzed the development of economic indicators over the last ten years used quantitative methods and descriptive statistics. Based on results of economic analysis and seniors opinions we stated some proposals for improvements of situation in social facilities and in dealing with the lack of places in them.

**KEYWORDS**: social facility, funding, economic efficiency, level of pensions, structure of resources and costs

**JEL CLASSIFICATION:** D20, D40, M10

## INTRODUCTION

In our society every human being goes through the process of aging and several natural periods of life. The last period of life is the age of the old which can vary from person to person. Mostly, however, it is accompanied by waning strength and the appearance of various serious diseases. It is often the period of life when people lose their ability to take care of themselves and they are dependent on family or state support. If a person is dependent on constant care, s/he must go to an institutional care. There are different types of facilities that provide this care as a form of social service. In this paper we focus on one facility which provides year-round care for seniors. Anyway, this task is a combination of economic, social and health issues. We analyze the economic functioning of facility for seniors, explain the principle of financing, the share of clients, the state and other sources, and find out the availability of vacancies in a particular facility. Obtained results could be used in the next research for the comparison of such facilities in different regions of Slovakia.

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Along with increasing of the aging population and the number of elderly facilities, the existing unsustainable patterns of building operations have caused a series of space-related problems such as space waste, increasing operational costs and poor living environment [8].

According to Radičová [5], the state uses public social security to deal with the consequences of social events and risks and to guarantee social security. State intervention instruments are specified by the state social policy (employment policy, family policy and social security policy) in the context of social solidarity at national level, with the emphasis on the social justice effects realized through the allocation of resources.

The system of social security is, according to Rievajová [6], a set of legal, financial and organizational tools and measures aimed at compensating the adverse financial and social consequences of various life situations and events threatening recognized social rights or preventing such situations. Gejdošová [2] defines social services as a special set of activities aimed at satisfying the individual needs of the social services, as well as collective needs, which are done in a different way not as the transfer of material goods.

According to § 8 part 1 of the Act No. 448/2008 Coll. on Social Services[9], the Municipality and the Higher Territorial Unit, in the scope of its competence, ensures the availability of the social service for a person who is dependent on it and the right to choose the social service under the conditions stipulated by the law. On the territory of the Slovak Republic, according to the Social Services Act, social service can be provided in three forms:

1. Outpatient social services provided to a person who is attending, accompanying or being transported to the place of social service provision. The place of providing outpatient social services may also be a facility.
2. Field social service that is provided to a person in his/her natural social environment.It can also be provided through field programs designed to prevent the social exclusion of persons, families and communities in a socially disadvantaged situation.
3. The social welfare service provides accommodation, either as a year-round or as a weekly social service.

Under Section 71 of the Social Services Act No. 448/2008 Coll. [9], social services provided by a public social service provider can be financed:

(a) from the budget of the public service provider;
(b) payments for social services from the recipient of the social service under a contract for the provision of the social service and from reimbursement for other activities;
(c) the reimbursement of economically justifiable costs associated with the provision of the social service;
(d) the financial contribution provided by the Ministry to finance the social service in an establishment;
(e) funds received under a written grant agreement;
(f) funds from associations of municipalities, associations of higher territorial units and associations of persons;
(g) as a result of the management from secondary activities carried out by facilities with the constitutional competence or founding competence of a municipality or a higher territorial unit with its consent;
(h) income from a social enterprise;
(i) and from other sources.

Seniors go to different social care facilities when they stay alone at home, when they have no jobs for a longer time or their relatives are afraid of their safety. In general, they get into the welfare facility for the sake of helping someone else but also for social reasons.

Many seniors often associate going to the retirement home with the expected end of their lives, with death. Care in social care facilities is associated with the risk of age segregation or deprivation of the seniors. The state, the company can provide seniors with accommodation, food, medical care, they can try to organize their day, but they cannot provide emotional background and feeling that they are needed and able to help someone. In particular, women are less well adapted to staying in seniors, mainly because they lose their lifelong role [4].

Senior facilities should provide decent housing for older people. Every old person deserves to be well cared of. Every facility should provide high-quality services, both medical and social. Seniors should not only have a roof overhead and hot food, they deserve much more. Senior care should be provided to suit every individual.

Table 1 Number of inhabitants and pensioners in social services facilities in SR

| Region | Population | Number of pensioners | Number of people in facilities | Number of pensioners in facilities | Share of pensioners per inhabitant in % | Share of pensioners in facilities per people in facilities in % |
|---|---|---|---|---|---|---|
| **SR** | **5 410 836** | **1 312 205** | **45 720** | **24 061** | **24.3** | **52.6** |
| Bratislava | 612 682 | 150 881 | 5 377 | 3 082 | 24.6 | 57.3 |
| Trnava | 556 577 | 138 583 | 5 249 | 2 903 | 24.9 | 55.3 |
| Trenčín | 593 159 | 156 596 | 4 978 | 3 376 | 26.4 | 67.8 |
| Nitra | 688 400 | 180 927 | 6 855 | 3 655 | 26.3 | 53.3 |
| Žilina | 690 121 | 162 744 | 6 277 | 2 885 | 23.6 | 46 |
| Banská Bystrica | 658 490 | 163 956 | 5 850 | 2 987 | 24.9 | 51.1 |
| Prešov | 817 382 | 179 015 | 5 614 | 2 673 | 21.9 | 47.6 |
| Košice | 794 025 | 179 503 | 5 520 | 2 500 | 22.6 | 45.3 |

Source: [3]

The facility for seniors as a form of social assistance is only applied when the seniors are not able to liveon their own in their natural environment, even with the aid of terrain social services, in particular nursing services and organized communal meals. The “Social Services Act” stipulates in Section 35 that a social service is provided in a senior institution to:

(a) a person who has reached the retirement age and is dependent on the assistance of another person and the degree of dependency is at least IV in accordance with the attachment 3 and

(b) a person who has reached retirement age and needs social security services of this facility for some other serious reasons.

In the facility the seniors:

a) are provided with

1. assistance in reliance on the assistance of other person,

2. social counseling,
3. social rehabilitation,
4. accommodation,
5. board,
6. cleaning, washing, ironing and maintenance of laundry and clothing,
7. personal needs;

b) have the conditions for the safe-keeping of valuablescreated;
c) have leisure activities secured.

## MATERIAL AND METHODS

The first part of the paper contains the data taken from the "Report on the Social Situation of the Population of the Slovak Republic in 2018" [7], which depicts the level of pensions. An important source for the analysis of the economic situation of seniors was the material provided by the Facility for seniors in Gabčíkovo. Based on internal documents, annual reports and financial statements, we assessed the economic development and operation of the facility over the last ten years, 2009 – 2018. We analyzed the balance sheet and profit and loss statements, a property and capital structure, and compared the assets and liabilities in years 2009 – 2018. A cost-benefit structure was also prepared to provide an overview of their development in this period. In the facility we conducted interviews and questionnaire survey in 2016. We used descriptive statistics to evaluate answers of participants of the survey.

## RESULTS AND DISCUSSION

### Level of pensions in Slovakia

The pension insurance comprises old-age insurance and invalidity insurance. The old-age pension, early retirement pension, disability pension, widow's pension, widower's pension and orphan's pension are provided by the pension insurance scheme. Until 31 December 2017, 1,697,880 pension benefits were paid (excluding state-funded pensions, e.g. so-called youth invalids, wage pensions, social pensions, and non-retirement pensions and pensions not covered by automated evidence). There were 11 pensions not included in the automated evidence and the number of pensions paid to foreigners was 26 534 [7].

Table 2 Development of old-age pensions in Slovakia

| | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Level of pension | 339.73 | 352.54 | 362.08 | 375.89 | 390.51 | 400.18 | 411.06 | 417.4 | 428.3 | 444.2 |

Source: [7]

Compared to December 31, 2016, the number of disbursed pensions in December 2017 was higher by more than 11 thousand, a year-on-year increase of 0.7%. There was a decrease in the number of orphan´s and widow´s benefits (by approximately 0.6 thousand and 1.7 thousand respectively) in comparison with 2016.The substantial increase of old-age pensions was noted in 2017 comparing with 2016 (by circa 17 thousand).The reason of the increase dwelled in valorization of the pensions by fixed sum consisting of 2 % of the average pensions or benefits of particular type.

Costs on pensioner insurance fees in 2017 were represented by nearly 6.8 billion Euros, which is by 4.3% more than in 2016. National benefits of 82.5 million Euros were given in the forms of spousal pension, social pension, youth invalids, disability pension, pension increase due to the only source of income, to resistance and rehabilitation, including the premium for state service and compensatory bonus, minimum pension. The sum of 18.2 million Euro out of 82.5 million Euros was paid to increase old-age pensions and disability pensions of the people who achieved the age of old-age pension in order to equal to a minimum pension.

The expenses of Christmas benefit to some of the retired in 2017 (without any surcharge additionally paid for the year 2016) were 75.8 million Euros while in 2016 it was 77.4 million Euros. Total amount of the retired who received Christmas benefit in 2017 reached the number of 1 175 964 retired people, in 2016 it was 1 166 346 people. The ratio of the average old-age solo pension paid on 31 December 2017 (428.3 Euro) to the average monthly wage for the year 2017 in the economy of the Slovak Republic (954 Euro) was 44.9%, a slight decrease compared to 2016 (by 0.9).

**Facility for seniors in Gabčíkovo – characteristics and funding process**

Facility for seniors in Gabčíkovo is a non-profit organization that was established in 1956. The facility is located in the northeastern outskirts of the town in a beautiful and pleasant environment, in the mansion of Amade´s family (former owner in the 17th century), and has a capacity of 110 persons.

The financing process of the facility begins in September (in the current year), when a new money application is submitted for the next year. The contract is signed in mid-February and within ten days from signing the city receives the money for the first quarter. The city then sends the money to the facility account. Additional contributions from the state come quarterly, mid-April, July, and the last contribution come on October, 10$^{th}$. If the facility does not spend the budget by December, 31, the money must be returned to the state. The problem arises at the beginning of the year. From January, 1$^{st}$, until the signing of the contract and the receipt of the contribution, the facility has available cash only from the clients' remittances. These are, however, insufficient to cover the running of the facility. This situation needs to be resolved by a loan from the founder.

The facility budget covers not only wages, energy, meals, etc., but also some investment into the maintenance of the building and the surrounding park. A recent investment has been put into the building of another closed department because of the great interest in placing clients who need constant supervision. The investment was funded by facility own budget.

Table 3 Economic eligible costs

| Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Total costs | 709 823 | 739 017 | 808 024 | 739 662 | 748 903 | 705 066 | 709 489 | 709 614 | 713 59 | 712 332 |
| Number of clients | 91 | 95 | 104 | 95 | 96 | 91 | 91 | 92 | 90 | 93 |
| Average economic eligible costs per month | 650.02 | 648.26 | 647.45 | 648.82 | 650.9 | 645.66 | 649.71 | 651.11 | 652.45 | 659.14 |

Source: [1], own processing

In terms of finance, in the seniors' facility the average daily economically eligible costs are 11.73 Euro. The amount of the payment ranges from 305.1 Euro to 398.4 Euro. As stated the director of the Facility for seniors in Gabčíkovo, the average payment is between 320 Euro and 350 Euro. The financial burden and the impact on the economic situation of the clients were examined in the questionnaire survey (conducted in 2016); selected results are presented in Table 4. In terms of client satisfaction, there was a scale with five options for respondents: 1 - very satisfied, 5 - very dissatisfied.

Table 4 Results of the questionnaire survey

| | Age of seniors | Years in facility | Amount of the pension | Payment amount | Total satisfaction |
|---|---|---|---|---|---|
| Average | 79 | 6.2 | 407.33 | 347.33 | 1.7 |

Source: own processing

Funding consists of two components. The first component is a government contribution of 320 Euro per month, i.e. 3.840 Euro per year for a beneficiary of social services. Table 3 shows economic eligible costs which amount to an average of 640 to 650 Euros per month per social service recipient. These are all reasonable and cost-effective costs of providing social services.

The second component of funding is payments from social service recipients. This payment compensates the difference between economically justified costs and the state contribution. This payment for social services is calculated for each client individually and depends on the size of the rooms, the degree of dependence on the social service, the severity of the illness or whether the client needs a special diet.

Most clients pay for their retirement, but many clients have such low pension that they are notable to pay. From results of questionnaire follow that 30% of clients are inadequate for their income. This result was confirmed by the director of the facility, according to which 30 clients out of 90 have insufficient pensions. In these cases, the family has to pay. The facility has the obligation to provide customers with pocket money for their own use, which accounts for 25% of the subsistence minimum, which is currently 198.09 Euro, representing 49.52 Euro. This pocket money has to be paid by the facility to each client even though his or her pension is insufficient to pay for the social service payment.

## Financial – economic situation in Facility for seniors in Gabčíkovo

In this part we present the development of non-current and current assets of the facility.

Table 5 Development of the property structure

| Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Non-current Assets | 204 849 | 182 680 | 160 513 | 192 202 | 210 937 | 191 486 | 172 294 | 153 159 | 142 034 | 135 533 |
| Current Assets | 55 042 | 54 089 | 71 720 | 127 052 | 188 914 | 220 750 | 246 958 | 270 665 | 274 930 | 281 899 |
| Accruals | 796 | 1 446 | 1 545 | 1 183 | 1 125 | 1 000 | 1 287 | 1 190 | 1 258 | 1 075 |

Source: [1], own processing

As for assets, in 2009 the total assets amounted to 260 687 Euro. 79% of the assets were tangible fixed assets, including land, buildings and separate movables. Ordinary assets accounted for 21% of total assets. The value of tangible fixed assets has decreased by 16%. The value of current assets increased by 191,916.64 Euro and its share in total assets increased to 58.7%. This change in the ratio of current and non-current assets is shown in Table 5. Change was caused by a slight increase in inventories, almost doubling the state of bank accounts, but above all an enormous increase in receivables from non-tax revenues of municipalities and higher territorial units and budget organizations set up by the municipality and higher territorial units.

Table 6 Development of the capital structure

| Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Equity | -1 266 | -5 397 | -3 291 | 60 984 | 93 736 | 129 524 | -16 558 | -20 992 | -36 758 | -25 705 |
| Liabilities | 109 942 | 111 165 | 127 415 | 116 360 | 176 251 | 166 445 | 329 752 | 348 577 | 366 539 | 366 435 |
| Accruals | 152 011 | 132 447 | 109 654 | 143 093 | 130 989 | 117 268 | 107 346 | 97 430 | 88 441 | 77 777 |

Source: [1], own processing

Table 6 shows development of the capital structure of the facility, which is very important to assess the coverage of the assets. Capital is the source of funding the organization uses to secure its operations. Large increase in commitments is visible, especially among public administrations. The cost structure consists of consumables, service costs, personnel costs, other operating expenses, depreciation, provisions, finance costs and income tax expense. Consumed purchases include energy consumption, medical supplies, food, and so on. These costs have been declining in the last four years.

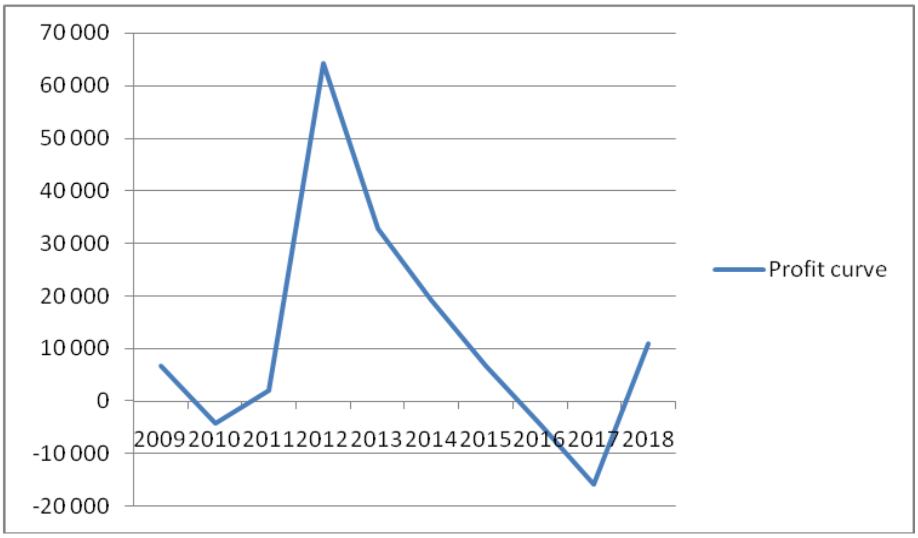
The development of costs and revenues in the facility in Gabčíkovo is summarized in the Table 7. The revenue structure consists of sales of services, other revenues from operating activities, settlement of reserves, financial revenues and revenues from transfers and budget revenues in municipalities, higher territorial units and in budgetary organizations and contributory organizations established by the municipality or higher territorial unit. The difference between costs and yields is the result of the management. Its development is illustrated in Figure 1 and we can see fluctuating character of a profit curve.

Table 7 Development of costs, revenues and profit

| Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Total costs | 857 199 | 884 088 | 1 008 657 | 1 033 458 | 1 020 644 | 999 791 | 1 046 863 | 1 100 022 | 1 110 123 | 1 299 435 |
| Total revenues | 863 778 | 879 958 | 1 010 764 | 1 097 733 | 1 053 397 | 1 019 081 | 1 053 640 | 1 095 588 | 1 094 357 | 1 310 488 |
| Profit | 6 779 | -4 130 | 2 107 | 64 275 | 32 752 | 19 289 | 6 777 | -4 434 | -15 765 | 11 053 |

Source: [1], own processing

Figure 1 Development of profit at the facility




Source: [1], own processing

Based on the results of the analysis, we propose changes in the following areas:

- Problem in the financing of facilities for seniors from the state budget, caused by the late conclusion of the contract, during the period from January to mid-February, when there is lack of funds. It would be advisable to conclude contracts sooner, preferably in January.
- Insufficient financial assessment of personal staff in facilities for seniors, due to being included in the social sphere and not in the health care sector.
- Insufficient promotion of facilities for seniors due to the lack of available information. It is advisable to create a website with all the necessary information about the facility.
- The biggest problem is not the lack of facilities and places in social services facilities, but their uneven deployment within individual regions. It would be appropriate to deploy these facilities more appropriately and more evenly.

## CONCLUSIONS

It should be remembered that the seniors will always reach the age at which they may need help and care. It is important to address this issue, discuss it and look for ways to improve the situation of the facilities and create conditions for improving the services provided. This can be a good investment for the future. For these reasons, the issue has become a major topic of this paper.

The main objective was to analyze the economic functioning of facilities for seniors. First of all, we analyzed the economic situation of seniors in the context of their level of pensions in Slovakia. Then the research followed in The Facility for Seniors in Gabčíkovo; we conducted an interview with the director of the facility, staff and facility clients. The facility director explained us the process of financing the facility; the share of recipient payment and state funding is on average about 1:1. According to the obtained responses of the clients of the facility, it can be conculded that they are satisfied with the provided social services. From financial point of view, the pension amount is sufficient for most of clients for the required payments.

## REFERENCES

[1] *FinStat.* Financial statements. Facility for seniors, *Gabčíkovo* (in Slovak). Retrieved 2019-03-11 from: https://finstat.sk/00655716

[2] Gejdošová, Z. (2012). *Sociálne zabezpečenie v systéme verejnej správy na Slovensku*. Ružomberok: VERBUM, 2012. 218 p. (in Slovak).

[3] Hanáčková, D. (2015). Sociálne služby pre seniorov v podmienkach Slovenskej republiky. *Forward – Moving frontiers of rural and regional development: proceedings of research conference*, *in Nitra - Slovakia, December 4-5, 2014.* Nitra: SPU, 2015, pp. 73-83 (in Slovak).

[4] Hrozenská, M. a kol. (2008). *Sociálna práca so staršími ľuďmi a jej teoreticko-praktické východiská*, Martin: Osveta, 2008. 181 p. (in Slovak).

[5] Radičová, I. (2003). *S.O.S. – Sociálna ochrana na Slovensku*. Bratislava: S.P.A.C.E., 2003. 197 p. (in Slovak).

[6] Rievajová, E. a kol. (2000). *Sociálne zabezpečenie*. Bratislava: SPRINT (in Slovak).

[7] *Report on the Social Situation of the Population of the Slovak Republic in 2018* (Správa o sociálnej situácii obyvateľstva Slovenskej republiky za rok 2018), (in Slovak). Retrieved 2019-02-06 from https://www.employment.gov.sk/sk/ministerstvo/vyskum-oblasti-prace-socialnych-veci-institut-socialnej-politiky/spravy-socialnej-situacii-obyvatelstva/rok-2018.html

[8] Yuan, J. et al. (2019). Examining sustainability indicators of space management in elderly Facilities - a case study in China. *Journal of Cleaner Production*, vol. 208, no. 20, pp. 144-159. Retrieved 2019-02-06 from: https://doi.org/10.1016/j.jclepro.2018.10.065

[9] *Zákon č. 448/2008 Z.z. o sociálnych službách* a o zmene a doplnení zákona č. 445/1991 Zb. o živnostenskom podnikaní (živnostenský zákon) v znení neskorších predpisov (Act No. 448/2008 Coll. on Social Services) (in Slovak).

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***Original Paper***

# Differential equations in mathematics education at the Faculty of Engineering SUA in Nitra

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**ABSTRACT**

Some properties and relations can be described by functions and their derivatives, which can be written as an equation containing the variables x and y and derivatives $y', y'', \dots, y^{(n)}$. Such equations are called differential equation of *n*-th order, where the order of the differential equation is determined by the highest derivative present in that equation. We can write this equation in the form $F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0$ . Only students of the Faculty of the Engineering SUA in Nitra in the subject Mathematics for Technicians study the topic on differential equations in their first year of study. Since the scope of teaching in this subject is very small, so we deal only with certain types of differential equations. In this paper, we set a goal to verify the level of students' knowledge about differential equations at this faculty. We used methods of mathematical descriptive statistics to analyze study outcomes. We found that if the student was successful in solving differential equations, he was also successful in the final test. It has not been confirmed that if a student is able to calculate one kind of differential equations, s/he can calculate another type.

**KEYWORDS**: differential equations, average, correlation

**JEL CLASSIFICATION:** C02, C10, I21

## INTRODUCTION

Different economic, statistical, physical and other phenomena and processes can be characterized by quantities that can be mathematically expressed by functions and their derivatives. The function $y = f(x)$ expresses the dependence between the variables $x$ and $y$. We often look for relationships between a function $y = f(x)$ and its derivatives: $y' = f'(x), y'' = f''(x), \dots, y^{(n)} = f^{(n)}(x)$. These relations can be written using differential equations. Differential equations constitute a part of mathematical analysis.

According to Kulička and Machalík [3] "The study of differential equations is an attractive illustration of the application of the ideas and techniques of calculus in our everyday live".

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Differential equations have also application in many practical areas, for example in technical practice when harvesting the sugar beet. In [1], [2] there are presented analytical expressions for finding the first and second eigenfrequency and expressions for finding the amplitude of forced vibrations of an elastic body with the usage of differential equations.

At each of the six faculties of the Slovak University of Agriculture (SUA) in Nitra students have compulsory mathematics for one or two semesters. However, only the first year students of the Technical Faculty are familiar with the differential equations. In our paper we want to find out how students of this faculty can understand this area of mathematics.

## MATERIAL AND METHODS

We call the differential equation an

$$F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0,$$

where $x$ and $y$ are variables and $y', y'', \dots, y^{(n)}$ derivatives of $y$.

Students of the Faculty of Engineering have surprisingly only one semester of compulsory mathematics. Since the scope of teaching is very small, we teach students only certain types of differential equations. Namely separable first-order differential equations, higher-order differential equations, but we only deal with those where the order can be reduced by multiple integration, and homogenous higher-order linear differential equations with constant coefficients. We will describe in more detail the above-mentioned types of differential equations.

*A first order differential equation* is an equation of the form

$$F(x, y, y') = 0.$$

If it is possible to express the derivative from this equation, we can get a separable, homogeneous or a linear differential equation.

*A separated differential equation of first order* has the form

$$p(x) + q(y) \cdot y' = 0,$$

where $p(x)$ and $q(y)$ are functions of $x$ and $y$.

*A separable differential equation of first order* has the form

$$p_1(x) \cdot p_2(y) + q_1(x) \cdot q_2(y) \cdot y' = 0,$$

where $p_1(x)$ and $q_1(x)$ are functions of $x$ continuous on $(a, b)$, $p_2(y)$ and $q_2(y)$ are functions of $y$ continuous on $(c, d)$. If $q_1(x) \cdot p_2(y) \neq 0$ is true, this equation can be transformed to a separated differential equation of first order.

If $y = f(x)$ is continuous function on the interval $I$, we call the differential equation of $n-$th order

$$y^{(n)} = f(x)$$

*a differential equation of higher order where the order can be reduced by multiple integration.*

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*A homogenous linear differential equation of higher order with constant coefficients* is an equation of the form

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = 0,$$

where $a_1, a_2, \ldots, a_n$ are constants, $a_i \in R, i = 1, \ldots, n$.

Our goal was to find out how students of the Faculty of Engineering of the Slovak University of Agriculture in Nitra understood this part of mathematics, i.e. if they are able to solve examples with differential equations. Two hypotheses arose from this goal:

1. If a student is able to solve one type of differential equations, he / she can solve also the other type.
2. If a student is successful in solving differential equations, he / she is also successful in a general test containing these equations.

We used methods of mathematical descriptive statistics in order to achieve this goal. We put the collected data into MS Excel tables, calculated arithmetic means and used correlation coefficients to verify the validity of both hypotheses.

## RESULTS AND DISCUSSION

Students had five problems to solve in a written exam test, one of which was always a first order differential equations and another one a higher order differential equations, but only from among the above-mentioned types. We chose examples from the textbook Chapters from Higher Mathematics [5] and Tutorials on Mathematics [4].

Here is example of such test.

1. Find the domain of the function $f: y = \ln \dfrac{5-x}{2+x}$.
2. Find the intervals of monotonicity of the function $g: y = 2x^3 + 6x^2 - 18x + 7$.
3. Evaluate $\int 4x \cdot \ln x \, dx$.
4. Solve the differential equation $3 + 2x - yy' = 0$.
5. Solve the differential equation $y'' + 3y' + 2y = 0$.

The test was written by 132 students in academic year 2018/2019. Each example was awarded 10 points, so the student could obtain 50 points in total. We evaluated our hypotheses with tools of MS Excel 2016. We created two tables. The letters in the tables mean the following: the letter A stands for the arithmetic mean and the corresponding percentage of total points for the first three problems, B for the problem on the first order differential equation and C for the problem on the higher order differential equation. Line X shows the average number of earned points and line Y the percentage (Tab. 1).

Table 1 Average number of points

| | *A* | *B* | *C* | *B+C* | *A+B+C* |
|---|---|---|---|---|---|
| ***X*** | 17.9 | 5.8 | 6.5 | 12.3 | 30.2 |
| ***Y*** | 59.67 | 58.03 | 64.84 | 61.44 | 60.42 |

Source: authors' calculations

The best results were with problems on a higher order differential equation, where the success rate was 64.84%. Students earned 59.67% for the first three problems, and the worst results were in the case of a first order differential equation, where students got on average only 58.03% points. Both differential equations were solved on average with success 61.48%. The overall average test success rate was 60.42%.

The validity of the above hypotheses was determined by correlation coefficients. Using the correlation coefficients we determine three degrees of dependence between two investigated objects. The weak dependence is when the correlation coefficient is from the interval $\langle 0, 0.\overline{3} \rangle$, the mean for values from the interval $\left( 0.\overline{3}, 0.\overline{6} \right\rangle$ and strong for values from the interval $\left( 0.\overline{6}, 1.0 \right\rangle$. Results in table 2 show correlation coefficients for each relationship.

Table 2 Correlation coefficients

| | *A* | *B* | *C* | *B+C* | *A+B+C* |
|---|---|---|---|---|---|
| ***A*** | | 0.235117 | 0.355803 | 0.375336 | 0.88666 |
| ***B*** | 0.235117 | | 0.231563 | 0.792019 | 0.559578 |
| ***C*** | 0.355803 | 0.231563 | | 0.777305 | 0.636646 |
| ***B+C*** | 0.375336 | 0.792019 | 0.777305 | | 0.761409 |
| ***A+B+C*** | 0.88666 | 0.559578 | 0.636646 | 0.761409 | |

Source: authors' calculations

Table 2 shows that, the first hypothesis was not confirmed at all. This means that if a student is able to solve one kind of differential equation, he/she is not able to solve another type automatically. This is the relationship between B and C. Here he was the smallest correlation coefficient in the whole table, only 0.231563, which represents a weak dependence.

The second hypothesis was proved. That is if a student is successful in solving differential equations, he/she is also successful in the whole test. This is the relationship between B+C and A+B+C, where the correlation coefficient was 0.761409, which falls into the interval of strong dependence.

Using this table we can also make the following statement: If a student is able to solve the first three problems that do not contain differential equations, he/she is also successful in the whole test. This is the relationship between A and A+B+C, where was the highest correlation coefficient 0.88666.

## CONCLUSIONS

Mathematics with its system has a firm place in the university education, and therefore also in the other "non-mathematical" fields. In our paper we tried to find out how students of the Faculty of Engineering of the Slovak University of Agriculture in Nitra studied and understood the topic on various types of differential equations. We found that they best understood the solution procedure for higher order differential equations. Also, we found that if students are able to solve differential equations, they are likely to succeed in the final math test. It can be concluded that differential equations are one of the most important parts of mathematical analysis, and they have application in a variety of scientific disciplines.

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## REFERENCES

[1] Bulgakov, V. M., Adamchuk, V., Holovach, I. & Országhová, D. (2016). The theory of longitudinal vibrations of a conical elastic body in an elastic medium. *Agricultural Science and Practice*, 3(1), pp. 27-35.

[2] Bulgakov, V. M., Adamchuk, V., Olt, J. & Országhová, D. (2015). Use of Euler equations in research into three-dimensional oscillations of sugar beet root during its vibration-assisted lifting. *Agronomy Research,* 13(1), pp. 33-45.

[3] Kulička, J. & Machalík, S. (2016). Differential equations in the Education of the Future Transport Engineers with Support of MATLAB. *ICERI 2016*, Proceedings, pp. 7471-7478, Seville/Spain: IATED.

[4] Országhová, D. & kol. (2016). *Návody na cvičenia z matematiky* (Tutorials on Mathematics). Nitra: SPU v Nitre (in Slovak).

[5] Pechočiak, T. & Matušek, V. (2012). *Kapitoly z vyššej matematiky* (Chapters from Higher Mathematics). Nitra: SPU v Nitre (in Slovak).

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***Original Paper***

# Evaluation of specific integrals by differentiation

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**ABSTRACT**

Differentiation and integration (anti-differentiation) constitute one of the fundamental techniques used in higher mathematics. These operations are inverse of each other. While differentiation (to the extent of school mathematics) is relatively simple and straightforward, integration, in general, is a much more involving task. There are various classical methods to evaluate elementary integrals, e.g. substitution, integration by parts, partial fraction decomposition or more advanced techniques like the residue theorem, or Cauchy's integral formula. The paper deals with some types of elementary functions whose integrals can be evaluated by intelligent guess and differentiation.

**KEYWORDS**: higher mathematics, differentiation, integration

**JEL CLASSIFICATION:** I 20, C20

## INTRODUCTION

Integration, i.e. evaluation of the indefinite integral is one of the basic operations in higher mathematics. If we have a continuous function $f(x)$ on $(a,b)$, then there exists a function $F(x)$ having the property that $F'(x)=f(x)$ on $(a,b)$. The function $F(x)$ is called an antiderivative of $f(x)$. In order to find the function $F(x)$, we have to integrate or anti-differentiate the function $f(x)$. So simply speaking, integration is a reverse operation to differentiation. Differentiation is a relatively simple and routine operation, since there are general rules available [1], [2], [4]. On the other hand, its reverse, integration is generally much more intricate and a tedious task. In the paper we focus on some functions whose integrals can be guessed and evaluated by subsequent differentiation and comparison. The idea was discussed by Dawson in [3]. Some functions, when differentiated, do not change qualitatively. These functions are polynomials, exponentials and trigonometric functions such as $\sin ax$, $\cos bx$ and various combinations of them. Differentiation of these functions gives back qualitatively the same functions.

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## MATERIAL AND METHODS

In some cases we can guess the form of the antiderivative of the function $f$. The idea behind is the property:

$$\int f(x)\,dx = something \quad \Rightarrow \quad [something]' = f(x) \tag{1}$$

So if "*something*" differentiated is qualitatively the same as the function being integrated, we can equate both sides of (1), make a comparison and obtain a solution. The usual method that works here is the method of undetermined coefficients. We illustrate the idea with a simple example. Let's consider the integral $\int (x^2+x+1)\,dx$. We know that derivatives of polynomials are polynomials of degree less by one, hence the antiderivative of $x^2+x+1$ must be a polynomial of degree 3, i.e. of the form $Ax^3+Bx^2+Cx+D$.

$\int (x^2+x+1)\,dx = Ax^3+Bx^2+Cx+D \Rightarrow x^2+x+1=\left[Ax^3+Bx^2+Cx+D\right]'$ and

$x^2+x+1=3Ax^2+2Bx+C$ and a simple comparison yields $A=\frac{1}{3}, B=\frac{1}{2}, C=1$, and

$$\int (x^2+x+1)\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2+x+D$$

## RESULTS AND DISCUSSION

Now we try to develop this idea a bit more, primarily to integrals of the abovementioned functions. Hereinafter, we denote polynomials of degree *n, m* as $P_n(x), Q_m(x)$etc., respectively and their *k*-th derivatives as $P_{n-k}(x), Q_{m-k}(x)$ etc., respectively.

1. $\int P_n(x)\,e^{ax}\,dx.$

A judicious guess says that the antiderivative of $P_n(x)\,e^{ax}$ must have the form $Q_n(x)\,e^{ax}$. If we differentiate the function $P_n(x)\,e^{ax}$, we get (since we consider polynomials in general, we deliberately neglect the negative signs and the factor *a*):

$$\left[P_n(x)\,e^{ax}\right]' = P_{n-1}(x)\,e^{ax} + P_n(x)\,e^{ax}$$

and further

$$\left[P_{n-1}(x)\,e^{ax}\right]' = P_{n-2}(x)\,e^{ax} + P_{n-1}(x)\,e^{ax}$$

$$\left[P_{n-2}(x)\,e^{ax}\right]' = P_{n-3}(x)\,e^{ax} + P_{n-2}(x)\,e^{ax}$$

By combining these equations we obtain

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$$P_n(x)e^{ax} = \left[P_n(x)e^{ax}\right]' + \left[P_{n-1}(x)e^{ax}\right]' + \left[P_{n-2}(x)e^{ax}\right]' + P_{n-3}(x)e^{ax} \text{ or}$$

$$P_n(x)e^{ax} = \left[\left( P_n(x) + P_{n-1}(x) + P_{n-2}(x) \right) e^{ax}\right]' + P_{n-3}(x)e^{ax}$$

It is obvious that after performing $n$ steps the polynomial $P_n(x)$reduces to a constant and we will have

$$P_n(x)e^{ax} = \left[\left( \sum_{k=0}^{n} P_{n-k}(x) \right) e^{ax}\right]'$$

$$\int P_n(x)e^{ax}\,dx = \left( \sum_{k=0}^{n} P_{n-k}(x) \right) e^{ax} = Q_n(x)e^{ax}, \text{ where } \sum_{k=0}^{n} P_{n-k}(x) = Q_n(x)$$

**Example 1:** Evaluate $\int (3x^2 - 2x + 4)e^{2x}\,dx$.

Solution:

$$\int (3x^2 - 2x + 4)e^{2x}\,dx = (Ax^2 + Bx + C)e^{2x}$$

Now we take the derivatives of both sides

$$(3x^2 - 2x + 4)e^{2x} = (2Ax + B)e^{2x} + 2\,(Ax^2 + Bx + C)e^{2x}$$

We immediately see that $A = \frac{3}{2}, 2A + 2B = -2, B + 2C = 4 \;\Rightarrow\; B = -\frac{5}{2}, C = \frac{13}{4}$, hence

$$\int (3x^2 - 2x + 4)e^{2x}\,dx = \left( \frac{3}{2}x^2 - \frac{5}{2}x + \frac{13}{4} \right) e^{2x} + const$$

2. $\int P_n(x)\sin ax\,dx, \int P_m(x)\cos ax\,dx.$

In like manner as in the previous case (by subsequent differentiation and reduction of the polynomial $P_n(x)$ to a constant) we can derive that

$$\int P_n(x)\sin ax\,dx = Q_n(x)\cos ax + R_{n-1}(x)\sin ax \tag{2}$$

$$\int P_m(x)\cos ax\,dx = Q_m(x)\sin ax + R_{m-1}(x)\cos ax \tag{3}$$

**Example 2:** Evaluate $\int (4x^3 - x + 1)\sin 4x\,dx$.

Solution:

$$\int (4x^3 - x + 1)\sin 4x\,dx = (Ax^3 + Bx^2 + Cx + D)\cos 4x + (Ex^2 + Fx + G)\sin 4x$$

$$\left(4x^3 - x + 1\right)\sin 4x = \left(3Ax^2 + 2Bx + C\right)\cos 4x + 4\left(-Ax^3 - Bx^2 - Cx - D\right)\sin 4x + 4\left(Ex^2 + Fx + G\right)\cos 4x + \left(2Ex + F\right)\sin 4x$$

And we have

$$A = -1,\ B = 0,\ -3 + 4E = 0,\ -4C + 2E = -1,\ 2B + 4F = 0,\ -4D + F = 1,\ C + 4G = 0$$

$$\int \left(4x^3 - x + 1\right)\sin 4x\, dx = \left(-x^3 + \frac{5}{8}x - \frac{1}{4}\right)\cos 4x + \left(\frac{3}{4}x^2 - \frac{5}{32}\right)\sin 4x + const$$

Adding up (2) and (3) yields

$$\int P_n(x)\sin ax + Q_m(x)\cos ax\, dx = R_n(x)\cos ax + S_{n-1}(x)\sin ax, \text{ if } n > m$$

$$\int P_n(x)\sin ax + Q_m(x)\cos ax\, dx = R_n(x)\cos ax + S_n(x)\sin ax, \text{ if } n = m$$

$$\int P_n(x)\sin ax + Q_m(x)\cos ax\, dx = R_{m-1}(x)\cos ax + S_m(x)\sin ax, \text{ if } n < m$$

**Example 3:** Evaluate $\int \left(\left(x^2 + 3x - 2\right)\sin 6x + \left(x^3 + x^2 + 4\right)\cos 6x\right) dx$.

Solution:

$$\int \left(\left(x^2 + 3x - 2\right)\sin 6x + \left(x^3 + x^2 + 4\right)\cos 6x\right) dx = \left(Ax^2 + Bx + C\right)\cos 6x + \left(Dx^3 + Ex^2 + Fx + G\right)\sin 6x$$

$$\left(x^2 + 3x - 2\right)\sin 6x + \left(x^3 + x^2 + 4\right)\cos 6x = \left(2Ax + B\right)\cos 6x + 6\left(-Ax^2 - Bx - C\right)\sin 6x + 6\left(Dx^3 + Ex^2 + Fx + G\right)\cos 6x + \left(3Dx^2 + 2Ex + F\right)\sin 6x$$

$$D = \frac{1}{6},\ E = \frac{1}{6},\ 2A + 6F = 0,\ B + 6G = 4,\ -6A + 3D = 1,\ -6B + 2E = 3,\ -6C + F = -2$$

$$\int \left(\left(x^2 + 3x - 2\right)\sin 6x + \left(x^3 + x^2 + 4\right)\cos 6x\right) dx = = \left(\frac{-1}{12}x^2 - \frac{4}{9}x + \frac{73}{216}\right)\cos 6x + \left(\frac{1}{6}x^3 + \frac{1}{6}x^2 + \frac{1}{36}x + \frac{20}{27}\right)\sin 6x + const$$

3. $\int e^{ax}\sin bx\, dx, \int e^{ax}\cos bx\, dx.$

$$\left[e^{ax}\sin bx\right]' = a\, e^{ax}\sin bx + b\, e^{ax}\cos bx \qquad (4)$$

$$\left[e^{ax}\cos bx\right]' = -b\, e^{ax}\sin bx + a\, e^{ax}\cos bx \quad / \cdot \frac{a}{b} \qquad (5)$$

Adding up (4) and (5)

$$\left[e^{ax}\sin bx\right]' + \frac{a}{b}\left[e^{ax}\cos bx\right]' = \left(\frac{a^2}{b}+b\right)e^{ax}\cos bx$$ and relabeling the constants gives

$$\int e^{ax}\cos bx\,dx = A\,e^{ax}\sin bx + B\,e^{ax}\cos bx$$

Analogously, solving for $\int e^{ax}\sin bx\,dx$ yields the same general form of solution

$$\int e^{ax}\sin bx\,dx = A\,e^{ax}\sin bx + B\,e^{ax}\cos bx$$

**Example 4:** Evaluate $\int e^{4x}\sin 5x\,dx$.

Solution:

$$\int e^{4x}\sin 5x\,dx = A\,e^{4x}\sin 5x + B\,e^{4x}\cos 5x$$

$$e^{4x}\sin 5x = 4A\,e^{4x}\sin 5x + 5A\,e^{4x}\cos 5x + 4B\,e^{4x}\cos 5x - 5B\,e^{4x}\sin 5x$$

$$4A - 5B = 1,\ 5A + 4B = 0$$

$$\int e^{4x}\sin 5x\,dx = \frac{4}{41}e^{4x}\sin 5x - \frac{5}{41}e^{4x}\cos 5x + const$$

4. $\int P_n(x)e^{ax}\sin bx\,dx,\ \int P_n(x)e^{ax}\cos bx\,dx.$

This case is the combination of all the previous cases. Pursuing the same idea shows that the general form of a solution in this case is

$$\int P_n(x)e^{ax}\sin bx\,dx = Q_n(x)e^{ax}\sin bx + R_n(x)e^{ax}\cos bx$$

$$\int P_m(x)e^{ax}\cos bx\,dx = Q_m(x)e^{ax}\sin bx + R_m(x)e^{ax}\cos bx$$

**Example 5:** Evaluate $\int (x+4)e^{2x}\cos 3x\,dx$.

Solution:

$$\int (x+4)e^{2x}\cos 3x\,dx = (A\,x+B)e^{2x}\cos 3x + (C\,x+D)e^{2x}\sin 3x$$

$$(x+4)e^{2x}\cos 3x = A\,e^{2x}\cos 3x + 2(Ax+B)e^{2x}\cos 3x - 3(Ax+B\ )e^{2x}\sin 3x +$$
$$C\,e^{2x}\sin 3x + 2(Cx+D)e^{2x}\sin 3x + 3(Cx+D)e^{2x}\cos 3x$$

After canceling $e^{2x}$ we have

$A+2B+3D=4, 2A+3C=1, -3A+2C=0, -3B+C+2D=0$,

and solving this system yields

$$\int (x+4)e^{2x}\cos 3x\,dx = \left(\frac{2}{13}x+\frac{109}{169}\right)e^{2x}\cos 3x+\left(\frac{3}{13}x+\frac{144}{169}\right)e^{2x}\sin 3x + const$$

## CONCLUSIONS

Note that all the above mentioned integrals can be evaluated by means of the "by parts" integration method, which is also a formal justification of the results, but employing this method to solve Example 3 or Example 5 is a pretty formidable task, to say the least. There are other types of functions whose antiderivatives can be found without the "necessity" of integration. We will investigate such functions in the upcoming paper. The use of the presented method is left to the reader in every particular case.

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## REFERENCES

[1] Appel, W. (2017). *Mathematics for Physics and Physicists*. Princeton Univ. Press, Princeton.

[2] Ayres, F. & Mendelson, E. (2009). *Calculus*. Schaum's Outlines, Mc Graw Hill.

[3] Dawson, R. *Integration by undetermined coefficients*. Retrieved 2019-10-09 from http://cs.stmarys.ca/~dawson/Undetermined.pdf

[4] Nagy, J., Nováková, E. & Vacek, M. (1984). *Integrální počet* (Integral calculus). MVŠT, SNTL Praha (in Czech).