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# MODELLING OF SOUND IMPACT ON PRIMARY CANVASES FROM BASALT FIBER

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***Abstract:*** Basalt fiber is a heat-insulating non-combustible material that has excellent heat conductivity, hygroscopicity and chemical stability characteristics, and is widely used in many branches of engineering and other human activities. However, basalt fiber canvases have a significant disadvantage in the form of non-fibrous inclusions contained in them, which reduce the productquality and can cause minor injuries due to their prickliness.

## 1 Introduction

Basalt fiber is a heat-insulating non-combustible material that has excellent heat conductivity, hygroscopicity and chemical stability characteristics, and is widely used in many branches of engineering and other human activities. However, basalt fiber canvases have a significant disadvantage in the form of non-fibrous inclusions contained in them, which reduce the product quality and can cause minor injuries due to their prickliness. It was found in research [1] that when an acoustic field is applied to the formed primary canvases from basalt fiber, separation of non-fibrous inclusions from them is observed. The task of modelling this phenomenon and process is worthwhile.

Basalt fiber can be represented as an oscillatory system, the scheme of which is shown in Figure 1. Some of the elementary fibers can be represented as elastic beams resting on adjacent curved fibers. We assume that non-fiber inclusions of different mass and geometric shape are held by the forces of natural adherence to each of such elementary fibers. Such elastic beams with inclusions attached to them are under the influence of the acoustic field.

## 2 Calculation scheme

General theoretical principles of the vibrational systems investigation are considered in works [1], [2], [3], on the basis of which this research is made.

Let's make a mathematical model of the system. We consider a system of interacting bodies connected by elastic coupling (Fig. 1).

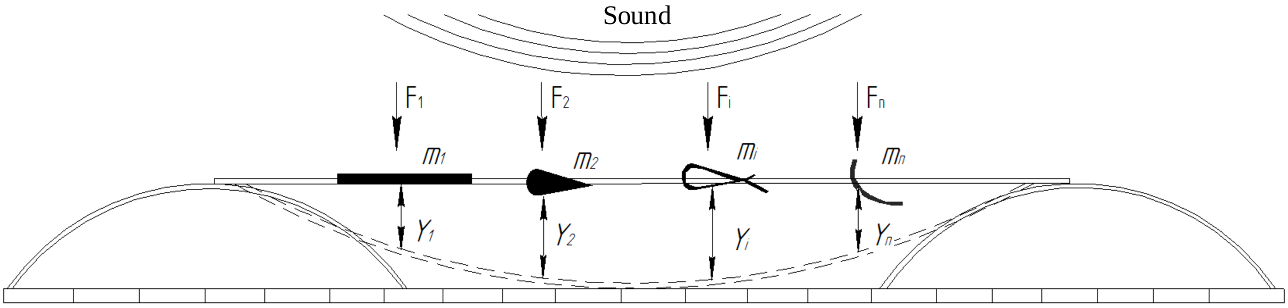

*Figure 1 The calculation scheme*

In Fig. 1: *m* is the mass of the element, kg; *c* - rigidity of elements connection, N / m; *a* – a damping coefficient , which determines energy losses for viscous friction, kg/s; *y* - elements motion from the static equilibrium position, m; *x* – forced motion of the elastic base as a result of the sound impact, m.

Based on Newton's second law, we consider the forces balance, impacting on each of the four masses. In this model, two forces are considered:

- the elastic resistance force, proportional to the motion,
- the viscous resistance force, proportional to the motion speed.

Four forces equilibrium equations form a system of four differential equations according to the calculation scheme in Fig. 1:

$$\begin{cases} m_1y_1'' = c(x-y_1)+c(y_2-y_1)+a(x'-y_1')+a(y_2'-y_1') \\ m_2y_2'' = c(y_1-y_2)+c(y_3-y_2)+a(y_1'-y_2')+a(y_3'-y_2') \\ m_3y_3'' = c(y_2-y_3)+c(y_4-y_3)+a(y_2'-y_3')+a(y_4'-y_3') \\ m_4y_4'' = c(x-y_4)+c(y_3-y_4)+a(x'-y_4')+a(y_3'-y_4') \end{cases} \quad (1)$$

The free motion of each mass separately can be described by the following second-order differential equation:

$$m_iy_i''+ay_i'+cy_i=0 \quad (2)$$

Or in the form of a typical oscillatory link [4] of the dynamic system:

$$T_i^2y_i''+2T_i\xi_iy_i'+y_i=0 \quad (3)$$

Where Ti - the time constant or period of free oscillations, s;
ξi i- the damping coefficient (0 <ξi <1).

These parameters in the joint consideration of (2) and (3) are the following:

$$T_i=\sqrt{\frac{m_i}{c}} \quad (4)$$

$$a=2T_i\xi_ic \quad (5)$$

Then the fundamental frequency of each of the four elements is determined from the known formula [4]:

$$\omega_i=\frac{\sqrt{1-\xi_i^2}}{T_i}=\frac{\sqrt{4m_ic-a^2}}{2m_i} \quad (6)$$

The analytical solution of the system (1) leads to the single linear differential equation with constant coefficients of the eighth order. A further solution involves finding the roots of the characteristic equation of the eighth degree and considering various solutions for real or complex roots, which causes certain difficulties.

To study the object in the frequency domain and also to consider alternative ways of system solution (1), it is suggested to use the methods of [4] Automatic Control Theory (TAU). Applying the Laplace transforms, and going over to the images (7), the system (1) can be represented in the form (8):

$$\begin{cases} y_i'' \to p^2Y_i \\ y_i' \to pY_i \end{cases} \quad (7)$$

$$\begin{cases} m_1p^2Y_1+2apY_1+2cY_1=(c+ap)(X+Y_2) \\ m_2p^2Y_2+2apY_2+2cY_2=(c+ap)(Y_1+Y_3) \\ m_3p^2Y_3+2apY_3+2cY_3=(c+ap)(Y_2+Y_4) \\ m_4p^2Y_4+2apY_4+2cY_4=(c+ap)(Y_3+X) \end{cases} \quad (8)$$

Or as a system of operator equations with transfer functions:

$$\begin{cases} Y_1=W_1(X+Y_2) \\ Y_2=W_2(Y_1+Y_3) \\ Y_3=W_3(Y_2+Y_4) \\ Y_4=W_4(Y_3+X) \end{cases} \quad (9)$$

Where the operator transfer functions are defined as follows (10) and are mathematical models of the structural elements of the dynamic system.

$$\begin{cases} W_1=\frac{c+ap}{m_1p^2+2ap+2c} \\ W_2=\frac{c+ap}{m_2p^2+2ap+2c} \\ W_3=\frac{c+ap}{m_3p^2+2ap+2c} \\ W_4=\frac{c+ap}{m_4p^2+2ap+2c} \end{cases} \quad (10)$$

Then, solving the system (9) about $Y_i$ for a given X, we obtain a system solution (1) for all masses in the following operator form (11).

$$Y_i=W_{pi}(p)X \quad (11)$$

Performing the inverse Laplace transform, we can also obtain an analytic solution of the problem.

Passing to the complex transfer function (12), according to the known formulas [4], we have the amplitude-frequency *A* (ω) and the phase-frequency φ (ω) characteristics

$$\begin{cases} W_P(\omega i)=Re(\omega)+Im(\omega)\cdot i \\ A(\omega)=|W_P(\omega i)|=\sqrt{Re(\omega)^2+Im(\omega)^2} \\ \varphi(\omega)=\arg(W_P(\omega i))=arctg(Im(\omega)/Re(\omega)) \end{cases} \quad (12)$$

The critical frequencies themselves can be calculated without amplitude response, as the square roots of the eigenvalues of the matrix (13) coefficients of the original (1) undamped (ξi = 0, ai = 0) differential equations system.

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$$\begin{bmatrix} \frac{-2c}{m_1} & \frac{c}{m_1} & 0 & 0 \\ \frac{c}{m_2} & -\frac{2c}{m_2} & \frac{c}{m_2} & 0 \\ 0 & \frac{c}{m_3} & -\frac{2c}{m_3} & \frac{c}{m_3} \\ 0 & 0 & \frac{c}{m_4} & \frac{-2c}{m_4} \end{bmatrix} \quad (13)$$

## 3 Results and Conclusions

The analytical calculation of the eigenvalues of a fourth-order matrix again requires the solution of high-order algebraic equations. However, there are a lot of applied software solutions for such problems.

In accordance with the proposed model, we solve the problem of obtaining the dynamic characteristics of the system with such physical parameters:

- elementary fiber diameter $d$ = 0.003 mm;
- elementary fiber length $L$ = 35 mm;
- size (the ball diameter) of inclusions $v_1$ = 0.3, $v_2$ = 0.1, $v_3$ = 0.5, $v_4$ = 0.2 mm;
- Basalt density p = 2300 kg / m3 = 2.3 mg / mm3;
- basalt elasticity modulus $E$ = 100 GPa.

Then the fiber rigidity coefficient is the following:

$$c = E \cdot \frac{S}{L} = 20.20 \frac{\text{Н}}{\text{м}} = 2020 \text{ мг/мм}$$

Where $S$ - the elementary fiber cross-sectional area for the known d.

The inclusion masses for known $v$ and p are respectively equal to $m_1$ = 0.01035, $m_2$ = 0.00038, $m_3$ = 0.04792, $m_4$ = 0.00307 mg.

The damping coefficient does not affect the critical frequency, but affects the amplitude of the mass oscillation and is assumed to be $a$ = 1 mg / s.

The critical frequencies of the system with respect to the matrix (13) were thus ωK1 = 517, ωK2 = 86, ωK3 = 29, ωK4 = 183 Hz.

The natural frequencies of the individual masses according to the formula (6) are respectively equal to ω1 = 70, ω2 = 301, ω3 = 33, ω4 = 127 Hz.

The system solution (1) with frequency impact at the lower critical frequency $x = \sin(29t)$ is shown in Fig. 2.

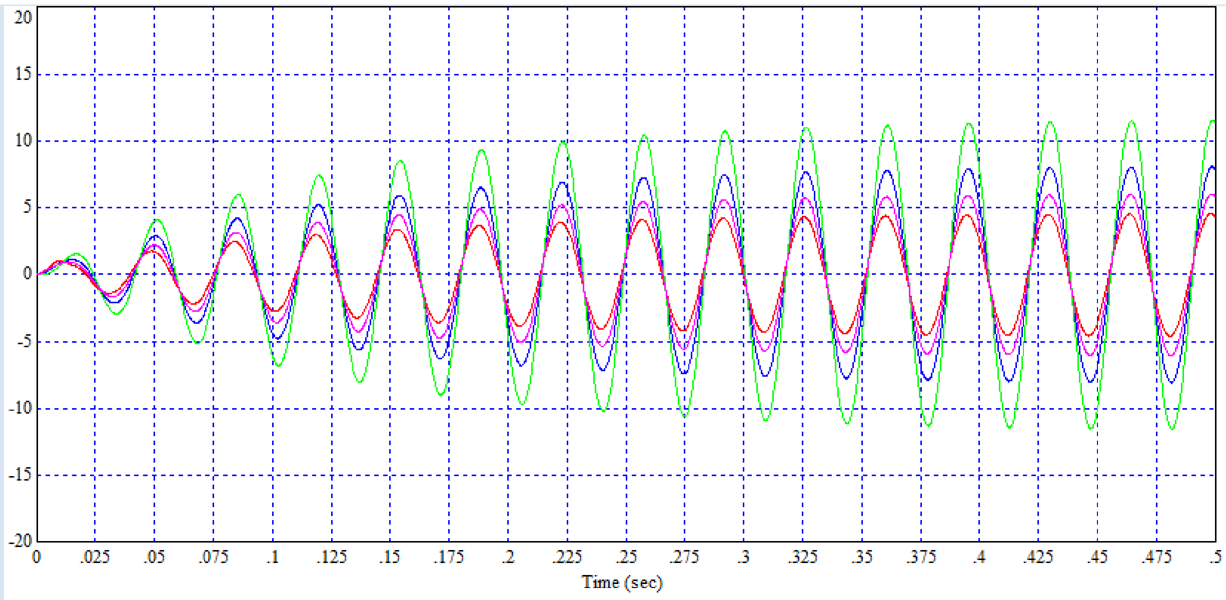


*Figure 2. Dynamic characteristic for x = sin(29t)*

The system solution (1) with frequency impact at the second critical frequency $x = \sin(86t)$ is shown in Fig. 3.

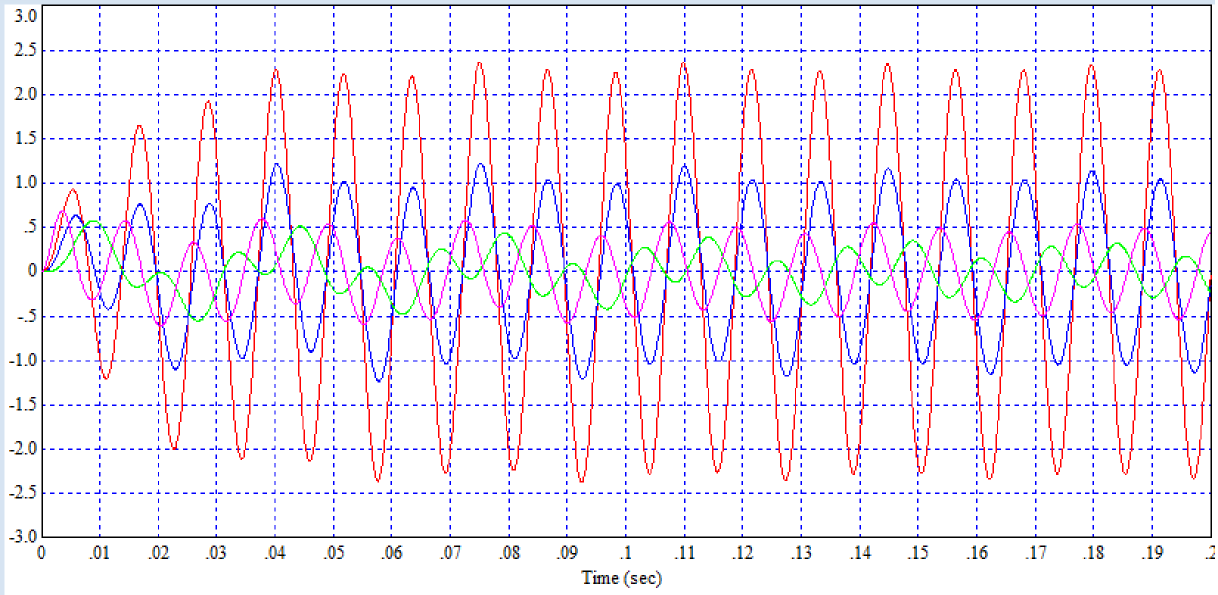


*Figure 3. Dynamic characteristic for x = sin (86t)*

The system solution (1) with frequency impact at the next critical frequency $x = sin\ (183t)$ is shown in Fig.4.

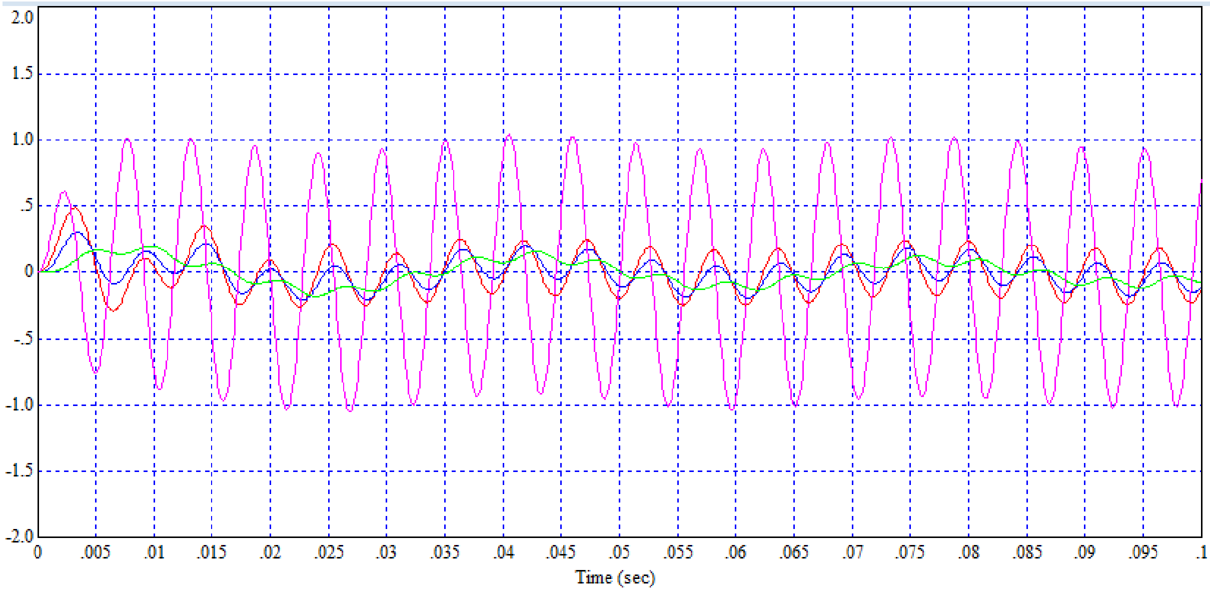


*Figure 4. Dynamic characteristic for x = sin(183t)*

Figure 5 shows the amplitude-frequency characteristics for all masses, calculated from formulas (12).

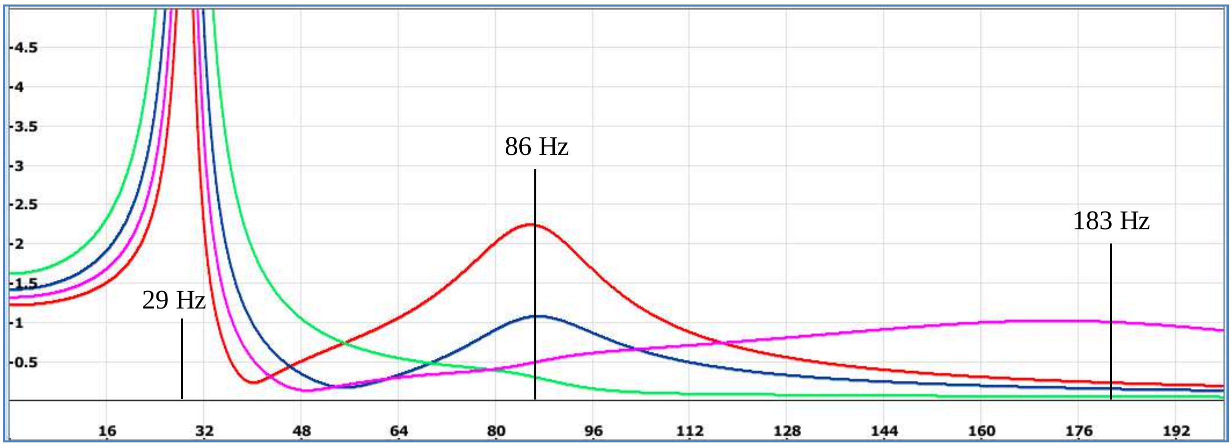

Figure 5. Amplitude-frequency characteristics

It is obvious that the largest resonance amplitude is observed at the very first (lowest) critical frequency of the system (Figures 2 and 5).

However, the inertia force that detaches the inclusion from the fiber, depends not only on the amplitude, but also on the oscillation frequency:

$$F_i = m_i A_i \omega^2 \tag{14}$$

Figure 6 shows the dependence of the detachment force of all masses on the frequency, calculated by formula (14).

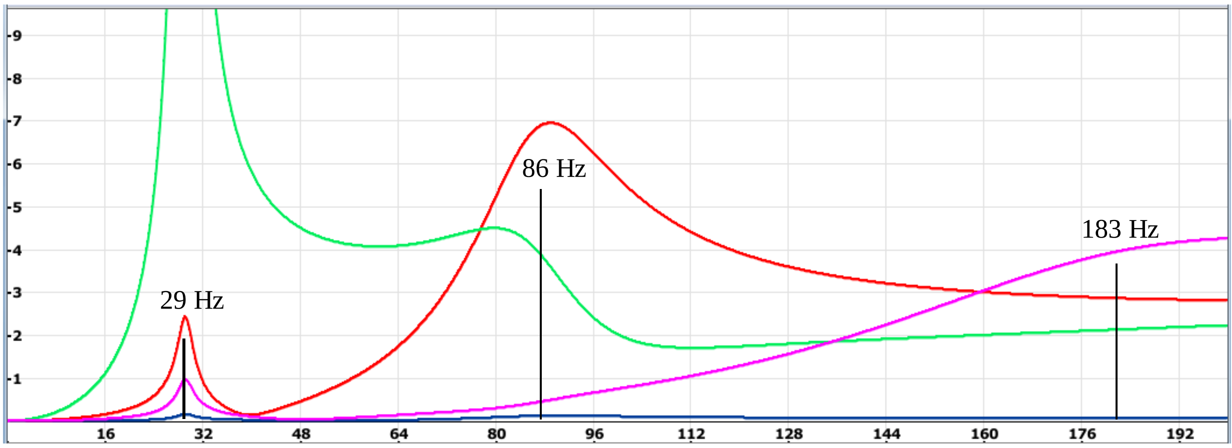

*Figure 6. Detachment force of masses*

So for the first mass (the red line in the graphs), the amplitudes at the first (ascending) three critical frequencies will be 5, 2.3 and 0.2 mm (Figures 2, 3, 4 and 5). The corresponding detachment forces (Fig. 6) will be 2.5, 7 and 3 µN. That is, the greatest detachment force is not at the largest amplitude and not at the highest frequency.

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**Review process**

Single-blind peer review process.