Environmental Modeling - Alternatives and Use in Interdisciplinary Education

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***Abstract*— Quality assessment of higher education belongs to the long term priorities of the EU school policy. Higher education, and its quality, is a key factor determining the future career of the youth. A natural part of the higher education should be also the involvement of students in various research and development activities. The article presents an example how this can be done by the means of the undergraduate students' research and professional activities focused on the environmental issues, what is a topics the students are interested in very much. Moreover it is a topic supporting also interdisciplinary relations of different research and study fields. In our case the interdisciplinary dimension was related to the anthropogenic sources of emissions including industrial processes, agriculture, traffic, mining, energetics and others.**

Keywords - education; interdisciplinary relationship; chemistry; environment; air pollutant; model, student´s research

I. INTRODUCTION

We live in a knowledge economy, a knowledge society [1-2]. Education enables us to integrate knowledge and extend cognition [3-5]. An effort to increase the quality and outcomes of school work is being given various terms [1]. Efficiency equals effectiveness [6]. It is contingent on the full package of modernization elements. Effectiveness and modernization are closely connected to optimization from the points of view of both meaning and function [1]. The optimization of educational processes is represented by the choice of the best alternative for the process arrangement that, in the course of the assigned time, enables maximal possible efficiency in terms of solving particular educational tasks [7]. If we want to respond to up-to-date social demands regarding contemporary school systems, we must keep in mind that engineering education has a tendency towards the integration of efforts, and this trend is also increasingly obvious in the area of teaching chemistry. From this point of view, chemical analysis, with regard to which cross-curricular attitudes which are implemented, indisputably plays a very important role.

II. ENVIRONMENTAL CHEMISTRY NOT ONLY THEORY BUT ALSO PRACTICE

Research activity is based on teamwork, the use of knowledge from particular scientific disciplines, as well as on the ability to use the obtained knowledge and information in real practice. The application of theoretical knowledge to activities focused on environment protection allows better understanding of sustainability rules, skills, understanding of biospheare, economical and ecological connections, problems of environment from local and global point of view and also their reasons. It closely connected with development of relationship between people and environment. It is not easy to apply chemical analysing methods with the aim to monitoring pollution in selected area. In this case the interdisciplinary approach is necessary.

At Department of Chemistry, Faculty Natural Sciences (FNS) in the Constantine the Philosopher University (CPU) in Nitra many years students are taught in bachelour study programm Environment Chemistry.

During bachelour studies (3–4 years) are daily and external students of study programme Environmental Chemistry taught in subjects-chemical; physical; mathematical; environmental; biological. Students studied different types of courses such as laboratories exercises General Chemistry, Inorganic Chemistry, Chemical Calculation, Organic Chemistry Analytical Chemistry, Biochemistry, Physical Chemistry, Safe Working with Chemicals, Computer Network, Computer-Assisted Molecular Modelling in Chemistry, Analysis of Components of the Environment, Physics for Scientists, Ecosystem Ecology, System Approaches in Ecology, Environment of the SR, Environmetal Monitoring, Bioethics, Basics of Plant Physiology and Genetics Laboratory Control of Biological Products and Environmental Education, Chemistry of Synthetic Polymers, Gardening, Fauna in Landscape Research and many laboratóries exercises.

Students of mentioned study programm were integrated to project focused on air pollution within Student Research and Professional Activities (SRPA). In the frame of the bachelor degree program dealt with a project orientated on the issue regarding selected components of the environment.

The project came from knowledge and practical skills which students acquired during their studies.

In the course of this project we used a long-term cooperation between the lecturers and students in various research tasks focused on different environmental problems. We monitored the state of emissions and imissions of selected pollutants (SO_2, NO_x) and the relationship between them, climate and the shape (profile) of the terrain.

The contamination model was designed and tested by using of the optical range. With these research activities being taken into consideration courses like transfer of innovation to the curriculum.

Students had an opportunity to take part in the solution of some selected sub-tasks associated with the current issue.

Based on this cooperation was some remarkable results which enriched both participating sides (students, and also their lecturers). This paper focuses on the aplication of theoretical knowledge and practical skills.

It is oriented on analysis of selected air pollutants and modeling of their concentration in the environment. Selected theme is very broad, open to innovation, acquiring and sharing experiences and technical and technological processes. It could be constant updated.

Presented work offer the possibilities of future mutual cooperation. It gave opportunities to better understand teoretical problems in praxis, presented possibilities of practical application oriented on chemical analysis and work in laboratory and also in terrain.

III. Research with Regard to Air Contamination (SO_2 and NO_x)

The air is one of the environment components, which is the irreplaceable and essential to almost all life forms on the Earth. One of the components of the natural environment adversely affecting industrial activity is air.

The sources of air pollutants in terms of global emissions, which result from human activities, differ in the type of pollutants [8-14]. Their impact on the environment is different; in that they can demonstrate a decrease, stay the same, or increase and cause economic loss [15, 16]. Traditional pollutants include contaminants that we know exist in several countries that have monitored changes in their their concentration [17, 18]. It is obvious that the worsening air quality is caused by gases such as sulphur dioxide (SO_2) [19, 20], nitrogen oxides (NO_x) [21, 22], carbon oxides (CO_x), hydrocarbons and particulate matters. Air pollution has been a serious global problem [23]**.** The air is exposed to many influences and pressures that reflects negatively on him not only itself, but also on other elements of the environment. Atmospheric pollution, which is a mixture of gas and mineral or organic particles, is complex and is mainly defined by the negative effect it causes to the environment and human health [24]. The increasing concentration of air pollutants has its impact on water, soil, flora and fauna. Each of mentioned themes provides a valuable guide for development of evironmetal chemistry and environmental modelling which support integrated assessments [25].

IV. Air Contamination Modeling with Focused on the Environment within Student Research and Professional Activities

Air contamination monitoring as a unit for integrated land-use assessment presents a number of methodological and technical challenges that exemplify many of the key issues involved in linking chemistry with environmental models.

The basis for environmetal modeling in land is fact that environmental, ecological, social and economic systems require integrated assessment [26-27]. It is connected with number of challenges for modelling, because these systems operate in fundamentally different ways [28].

Environmental models have focused on sustainability of resource use and the maintenance of species´ habitants [29].

Ongoing research focused on developing a new generation of environmental simulation models has significantly increased the demand for land surface data and the corresponding need for the expanded use of remote sensing and geographic information systems) technologies (GIS) for database development [24].

Student Research and Professional Activities (SRPA) is important for students in terms the students preparing for the practice. SRPA are open for students from different fields of study. The SRPA collaboration and participation process involving multiple disciplines, perspectives and stakeholders. They could learn how to present the results of their work.

Within Student SRPA at Department of Chemistry CPU in Nitra students are focused on monitoring of selected environmental element contamination-air.

Within SRA, several works focused on the concentration of sulphur dioxide and nitrogen oxides in the air environment closed to the chemical factory in Duslo Šaľa, Inc. (Slovakia).

The chemical factory Duslo Šaľa, Inc. is situated near the town Šaľa (in Nitra region), on the west of southern Slovakia, in the central part of Podunajská lowland. (Figure I.).

Students which participated on project they worked in laboratories (analysing, preparing solutions) and also in terrain (preparing air samples). They acquired knowledge about complexity and problems with analysing of samples. Application of theoretical knowledge in practical activities offer students opportunities better understand sustainability management. Practical presentation of causes and effects connected with the environmental monitoring help students understand to theoretical environmental problems. The theory and praxis point of view brings information which is important

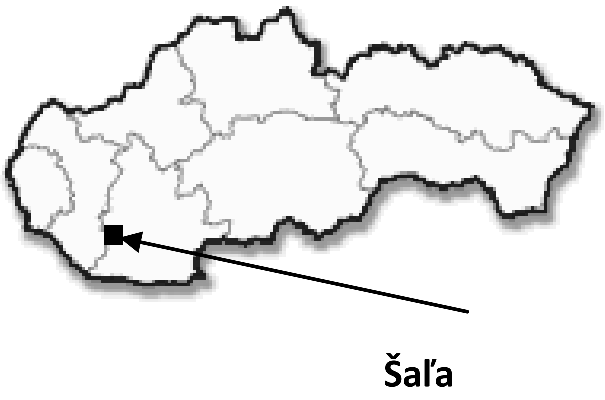

Figure 1. Monitoring Area

for air protection and also for sustainability management.

V. Materials and Methods

We monitored area where is one of the most important chemical factory in Slovakia located. Student´s research activities we could divided into five parts: collection of air sample, analysis of air sample, cartographical processing, using of method of differences and similarities and analysis of air quality.

Selected area (Sampling Area)

Chemical factory Duslo Šaľa Inc. is situated on south-western part of Slovakia, near Váh river. Its area is about 2,5 km^2. Location of chemical factory buildings was approved in 1958. There is large hygienic protected area because it is situated close to drinking water sources, important transport infrastructure and manpower. The basic row material is natural gas, industrial salts, phospate, potassium salt and phosphoric acid. Some of products from one operation could be used in working process of another one. Poisons which are potential dangerous for our environment and substants which are harmful to health, fuels and also solvents are processed in the factory. After 1976 the factory was focused on production of agrochemicals and materials for rubber industry. Development of processing was connected with resolving environmental problems and realisation of difficult ecologic programs. From widespread development programm were interesting parts which decrease pollutions of area f.e. substitution of coal by gas and environmental monitoring. Phased factory construction were close connected with solving of environmental problems and choosing environmental friendly technologies. Researches were focused on sources of air pollution. Ecological program of the factory was aimed to achievement of ecological parameters in accordance with legal standard.

Air Contamination (SO_2 and NO_x) in Student Research Activities

The basic material for the selected methodological research procedures and evaluation from the point of view of the environment, are maps and written documents, plans, cases and the results of their own analyses. Observing the atrophic effects, the objective has focused on the analysis of air samples. The research process we could divide into the following phases:

A. *Collection and analysis of air samples.*

- observation of SO_2 concentration by method of short-time cumulative monitoring [31],
- measuring of NO_2 concentration in solution of trietanolamin. The intensity increases in relation to the concentration of NO_2.

On the base of average daily concentrations were analyzed of SO_2 and NO_2, used for further interpretation of results; e. g. average year values of concentration of pollutants from the observed territory. In last years data from the monitoring station in Trnovec nad Váhom (SK405001) were used, from industrial station off-town region (N 17°55'44", E 48°09'00").

B. *Cartographical processing and defining the territory matrix.*

C. *Method of differences and similarities*-evaluated by the optical scale of Czekanowsky method displaying socio-economic phenomenon.

D. *Analysis of relationship of air quality in observations of antropic influences in the landscape-ecological evaluation of the region.*

VI. Results and Discussions

Differences in the concentration of SO_2 and NO_x during the observed period were recorded in steps and monitoring sites (Trnovec nad Váhom, Duslo Šaľa, Inc., Šaľa-Veča).

In 1999 the average mothly concentration of SO_2 in both monitoring places (Trnovec nad Váhom, Duslo Šaľa, Inc.) was in the interval 0.10-2.90 $\mu g.m^{-3}$. In 2000 the value of average monthly concentration was within 1.00-22.00 $\mu g.m^{-3}$. In 2004 the values of imissions decreased in half in comparison to 2003 (Table I.), while the volume of emissions was the same (Table II., year 2003=1196.76 t, year 2004= 1050.79 t) and average year temperature was lower than in 2003. In 2005-2012 the values were in the intervale from 4.54 $\mu g.m^{-3}$ to 27.78 $\mu g.m^{-3}$. (Table I.)

The data show the highest year average concentration of SO_2 was in 2007 (27.78 $\mu g.m^{-3}$) in Trnovec nad Váhom. The importance of the concentration of SO_2 is mainly in terms of the significant synergistic effect in connection with other matters [26, 27]. The monitoring of gaseous emissions enables not only the researcher to determine real access channels and follow the concentration of pollutants, but also to search for, and apply ways in which to eliminate them. The values relate to the development of the production of selected emissions in the monitoring period in Duslo Šaľa, Inc. and the climate conditions which influence the dispersion of monitored matters in the environment.

TABLE I. Concentration of SO_2 (1999-2012)

Measurement values [$\mu g.m^{-3}$]	1999	2000	2001	2002
Duslo Šaľa, Inc.	1.52 [a]	5.27 [a]	5.60 [b]	3.33 [b]
Trnovec n. Váhom	1.13 [a]	6.17 [a]	12.62 [b]	7.83 [b]
Trnovec n. Váhom	**2003**	**2004**	**2005**	**2006**
	12.51 [c]	6.09 [e]	10.02 [e]	6.62 [e]
	2007	**2008**	**2009**	**2010**
	27.78 [d]	7.65 [e]	17.24 [e]	8.75 [e]
	2011	**2012**		
	4.54 [e]	8.89 [e]		

Sources:

a. M. Feszterová, 2003,
b. M. Feszterová, 2004;
c. Regional Office – Departm.of Environment Nitra, 2010;
d. Departm. of Quality and Environment,, Duslo Šaľa, Ltd., 2010;
e. Department of Environment, Duslo Šaľa, Ltd. 2013.

TABLE II. Production development of selected emissions in DUSLO, ŠAĽA INC. [10^3 kg.rok^{-1}]

Year	**1999**	**2000**	**2001**	**2002**
NO_2	756.81	860.32	670.90	852.46
SO_2	1417.36	1503.49	1506.46	1148.82
	2003	**2004**	**2005**	**2006**
NO_2	665.3	789.51	803.21	812.32
SO_2	1200.86	932.09	1082.61	689.45
	2007	**2008**	**2009**	**2010**
SO_2	5.88	5.28	3.97	2.34
NO_2	606.66	568.87	593.77	567.50
	2011	**2012**		
SO_2	2.16	1.99		
NO_2	733.54	647.80		

TABLE III. Concentration of NO_2 (1999-2012)

Measurement values [μg.m^{-3}]	**1999**	**2000**	**2001**	**2002**
Trnovec n.Váhom	9.16 [a]	15.00 [a]	20.57 [a]	16.08 [a]
Duslo Šaľa, Inc.	14.57 [a]	14.26 [a]	23.91 [a]	15.99 [a]
Šaľa – Veča	16.52 [a]	14.47 [a]	23.46 [a]	16.76 [a]
Trnovec n.Váhom	**2003**	**2004**	**2005**	**2006**
	25.78 [d]	27.18 [d]	25.79 [d]	15.31 [d]
	2007	**2008**	**2009**	**2010**
	75.08 [c]	27.59 [d]	21.54 [d]	25.49 [d]
	2011	**2012**		
	22.42 [e]	23.17 [e]		

Sources:
a. M. Feszterová, 2003;
b. M. Feszterová, 2004;
c. Regional Office - Department of Environment in Nitra 2010;
d. Departm. of Quality and Environment, Duslo Šaľa, Ltd., 2010;
e. Department of Environment, Duslo Šaľa, Ltd. 2013.

In 1999 average year concentrations of NO_x flow 9.00 μg.m^{-3} to 17.00 μg.m^{-3} (Table III.) with increased values in winter. In 2000-2006 these values were in the intervale from 14.00 μg.m^{-3} to 27.00 μg.m^{-3}. By comparing the values of bad dispersion conditions (NO_x) we can see that they are close to the source. The collected data are related to weather conditions but it can also reflect changes in production, or good technical level of emmission catching devices. Nitrogen dioxide (NO_2) is a major indicator of road traffic [24].

The measured values show the highest year average concentration of SO_2 (27.78 μg.m^{-3}, Table I.) and NO_2 (75.08 μg.m^{-3}, Table III.) were detected in 2007 in Trnovec nad Váhom.

The volume of emissions of SO_2 and NO_x in 1999-2012 depends on the amount and quality of fuel and the production requirements (Table II.). In most cases, the chemical components are linked to anthropogenic activities, which are numerous in very populated areas; this is why atmospheric pollutants are most often measured in the urban areas [24].

We found negative correlation between emission NO_2, which was produced in Duslo Šaľa, Inc. and measured in air (r=-0.509; p=0.015). Between emission SO_2, which was produced in Duslo Šaľa, Inc. and measured in air (r=-0.453; p=0.059) was not found correlation.

The diffusion study has been figured out on the basis of selected emissions development in the monitored period from the chemical factory Duslo Šaľa, Inc. The concentrations of pollutants were counted with the help of Gaussian air pollution model [34, 35]. (Figure II.-III.)

The obtained data were analyzed using statistical software Statistics Portable. Correlation analysis (Spearman correlation) was used to determine the relationships between the NO_2, SO_2 emissions produced in Duslo, Šaľa, Inc. and measurement values NO_2, SO_2 in the air. Significant correlation coefficients were tested at $p<0.05$.

VII. SECONDARY STRUCTURE IN EMISSION AREA SO_2 AND NO_2

It is very difficult to evaluate the qualitative changes in the secondary landscape recalled by atrophic phenomenon. Our evaluation resulted from observations of the diversity of antrophic phenomenon, i.e. the more the phenomena appear, the more negative their impact will be on the region. This relationship is expressed in the system of phenomenon that is endangered and endangering.

The synthesis adopts the following steps: the evaluation of concentration of emissions in the region, the interpretation of the secondary structure, the danger to secondary structures by selected contaminants.

A. *Transport Intensity in the Region*–covered all communications and evaluative matrix with a number code

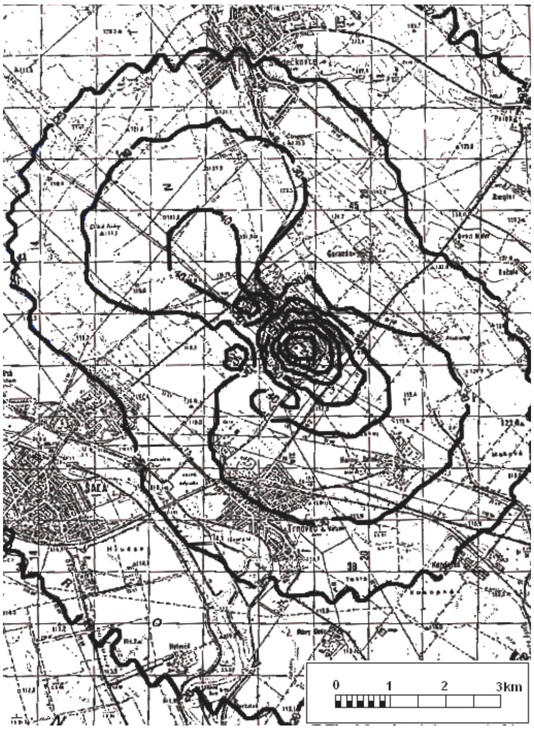

Figure 2. DISTRIBUTION OF THE MAXIMUM SHORTTERM NO_2 CONCENTRATION [μg.m^{-3}]

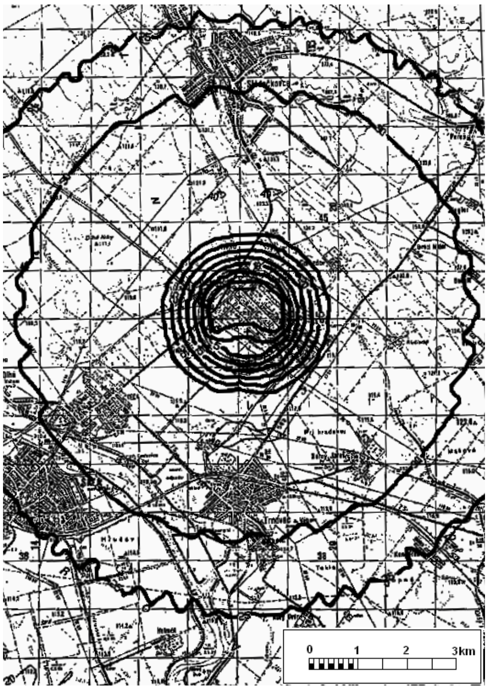

Figure 3. DISTRIBUTION OF THE MAXIMUM SHORTTERM SO_2 CONCENTRATION [$\mu g.m^{-3}$]

and graphic display of the transport density.

B. Occurrence and Intensity of Housing Development– covered the occurrence of housing stock in the region. Analytic maps with isolines of concentrations of NO_2 for secondary structure (transport intensity and housing) were synthetically processed by the method of parametric analysis.

C Evaluation of Secondary Structure and Selected Socio-Economical Phenomenon - synthesis results were mapped.

VIII. CONCLUSION

In the presented work, data with regard to air pollution due to SO_2 and NO_x and their impact on the environment were summarized. The basis of this work arises from the selection of sources (industrialisation, transportation, urbanization), which led to the increase in pollution and, at the same time, reflected the development of the chemical industry in the region. The concentration of air pollutants depends on both anthropogenic and climate factors. We observed factors of pollution concentration in the air and their transport, provided information and data which can be applied to an evaluation of the situation. The analyses of the region from the point of view of secondary landscapes adversely affected by SO_2 and NO_x emissions are what was expected, in terms of the demands of industry, and how these requirements are expressed in the region.

The urbanisation increases the pollution of our environment. The EU Directive (2008/50/EC) establish requirements for air quality modelling, including the definition of the modelling quality objectives, as a measure of modelling results acceptability. The public is not adequate inform about negativeš influences of air pollution. Based on studies people with heart or lung disease, older adults and children are at greater risk from air pollution. The impacts on public health and nature are disregarded.

Our article is oriented on the application of theoretical knowledge and practical skills to environmental monitoring. Due to closely connection it is possible to use it in the educational process, or Student Research and Professional Activities and other research activities in the field of environmental chemistry and environmental education. We used the knowledge of many the chemical, physical, mathematical, environmental and biological disciplines which were part of bachelor studies and laboratory exercises to individual disciplines. Mentioned activities are so interesting and they could positively motivated students.

The current educational concepts force us to search for more efficient ways to work with students. The requirements are increasingly higher over time, and schooling plays an important role. Scientific and research development has led to an increasing interest in scientific and educational innovation. Engineering education is the activity of teaching knowledge and principles related to professional practice. We are currently in the process of discussing what and how to teach, and which methods influence the quality and efficiency of the educational process. These discussions do not touch only upon education and personal development, but also reflect on practical life. This means that it is crucial for the learner to be able to apply knowledge in practical terms in their professional life. The technical characteristics of modern society means that schools should offer clarity of thinking and should develop personal experience, i.e. there is a strongly need to connect school to life. Education and training in science disciplines (e.g. chemistry, environment) is very closely related to OSH. The state of monitoring environmental element-air, affects the health of the population.

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The analysis of geomorphic indicators for the definition of the extension of Pleistocenic glaciers within alpine valleys

Method and applications

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Abstract— **The paper explains the use of geomorphic indicators (such as erratic blocks and tors) as indicators of presence/absence of glacier passage during the Pleistocene. Thanks to a census of over 600 erratic blocks, and their distinction from the tors, the reconstruction of glacier extension is possible also inside the alpine valleys. As example, the paper shows the reconstruction of Pleistocene glacier extension within the Lanzo Valleys (Western Alps, Italy). The reconstruction of the Pleistocene glaciers is enough accurate for applications to geology or biology. As example, we show the application of method to the problem of origin of endemic species living in caves: thanks to the definition of the relationships between caves and glaciers during the Pleistocene, it is possible subdivide the caves in five groups with different history of habitat and of climatic changes.**

Keywords: glaciers; geomorphic indicators; erratic blocks; tors; Western Alps

I. Aims of work

The extension maps of glaciers in the Pleistocene are very commons, but generally they have only purpose of disclosure, so they represent well only the glacial tongues at the mouth of the glacial valleys in lowlands. These maps often reconstruct the ice-covered part of the Alps more on the basis of the imagination than on geological evidences, especially where the glaciers are never arrived as far as the Po Plain.

However, a better reconstruction would have significant importance, e.g. for the biogeography studies. One of these is the study of areas where fauna can survive during the ice ages and the interglacial stages. For example, the "jardin des glaciers" hypothesis as the Orrido di Chianocco (Susa Valley) where can surviving species of thermophilic plants would not hold if we reconstruct, even approximately, the extent of glaciers.

In some modern studies, as the project CaveLab [17], the biologists study the speciation of the subterranean fauna. In fact, the glacier extension in the area surrounding the caves should influence the speciation: the cold climate and the deposition of glacial till (that is rich of interstices) can promote the spread of cryophilic species, or on the contrary the obstruction with ice of entrances of the caves and the cover of interstitial environments can isolate some species. A precise reconstruction of the ice cover during the Pleistocene is therefore always more useful.

The aim of this work is provide an objective tool to determine which areas have never had ice cover, and the contrary.

II. Introduction

The geologists have faced the problem of the extension of Alpine glaciers during the Pleistocene, immediately after the recognition of the validity of the theory of Ice Ages. Initially, with the recognition of morainic amphitheatres and erratic boulders, geologists have implicitly assumed that the Alpine glaciers during the ice ages were so large that their tongues have reached the Po Plain [10]. The current idea was a single ice cap from which outlet glaciers protruded. Within alpine valleys, Gastaldi [13], [14] and Sacco [27], [28], [29] have explained the moraines that lie below those of the "Little Ice Age", with periods of stop or glacial advance ("Dauniano", "Gleitzschiano") who have interrupting the slow and gradual retreat of the glaciers, at the end of the last ice age ("Würm").

The lack of certain glacial evidences in some valleys, the lack of morainic amphitheatre at the mouth of many alpine valleys (in Piedmont, particularly, the Cuneo, Lanzo and Sesia valleys), the discovery that the glacial retreat has been nearly instantaneous at geological time scale, have removed the theory of a single ice cap, at least as far as the Italian side of the

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This study is partially funded also by University of Turin project "Contrasti termici e umidità in microclimi di grotte di particolare interesse naturalistico" (MOTMRIL013).

The Piedmont Region in agreement with University of Turin (coordinator Michele Motta) has funded the census of erratic blocks for implementing the Regional Law n. 23 of 21.10.2010 "Development and preservation of erratic blocks that have high landscape, environment and history value".

Alps, in favour of the idea of large compound glaciers [26], [12] and [1].

With regard to the Piedmont, the papers of Sacco are even today in many valleys the best maps available for the glacial landforms, although Sacco has mistaken some accumulation of landslide or rock glacier for moraines. E.g. the blocks between Mondrone and Martassina (Lanzo Valleys, fig. 8), actually deposits of debris flow [10]; the group of blocks near Exilles (Susa Valley), believed of glacial origin still in the draft of Geological Map of Italy 1:50,000; actually, some rockfalls have deposed these blocks in historical age [6]. Roughly speaking, the recent mapping of the glaciers during the Pleistocene has still as basis, the moraines arrangement that Sacco has surveyed, and the distribution of glacial drift on the geological maps. Unfortunately, sometimes the geological maps are older of the Sacco works (Susa, 1890-1910, Oulx, 1910-11 ...) and all maps are inaccurate in the assignment of surface formations (e.g., the maps classify the deposits that are dubious as "moraine and debris mixed ").

In literature the identification of areas never covered by glaciers has focused almost exclusively on lowland areas (e.g. [11]). In particular the highly evolved soils are materials easily removable by the glaciers, so they mark absence of glacial passage after their developing age (almost always during Middle Pleistocene). In literature, the highly weathered rocks (grus, clays rich in kaolin) of certain pre-Alpine areas as Biella and Belmonte (Cuorgné), with a weathering that is starting in early Pleistocene (Gelasian), have the same meaning.

The main purpose of most recent research projects, such as PROGEO [15], [16], is the exact attribution of each moraine to its age, or glaciation. The recent works have used for this purpose: stratigraphy of wells, absolute dating... This work is almost always limited to morainic amphiteatres in plain. In the few cases in which papers have examined the alpine valleys (e.g. [8]), the main purpose was almost always chronological reconstruction of the glacial retreat at the end of the last Ice Age (e.g. in Aosta Valley [1], p. 25). This work (which led to the abolition of the late stages of Ice Age, proposed by Sacco) does not require the determination of the overall extension of the glacier, but only the position of the end moraines ([1], p. 29).

The study on the boulders of the morainic amphitheatre of Rivoli-Avigliana [3], and the subsequent surveys for the Piedmont register of the erratic blocks [19] have had the need to distinguish between tors [9] and erratic blocks. Therefore, their results have provided new elements for the delimitation of the areas covered by glaciers in the nearby mountains to the Po Plain. The tor distribution is clearly separate from that of the erratic blocks, suggesting a slope evolution entirely without glacial processes. The tors are small blocks easily removable by glaciers, and requiring a long period of shaping [2]: so, certainly they are among the oldest landforms in the Alps.

III. INDICATORS

The method employed start with the identification of forms and deposits that are indicators of a glacier passage or, conversely, that are indicators of the lack of glacial processes in the past. Indicators generally extend their information geometrically in an area, with limits defined according to the type of indicator (fig. 1). The limits of this area are:

– for the indicators of glacial passage, the part of mountain slope that is limited upstream by the contour line at the same altitude of the indicator, and that is limited downstream by the bottom of the valley and by line of maximum gradient between indicator and thalweg (i.e. the bottom of the valley).

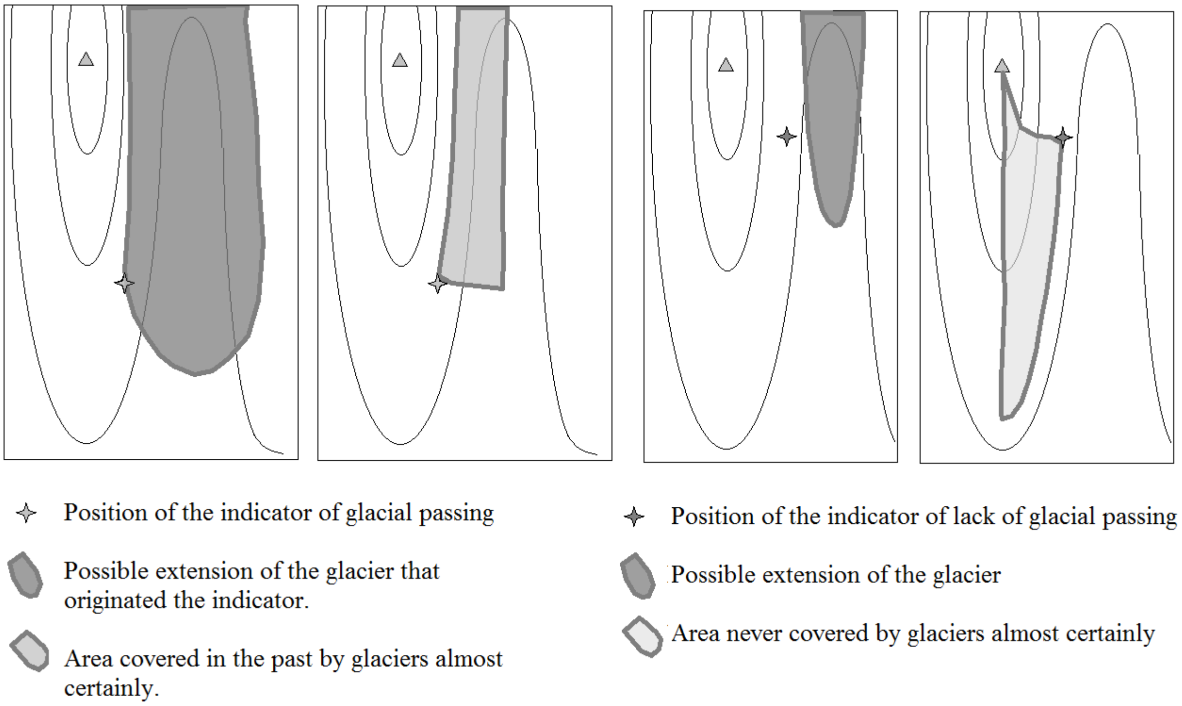

Figure 1. Extension criteria for the area subtended by an indicator.

– for the indicators of lack of glacial passage, the area that has the contour line at the same altitude of the indicator as downstream limit, the watershed and the line of maximum gradient between indicator and watershed as upstream limit.

Some indicators are unquestionable, because they may result solely from the presence (or absence) of glacial processes. Other indicators are doubts, because they may be interpretable, as well with low probability, as resulting from other geomorphic processes.

The indicators used are as follows.

A. *Unquestionable glacial indicators:*

– *Erratic blocks*, boulders transported by glaciers (fig. 2). The recognition criteria for the distinguishing between erratic blocks and others big blocks are [20]: lithology different from the bedrock; position independent from the bedrock structure; surfaces with traces of glacial transport (i.e. faceted rocks and glacial striae). The register created for the Piedmont Region by [22] lists the major erratic blocks of Piedmont: it contains the description of more than 600 blocks, or boulder groups.

Figure 2. Pera Aguà (Susa Valley) has all the characteristic of an erratic block: this block of amphibolite is resting on a base of white marble, the contact surface is irregular and with lenses of till, the lower part of boulder has glacial striae. This block shows that the ancient Susa Glacier in this point is arrived as high as the altitude of the block, a hundred of meters above the bottom of the valley.

– *Glacial till* and *Moraines*, deposits and forms of glacial accumulation that are obtained (with the limitations described above) from the Geological Map of Italy 1:100,000 or 1:50,000 (if available) and from the works of Sacco.

– *Faceted rocks* and *Polished bedrock surfaces with glacial striae* (fig. 3), forms of glacial abrasion. They have only some quotations in the works of Sacco and of Motta & Motta [18], [20], [22].

B. *Questionable glacial indicators:*

– *Diamicton* is a deposit with texture similar to the till, but of questionable origin. This list cites the diamicton for completeness, but the geological maps of Piedmont not report this term. These maps report sometimes the equivalent terms of “moraines mixed with debris” or “questionable moraines”.

– *Truncated spur*. The main glacier of the valley often has shaped several rocky walls on the arêtes of secondary watersheds. A database in [23] collects partly these landforms. Similar landforms are present in fluvial landscape, such as a triangular wall that interrupts a secondary watershed, because of the presence of a fault line, or due to linear erosion of a watercourse. Therefore, for the recognition is mandatory a complete analysis of the local geology, and of the landscape.

Figure 3. Polished bedrock with glacial striae and the typical triangular-shaped scars of the plucking. Susa Valley.

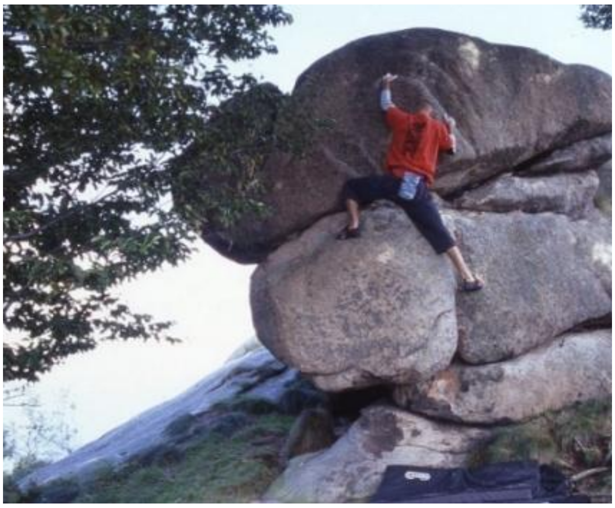

Figure 4. Typical (summit) tor: a pile of round blocks separated, but still closely in contact one another. Mottarone Mt., on the watershed between the glacial lakes of Orta and Maggiore.

- *Rocky steps* are on gentle slopes (e.g., bottom of the valleys). They are landforms of the subglacial environment [25]. If this landform not has glacial microforms, it is quite similar to a landform due to difference in rock resistance.
- *Cirques* are deep depressions on the upper part of mountain slope, under a steep wall. In Piedmont, they are relict landforms, developed almost only during the Pleistocene glaciations [5]. A range of mountains arranged by chance in semi-circle can (rarely) be confused with this landform.

C. *Unquestionable indicators of lack of glacial processes:*

- *"Vetusuoli"* are much evolved (under warm climate) soils. In the climate of Piedmont (temperate), they rest only on rocks of Middle or Lower Pleistocene: so, they are indicators of lack of glacial passage during the Upper Pleistocene (last glaciation). The vetusuoli of Piedmont are mapped in [31].
- *Tors* are piles of rounded blocks, born from the breaking up of a single rocky outcrop (fig. 4). The selective weathering has shaped these landforms [2]. Reference [9] has proposed a classification depending from the tor location: summit tor, on hill and ridge crests; spur tor, at the ends of ridges or spurs; valleyside tor, along valley sides. In Piedmont climate, the weathering can shape a tor only in a very long time. When a glacier overcame a tor, it removes easily the small blocks of the tor, and the bedrock becomes polished. Therefore, in the Alps the tors are only or in minor valleys that are flowing directly to Po Plain, or on the mountains close to the mouth of the glacial valleys in lowlands. The tor can be confused with an erratic block (fig. 5). For the recognition criteria, see [20]; the more important are the spacing frequency of the joints similar to that of the bedrock, and the parallelism of the fractures of tor and bedrock. A tor can be confused as well with the pillars resulting from landslides (e.g. in Piedmont the famous Torre delle Giavine, i.e. Landslides Tower, in Sermenza Valley), or ancient quarrying works (e.g. in Susa Valley the Roc di Pera Piana, near Avigliana). In Piedmont, the studies for the realization of the register of erratic blocks have faced largely the recognition of the tors. Reference [23] has reported a partial list of them.

Figure 5. The spur tor near Borna del Servaj (Ala Valley) very resembles an erratic block: it is located within an alpine valley, at 1450 m a.s.l., and on a rounded outcrop of the bedrock. Actually, a joint plane separates the block from the bedrock. Besides, this surface and the main sets of joints inside the block, and within the bedrock, are parallel one another; the bedrock surface has many weathering microforms; the rock texture of the boulder and of the bedrock is parallel.

D. Questionable indicators of the lack of glacial processes:

- Weathered debris. The geological maps report often these deposits, sometimes for filling of zones little known because of a lack of outcrops. The presence of thick layers of weathered debris (if in place, and if they are actually weathered debris!) has the same meaning of tors, but it is harder to define their age. In Northern Piedmont, the rocky hills around Biella have often thick layers of very weathered debris, containing kaolin, that are missing in the near Aosta Valley, because of glacial erosion [11]. In Southern Piedmont, the weathered debris are common on the pediments between Alps and Po Plain, underneath of slopes with tors. Some terraces covered by weathered debris are also on the summit of the mountains closer to lowlands; the authors consider them relicts of a large pediment of the "Plio-Pleistocene" (before of the glaciations, corresponding to current Gelasian) [4].

IV. Is the erratic block recognizable from the tors for its honeycomb weathering grade?

An example of an area where the glacial indicators are very close one another is the part of the Susa Valley closer to Po Plains. In this area, the higher slopes of the mountains were higher of the tongue of the glacier. However, these mountains are no very high, so they not had their glaciers at the top. Thus, the glacial forms are only down, near the bottom of the Valley, while the tors are on the higher slopes.

In this area, the position of the lateral moraines, which are all well preserved, also those of the Middle Pleistocene, well outlines the extension of the glacier. This situation is perfect to check if the weathering is stronger on the oldest surfaces, i.e.:

- rocks never covered by glaciers, versus rocks covered;
- rocks that a glacier has shaped during the last ice age (Upper Pleistocene), versus rocks modelled in previous glaciations (Middle Pleistocene).

Of course, the weathering has a variable speed of development in the different rocks. Therefore, we chose to compare the prasinite stone (albite-chlorite-epidote-actinolite schist), the most common rock in this area. To assess the weathering we used the scale of [24].

The fig. 6 is a cross section along the famous abbey of Sacra di S. Michele, the symbol of Piedmont.

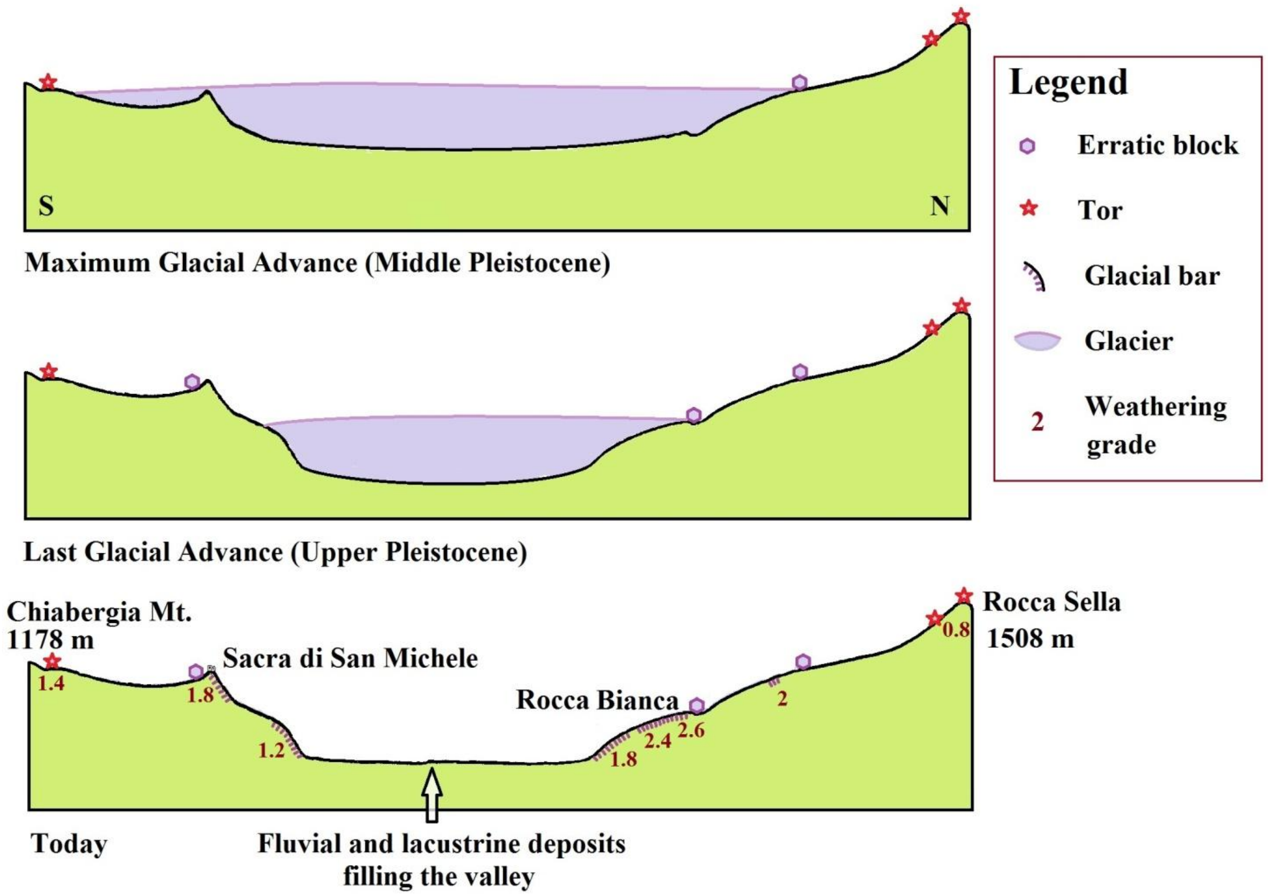

Figure 6. Cross section of the Susa Valley. According to [24], we have done an average of the weathering grade values of five walls that are at the same altitude. Honeycomb weathering grades employed in this study: 0: no visible weathering forms; 1: isolated circular pits; 2: pitting > 50% of area; 3: honeycomb present; 4: honeycomb > 50% of area; 5: honeycomb with some wall breakdown. The section shows that the weathering grade is independent of the geomorphic position or altitude.

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The results suggest that the ranking of alveolar weathering grade is unsuitable for recognise the glacial landforms.

In the climate of Piedmont, the weathering process is quite slow. The honeycomb weathering of the blocks of prasinite cutted for building, including those used a thousand years ago for the Sacra di San Michele, is not very developed (honeycomb weathering grade in the Mottershead classification = 1). However, the little time spent by the last ice age, is clearly sufficient to a stronger weathering of natural surfaces (grade 1-3). Probably, the difference of the weathering grade from one wall to another is due only to a local detachment of slabs from the surface, with the exposition of new rock surfaces without weathering forms.

V. Delimitation of area subtended by an indicator

We have extended the area subtended by the indicators, depending on the meaning of the indicators, upstream or downstream (fig. 1). In the valleys that hold the indicators, we have made always the extension. We extend the area subtended by indicators in tributary valleys only if we can exclude in these valleys the presence of minor glaciers in the past. Therefore in the tributary valleys have to be missing the following indicators:

– altitude of valley head over the 2000 m a.s.l.;
– glacial cirques, also if they are dubious;
– glacial drift and diamicton.

VI. Results of the application of the method to the Alps

Fig. 7 and 8 are reporting the distribution of major geomorphic indicators in Lanzo Valleys, and fig. 9 the consequent reconstruction of ancient Lanzo Glacier during the Pleistocene.

The extension of the weathered debris is not analysed (even if they are abundant near to tors), since their distribution is badly known in some parts of the area. However, the proximity between the areas subtended by the opposite indicators, allows a very good reconstruction of the major glaciers. Instead, the quality of the reconstruction is less for the small glaciers at high altitudes. The glacial deposits of small glaciers are not very extended, whereby frequently are not mapped on the geological maps; moreover, these deposits are often completely eroded or covered by recent deposits, such as landslides or talus cones. Even the count of the numbers of erratic blocks in these zones gives a number very less than the actual. In fact, the register of boulders had as objective to identify and protect the boulders of particular scenic interest, historical or natural: a boulder of high altitude hardly has features that fall between those of the boulders protected by law, while it is likely that a boulder near the bottom of the Valley, easily seen and appreciated by many people, has high historical value or landscaping.

A survey specifically dedicated to research of geomorphologic indicators would obtain definitely a high precision in the definition of the extension of the Pleistocene glaciers in higher altitude areas.

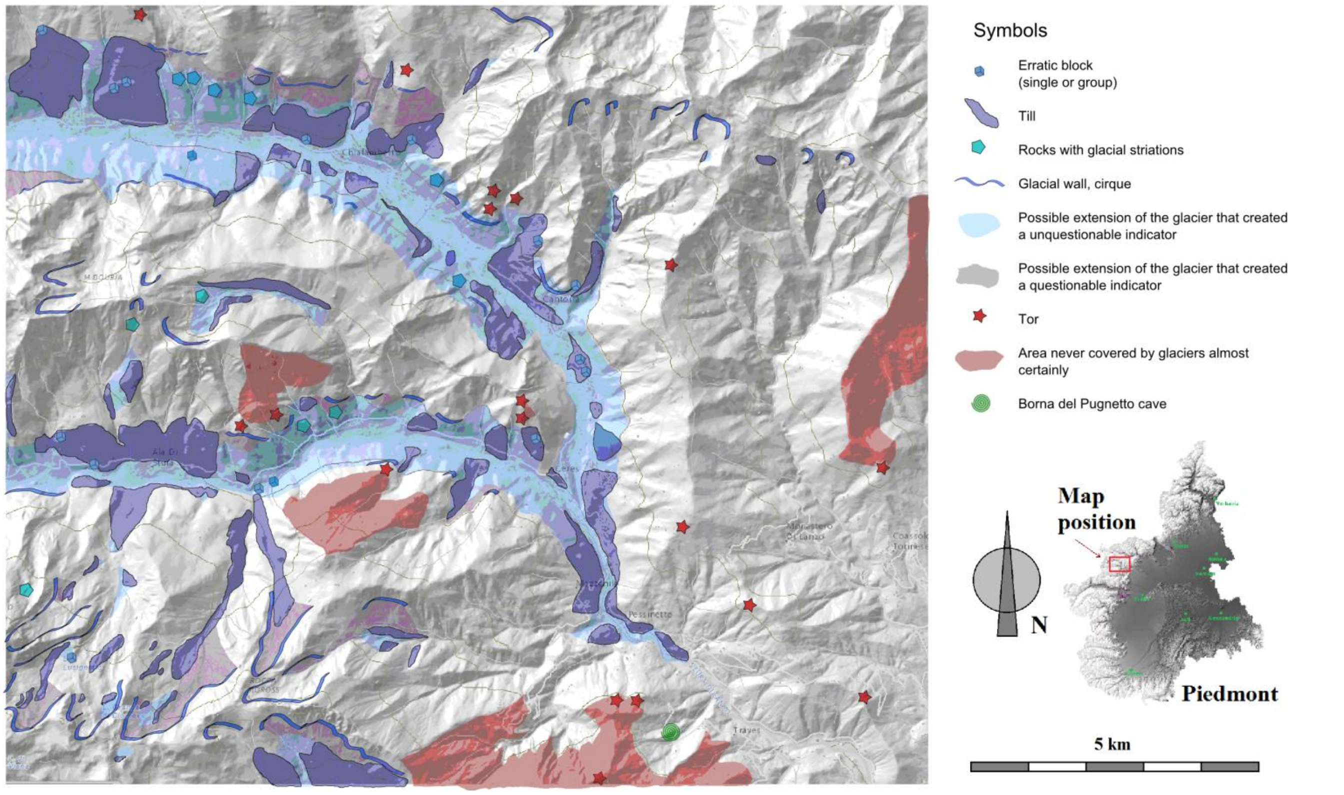

Figure 7. Distribution of geomorphic indicators of glacial passage (erratic blocks, till, rocks with glacial striae, glacial walls, such as truncated spur and rock step, cirques) and indicators of lack of glacial processes (tors) in Lanzo Valleys. Cartography of Arpa Piemonte.

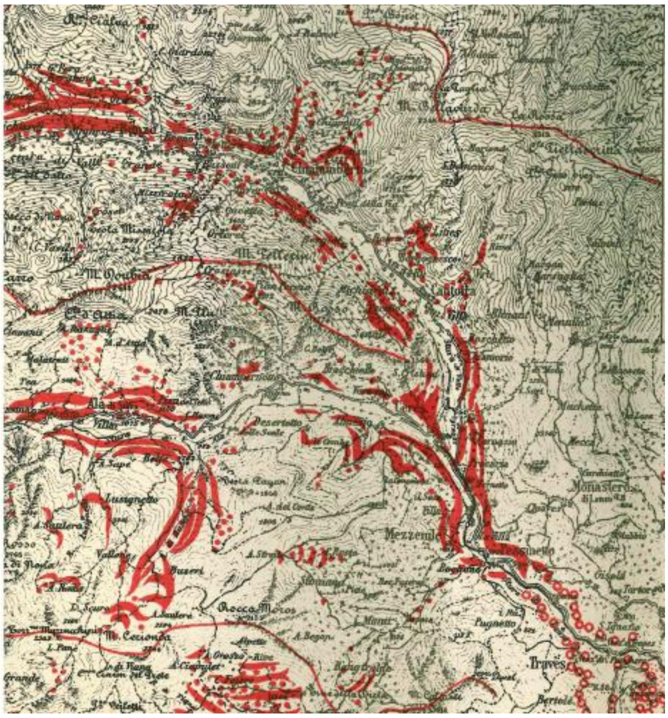

Figure 8. The “morenico sparso” (i.e., glacial deposits scattered, that are marked by dotted areas), the moraines (red lines) and fluvial deposits (red circles) in the Sacco survey of the Lanzo Valleys [28]

Despite the limits mentioned above, the card well highlights the difference in development of glacial processes between the two western valleys (Ala and Grande Valley) and the minor eastern valleys (Tesso and Malone) that are lower (only a few summits exceed 2000 m a.s.l.) and closer to the Po plain. In the major valleys, the only areas covered by glaciers are the slopes directly above the glacial tongues. In contrast, within the smaller valleys, the glacial traces are restricted to areas more shaded and high of the valley heads. At first glance, most of all the Lanzo valleys have the summits pointed and rocky ; but in the West, that is because these summits are a pyramidal peak surrounded by glacial cirques, while near to Po Plain (SE) the top of the rocky mountains is a summit tor.

In the valleys with numerous and generally unquestionable indicators, as Lanzo Valleys, Susa Valley or Aosta Valley, we never found contradictory indicators. In other words, the areas subtended by glacial flow indicators do not overlap ever, even partially, to the areas subtended by the indicators of lack of glacial passage.

In theory, then, in areas where the indicators are many but most of them are questionable indicators, the method might work to backwards. In other words, deposits of uncertain origin could be classified of glacial origin or not, depending on whether they are in areas subtended by the indicators of glacial passage or less.

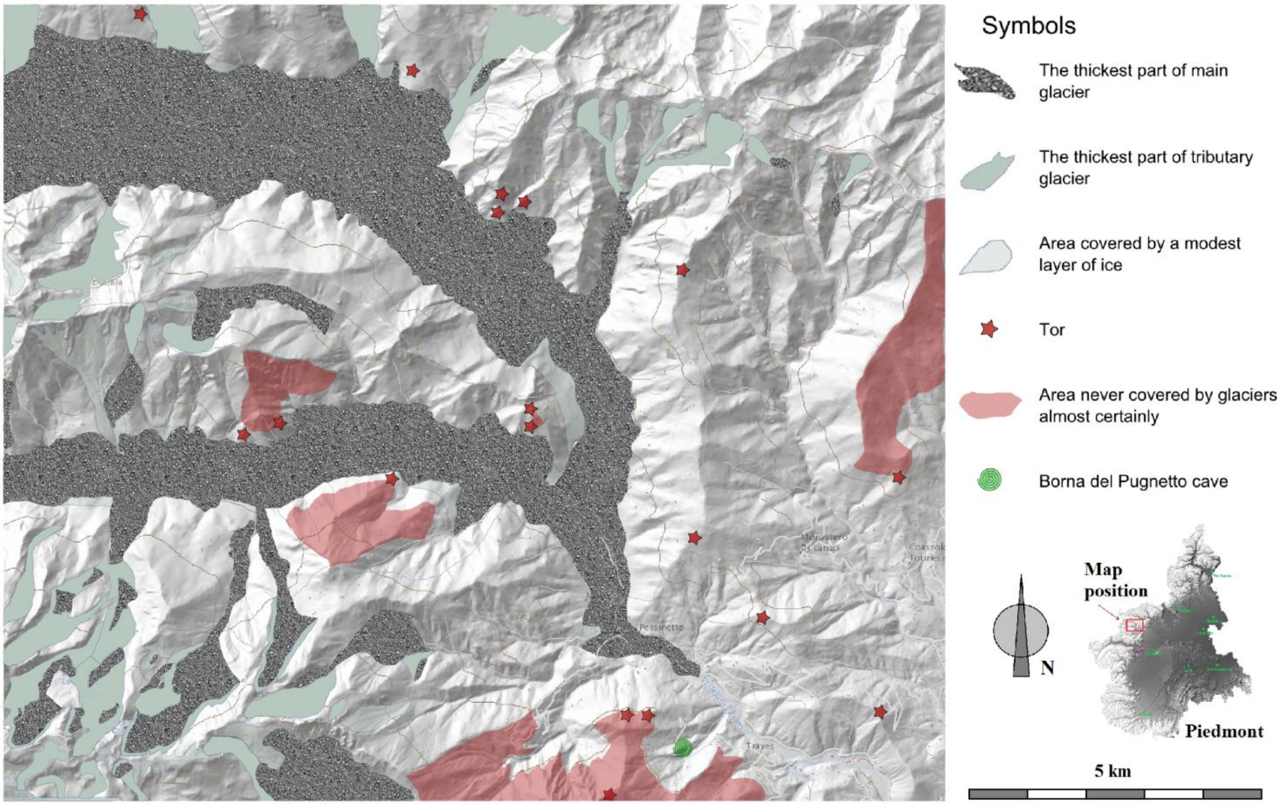

Figure 9. Reconstruction of ancient Lanzo Glacier during the Pleistocene, based on the analysis of glacial morphological indicators of fig. 7.

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TABLE I. CAVES STUDIED BY CAVELAB [21].

N° register	Cave	Municipality	Altitude	UTMED50_X	UTMED50_Y
118 Pi/CN	Grotta dell'Orso di Ponte di Nava	Ponte di Nava	808	410027	4885936
105 Pi/CN	Grotta delle Camoscere	Chiusa Pesio	1057	392974	4896975
108 Pi/CN	Grotta di Bossea	Frabosa Soprana	836	407374	4899582
art Pi/CN	Sotterranei del Forte A di Tetti Ruinas	Vernante	800	382565	4901169
1214 Pi/CN	Barun Litrun	Valdieri	1050	373117	4902365
122 Pi/CN	Grotte del Caudano	Frabosa Sottana	780	403537	4905362
1003 Pi/CN	Grotta Occidentale del Bandito	Roaschia	714	374716	4905527
151 Pi/CN	Tana della Dronera	Vicoforte Mondovì	525	408066	4910960
1102 Pi/CN	Buco dell'Aria Calda	Vignolo	840	377406	4911834
1239 Pi/CN	Grotta 1 di Argentera	Argentera	1795	335864	4918268
nc Pi/CN	Buco del Partigiano	Roccabruna	1170	364416	4929787
1024 Pi/CN	Grotta dei Partigiani	Rossana	615	375493	4932782
1019Pi/CN	Tana dell'Orso di Casteldelfino	Sampeyre	2357	349107	4935896
1148Pi/CN	Buco del Maestro	Paesana	750	360333	4949624
1538Pi/TO	Ghieisa d'la Tana	Angrogna	835	359634	4967937
1621Pi/TO	Tuna del Diau, Chiabrano	Perrero	1080	350672	4979013
1591Pi/TO	Tana del Diavolo, Roure	Roreto Chisone	1414	352140	4987790
1569 Pi/TO	Grotta Testa di Napoleone	Borgone	450	363263	4997329
1597 Pi/TO	Balma Fumarella	Gravere	864	345315	4999142
1501Pi/TO	Borna Maggiore del Pugnetto	Mezzenile	742	375551	5014621
art Pi/CN	Borna del Servais B	Ceres	1420	369011	5020340
1593Pi/TO	Grotta La Custreta	Sparone	1386	386340	5033760
1609 Pi/TO	Buca del Ghiaccio della Cavallaria	Brosso	1548	405988	5041429
2048Ao/AO	Grotta A di Ivery	Ivery	773	407363	5049504
2624Pi/BI	Caverna dell'Om Salvej	Sordevolo	1025	417306	5050186
2503Pi/BI	Grotta di Bergovei	Sostegno	415	442855	5056806
2509Pi/VC	Grotta delle Arenarie	Valduggia	780	446644	5062402
2001Ao/AO	Borna d'la Glace	Chabodey	1605	348747	5066007
2017Ao/AO	Fessura di Verrogne	Verrogne	1536	360703	5066012
2501Pi/VB	Caverna delle Streghe di Sambughetto	Sambughetto	670	446956	5084248

a. "art" in first column means man-made cavity, and obviously these "caves" not existed in Pleistocene. Also Buca del Ghiaccio della Cavallaria, Ghieisa d'la Tana, and Borna d'la Glace, tectonic caves formed by landsliding, probably not were existing before the Holocene. Tana dell'Orso di Casteldelfino, Buco del Maestro, Grotta 1 di Argentera, Tana del Diavolo, Tuna del Diau, Testa di Napoleone, Caverna dell'Om Salvej and Buco del Partigiano are tectonic caves, and perhaps not were existing during Pleistocene (or they exist only since the Upper Pleistocene).

b. UTMED50_X and Y are the UTM coordinates, European Datum 1950.

VII. APPLICATION OF THE METHOD TO THE STUDY OF SUBTERRANEAN FAUNA

The interdisciplinary project CaveLab "From microclimate to climate change: Caves as laboratories for the study of the effects of temperature on ecosystems and biodiversity" was born in 2013 at the University of Turin, with a team of several members of the departments of Life Sciences and Biology of the Systems, Earth Sciences, General Physics, Plant Biology and Analytical Chemistry [17].

The ultimate goal of the project is: from a scientific point of view, a better knowledge of the impact on ecosystems of sudden variations (in time of the evolution of species) of climate such as the deglaciation, or in a future scenario, the global warming; from a practical point of view, give advice on the good management of the tourism inside caves containing sensitive species.

The project begins with the characterization of the cave environment. The next step takes into account the influence direct and indirect of factors such as availability of trophic resources, the human disturbance, the structure of the biocoenosis and habitats, the Pleistocene climate, and, exactly, the extension of Quaternary glaciers.

Reference [7] has formerly analysed some of these caves, solely on the basis of the position in relation to glacial drift distribution on geological map of Italy 1: 100,000 (tab. 2).

Depending on the location of caves respect to position of glaciers, we can list different ecological conditions.

Caves at low altitude, without important flow of water and with the entrance on a sunny slope (e.g. Buco dell'Aria Calda): during glaciations the inner climate remains warm, so the interglacial fauna survives (nowadays they are become endemic and thermophilic species?).

TABLE II. CONDITIONS OF THE CAVES DURING PLEISTOCENE, ACCORDING TO [7].

Condition during Pleistocene	Caves
Covered by glacier	Abisso Artesinera, Balmovra, Barma di Grange Torre, Buco di Valenza, Caverna delle Streghe di Sambughetto, Elva Grotta dei Folletti di Novalesa, Grotta del Tiro a Volo, Grotta delle Meta (Borgone), Grotta di Bossea Grotta Testa di Napoleone, Rio Martino, Tana dell'Orso (Casteldelfino)
Maybe covered by glacier	La Custreta, Pertus dal Drai, Tana del Diavolo, Tumba d'Cucitt
Perhaps without a glacier covering	Fascette, Grotta dell'Orso di Ponte di Nava, Grotta di Candoglia
Certainly without a glacier covering	Barma dell'Argilla, Barma Scura, Buca del Ghiaccio della Cavallaria, Buco del Maestro (Paesana), Buco della Bondaccia Buco Dell'Aria Calda, Dronera, Garbo della Donna Selvaggia, Ghieisa d'la Tana, Grotta del Bandito Grotta del Pugnetto, Grotta della Marmorera, Grotta di Bergovei (Sostegno), Grotta di Levone, Grotta di Rio dei Corvi Grotta di Rossana, Grotta Superiore dei Dossi, Tana di Camplass, Tana of S.Luigi (Roburent)

Caves of the valleys without glaciers (e.g. Tana della Dronera) or caves at considerable distance from the glaciers, with important variations in the water and air temperature: only the little sensitive species can survive to thermic variations.

Caves that have been all the time without a glacier cover, but very close to a debris-covered glacier: during glaciations, the fauna that lives in the interstices of the moraine can colonize the cave. In the caves at high altitudes, survive glacial relicts; also at a lower altitude, on a shaded slope (e.g. Borna Maggiore di Pugnetto), several endemic species of cryophilic type, may living nowadays.

Animals that are living in the interstices of the moraine can colonize during the last glacial retreat a cave as soon as discovered by the glacier (e. g. The Custreta): so, these caves may contain the same fauna of the previous group, without however glacial relicts of the Middle Pleistocene.

Tectonic caves borne after the glacial retreat (e.g. Ghieisa of the Tana), man-made caves (e.g. Borna of Servais): the fauna can only have the species that live in the environment surrounding interstitial or that are able to spread rapidly.

VIII. CONCLUSIONS

The analysis of geomorphic indicators supplies an objective method for define the extension of Quaternary glaciers within alpine valleys. In normal conditions, in other words if the valley has a sufficient number of unquestionable indicators, the definition of the glacier extension is quite precise for the application of the results to other fields, such as biology. In the study case of relationship between caves and glaciers, the results are very improved compared with a former study, recent and well done. In fact, on the basis of the method proposed, we can change, or specify, the condition during Pleistocene of some caves formerly analysed by [7]:

- Grotta di Bossea: from covered to uncovered
- La Custreta: from maybe covered to certainly covered
- Buca del Ghiaccio della Cavallaria, Borna Maggiore del Pugnetto: the analysis with the method proposed has confirmed the condition during Pleistocene, and has specified the exact distance between cave and glacier.

The example demonstrates that the method employed gets better results than a traditional study based solely on analysis of geological maps [7] and allows, as shown in fig. 10 and 11, a very detailed and precise reconstruction of the glaciers extension during the Pleistocene.

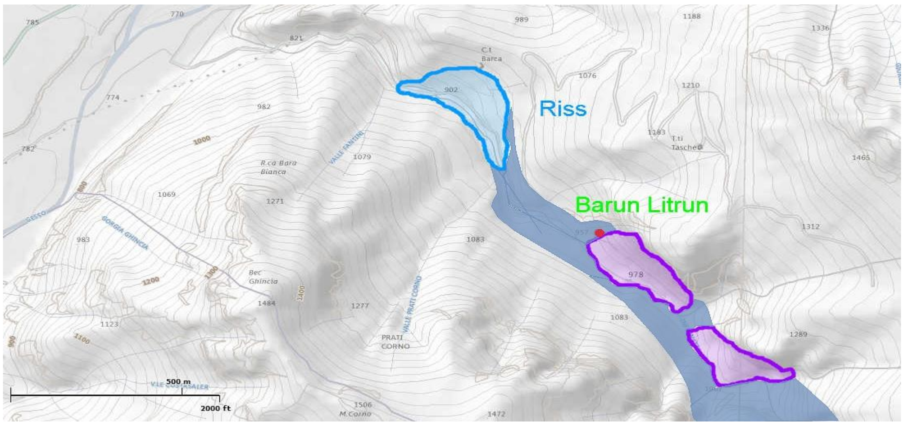

Figure 10. The Barun Litrun cave is between the glacial deposits of the Middle Pleistocene (pale blue) and the ones of the Upper Pleistocene (violet). The gray-blue area is the zone definitely covered by a glacier during Pleistocene.

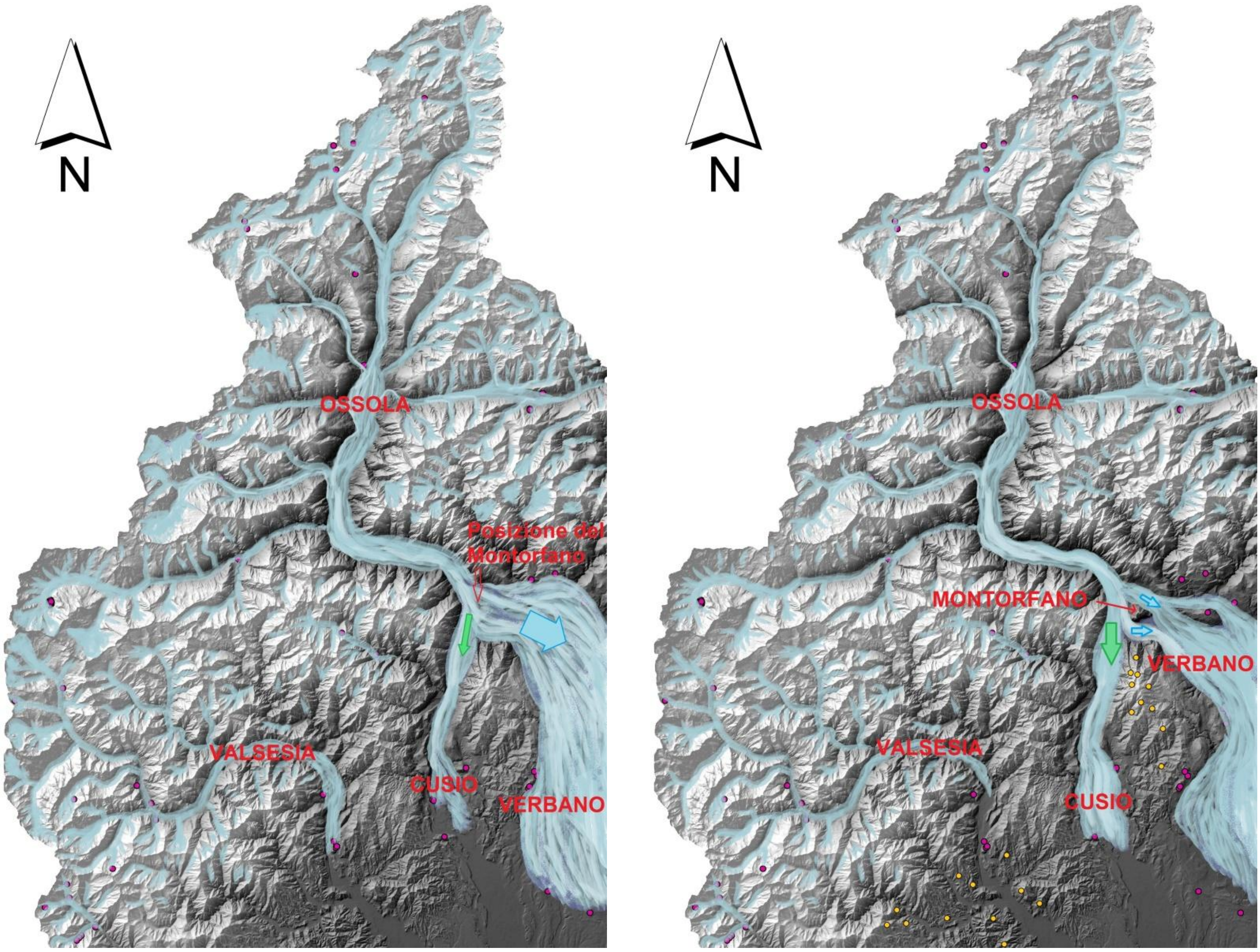

Figure 11. Reconstruction of the extension of glaciers (pale blue areas) in northern Piedmont during the Maximum Glacial Advance of Middle Pleistocene (left), and during the Last Glacial Advance of Upper Pleistocene (right), based on the analysis of glacial morphological indicators. Out of all these indicators, only the main erratic blocks and the tors (map on the right) are drawn (violet and orange circles). Width of map: 50 km.

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A decision aiding tool for multideciders

Based on a non transitive but asymmetric binary relation

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Abstract—**The paper presents a decision aiding tool that can be used by a set of decision makers (or *deciders*) of a set *D* for the ranking and selection of the alternatives of the set *A* according to the criteria of the set *C*. The method uses classical binary relations with classical properties but is grounded on a newly devised binary relation that satisfies asymmetry but not transitivity. This binary relation can be used to derive a partial order on the given set of alternatives according to the given set of criteria that, as a basic hypothesis, are not comparable among themselves. The proposed relation has been nicknamed *particular order* and is presented with the aid of two toy examples together with a somewhat detailed description of its properties and its applicability within the decision aiding tool.**

Keywords- partial order, decision making, multicriteria, multideciders, non-transitive asymmetric binary relations, directed [multi]graphs, lexicographic order, Borda and Condorcet methods

I. Introduction

The present paper has two main aims. The former aim is to present a non transitive asymmetric binary relation that allows each decision maker or ***decider*** d_i to produce a directed graph G_i that best describes his assessment of the available alternatives according to the given set of criteria.

The latter and consequent aim is to show how the decides can, as a joint collective effort, merge such directed graphs in a unique directed multigraph ***MG*** that reflects their assessment of the available alternatives according to the given set of criteria and how they can use ***MG*** in order to perform the final selection of one of the best alternatives as the most preferred one.

More about all these features in the rest of the paper whose structure reflects such aims as well as the need to present and analyze some potential problems and failures of the proposed tools together with some preliminary and tentative solutions.

II. The Two Steps and The Structure

We have that the deciders perform two steps. The first step can be performed independently from the other, has the sets of the alternatives and of the criteria as its inputs and produces a directed graph G_i for each decider d_i. This allows each decider to have his personal ranking of the alternatives so to be able to perform the final selection of one of the best alternatives as the most preferred one for the single decider.

The second step has no autonomy and no meaning of its own since it needs the graphs G_i as its input and produces, as the output of a collective effort from the deciders, the multigraph ***MG*** that can be used by the deciders as a whole to perform the final selection of one of the best alternatives as the most preferred one for the whole set of the deciders.

In the first part of the paper, therefore, we focus our attention on the newly devised binary relation that can be used by each decider as a decision aiding tool for the selection of an alternative from the set ***A*** of ***a*** alternatives according to the ***c*** criteria of the set ***C***. Both sets must be seen as exogenous sets with respect to the selection process so that we are in the so called *static setting* as opposed to the *dynamic setting* where such sets may vary during the selection process itself. The proposed tool aims at producing a directed graph of all the alternatives in which it is possible to identify the *worst alternatives*, that can be discarded, and the *best alternatives* among which it is possible to perform the final selection. The binary relation introduces a partial order on the alternatives that is represented through a directed graph and that has been nicknamed *particular order* since it purposely fails transitivity while it satisfies asymmetry ([1], [2], [5], [6] and [16]).

After the presentation of the binary relation and of the associated partial order, also with the aid of some toy examples, we present and discuss two possible sources of difficulty for which we propose some tentative solutions: the presence of isolated nodes and the presence of cycles in at least one of the graphs G_i. We also present two techniques of analysis (the *forward* and *backward pruning* techniques, [4], [10] and [11]) that prove useful also at the level of ***MG***.

The presence of isolated nodes can be solved with a modified version of the Borda voting method whereas the presence of the cycles can be tackled according to either an ex-ante or an ex-post approach. According to the ex-ante approach we impose to each decider to produce an acyclic graph whereas, according to the ex-post approach, we deal with the presence of cycles at the level of the multigraph ***MG***.

In the third part of the paper we then turn our attention to the collective effort from the deciders and present the procedure for the merging of the graphs G_i in the multigraph ***MG***. We also present a method for the final selection of one of the best alternatives based on a technique of *lexicographic ordering*. As it can happen on the graphs G_i also in this case we can have the presence of either isolated nodes or of cycles that represent possible causes of failure of the proposed method.

In the last part of the paper we, therefore, present and discuss these disturbing features and show how the presence of cycles can be tackled, alas not always with success, through the use of a modified version of the Condorcet voting method.

III. THE BACKGROUND

A. A preliminary remark

The literature abounds of binary relations (see for instance [5], [6], [15] and [16]). So why devising another one? This need arose during the draft of my PhD thesis ([7]) when I faced the need to solve the problem of a set of deciders that have to rank a common set of alternatives according to a possibly but not necessarily common set of criteria. This need motivated me to devise a multideciders multicriteria method since the methods I was able to find in the literature that I analyzed proved to be unsatisfactory for my purposes. For these reasons I decided to devise a new method.

The method that I devised essentially works as follows: every decider defines his own *individual partial order* of the alternatives then these partial orders are merged in a single *final partial order* that is used by the deciders themselves as a whole as a decision aiding tool for the final selection of one of the best alternatives. Each individual partial order is the end product of the application of the particular order on the set $\boldsymbol{A}$ through the use of the criteria of the set $\boldsymbol{C}$.

B. A note on the deciders

The *deciders* $\boldsymbol{d_i}$ form a set $\boldsymbol{D=\{d_1, \dots\, d_d\}}$ of $\boldsymbol{d}=|\boldsymbol{D}|$ peers ([2], [3], [4] and [12]) since they are assumed to have the same power within the decision process. The deciders face the task of ranking the alternatives of the set $\boldsymbol{A}$ as both an individual and a collective effort in order to arrive at the identification of the best alternatives that form a subset $\boldsymbol{A_b} \subset \boldsymbol{A}$ from which they, as a *political decision*, have to choose the most preferred alternative. The final decision is termed to be political since the alternatives of the set $\boldsymbol{A_b}$ must be seen as incomparable (see section VIII), at least according to the criteria of the set $\boldsymbol{C}$, so that the final selection can be performed, for instance, with the use of the Borda method or any other voting method ([8], [9] and [14]) but in cases where we have $|\boldsymbol{A_b}|=1$ when we have only one alternative to be selected.

C. A note on the criteria and on the alternatives

The *criteria* of the set $\boldsymbol{C}$ ([1], [2]) are assumed to be independent and to produce $\boldsymbol{c}$ total orderings that are not comparable among themselves. On the other hand the set $\boldsymbol{A}$ of the *alternatives* is assumed to have been pruned of the *dominated alternatives* or of the alternatives that are indistinguishable from at least another alternative for a certain number of criteria and are worse than the same alternatives for the remaining criteria. This pruning can be performed independently by the authority that defines the sets $\boldsymbol{A}$ and $\boldsymbol{C}$ for the deciders and independently from the deciders before each of them evaluates his partial order.

D. A note on the orders

Both the individual orders and the final order are partial since, given any two alternatives a_i and $a_j \in \boldsymbol{A}$, it may be impossible to state, over the whole set of the criteria, either a strict preference condition of a_i over a_j or the opposite strict preference condition or an indifference condition between the two alternatives. We underline how, on the other hand, each criterion $c_k \in \boldsymbol{C}$ produces by itself a total transitive ordering (with possible ties) of the alternatives of the set $\boldsymbol{A}$ and so ([1], [2] and [5]) a *linear ordering* (see, for some instances, the linear graphs of Fig. 1).

E. A note on the method

The proposed binary relation represents the basic tool (together with the others that will be introduced as needed along the way) that the deciders can use within the proposed multideciders multicriteria method ([2], [3] and [4]). This method takes $\boldsymbol{d}$ ordered graphs $\boldsymbol{G_i}$, one for each decider (and so one for each individual order), and produces a single directed multigraph $\boldsymbol{MG}$ that corresponds to the final partial order. Each of the $\boldsymbol{d}$ ordered graph is obtained, as the outcome of an individual effort of every decider, through the composition of $\boldsymbol{c}$ linear graphs. Both the composing ordered graphs and the composite multigraph fail transitivity from the very way they are defined, as it will be shown shortly, whereas the linear graphs satisfy transitivity.

IV. THE PARTICULAR ORDER BINARY RELATION

The key element is, therefore, represented by a binary relation that satisfies asymmetry but fails transitivity and that we nicknamed as a *particular order* ([1], [2] and [4]).

The main features of this binary relation derive from the way we defined it starting from a relation of strict preference that satisfies transitivity and asymmetry ([5], [6], [15] and [16]) and from a relation of indifference that satisfies reflexivity, symmetry and transitivity ([5], [6], [15] and [16]) and, therefore, is an equivalence relation.

A. The primitive relations

For each criterion $c_i \in \boldsymbol{C}$ we may define a strict preference relation $>_i$ and an indifference relation $\sim_i$ between pairs of alternatives $a_i, a_j \in \boldsymbol{A}$. Such relations are the primitive relations on which we ground the particular order.

Such relations are endowed with the classical properties that we have already listed ([5], [6], [15] and [16]). We present both of them with a toy example involving only one decider.

In this case, for instance, if $\boldsymbol{A}=\{a_1, a_2, a_3\}$ and $\boldsymbol{C}=\{c_1, c_2\}$, we can obtain the following total orderings (with possible ties) for the two criteria and for that single decider:

$$a_1 >_1 a_2 >_1 a_3 \quad (1)$$

$$a_2 >_2 a_1 \sim_2 a_3 \quad (2)$$

From (1) we derive $a_1 >_1 a_3$ and, similarly, from (2) we derive $a_2 >_2 a_3$.

It is easily seen how such relations generalize to more than three alternatives and to more than two criteria as well as to more than one decider, each decider defining his own orderings. It is easily seen, moreover, how we can put in correspondence with each of such orderings a linear graph (as it is shown, for another case, in Fig. 1).

B. The particular order

On the basis of these two primitive relations we can define our binary relation to be represented simply as › and that allows us to define the *particular order* among the alternatives. In order to make such definition we have to define two numerical quantities denoted, for simplicity, as x and y for each pair of alternatives $a_i, a_j \in \boldsymbol{A}$.

For a given pair of alternatives $a_i, a_j \in \boldsymbol{A}$ we can use x to denote the number of criteria such that $a_i \succ_k a_j$ for all the $c_k \in \boldsymbol{C}$ whereas we can use y to denote the number of criteria such that $a_j \succ_k a_i$ for all the $c_k \in \boldsymbol{C}$.

At this point we have the following three mutually exclusive cases:

(o_1) $x > y$ so we can state that $a_i \succ a_j$

(o_2) $x < y$ so we can state that $a_j \succ a_i$

(o_3) $x = y$ so we cannot state any relation between a_i and a_j.

If z denotes the number of criteria $c_k \in \boldsymbol{C}$ such that $a_j \sim_k a_i$ we have, for each pair of alternatives, the following invariant relation:

$$x + y + z = c \tag{3}$$

where $\boldsymbol{c}$ denotes the cardinality of $\boldsymbol{C}$. If we introduce a threshold $\varepsilon > 0$ so to state a preference between two alternatives if we have $|x-y|>\varepsilon$ and no preference if we have $|x-y|\leq\varepsilon$ we only get a reduction of the number of arcs in the resulting graph and, at the same time, an increase of the probability of having isolated nodes. For these reasons we decided to put $\varepsilon = 0$.

The invariant relation (3) can be used to derive some useful consequences (see section VI for some hints). We underline, indeed, how the third case (o_3) has two sub-cases:

($o_{3,1}$) $x = y = 0$ so z=c

($o_{3,2}$) $x = y \neq 0$ so z<c

that can be used to handle the presence of isolated nodes in a graph $\boldsymbol{G_i}$.

We can represent both (o_1) and (o_2) with a directed arc from the preferred alternative to the other one but the possible occurrence of (o_3) prevents us from drawing any link between the two alternatives and this qualifies the ordering as partial. In order to verify its being particular we need to verify that it satisfies asymmetry and fails transitivity. For this verification and some further comments we refer to the next section.

C. Verification of the properties

First of all we have to verify the alleged properties of the › binary relation.

We start with *asymmetry*. Given $a_i, a_j \in \boldsymbol{A}$ we may know that $a_i \succ a_j$ so we have that there exist x and y such that $x > y$ from where we derive $\neg\,(y > x)$ or $\neg\,(a_i \succ a_j)$ and hence the asymmetry or, vice-versa, we know that exist x and y such that $x > y$ so $a_i \succ a_j$ and, therefore, we have $\neg\,(y > x)$ and, consequently, $\neg\,(a_i \succ a_j)$.

For what concerns *transitivity* we may reason as follows. Given any three alternatives $a_i, a_j, a_k \in \boldsymbol{A}$ we may consider the pair a_i, a_j so to get (for instance) $x > y$ or $a_i \succ a_j$ then we may consider the pair a_j, a_k so to get (for instance) $x > y$ or $a_j \succ a_k$.

At this point we have to consider the remaining pair of alternatives and so a_i and a_k. In order to have transitivity we should have $x > y$ and so $a_i \succ a_k$ but, instead, we may get either $x = y$ or $x < y$ and so either we cannot state any preference relation between the alternatives a_i and a_k or we get $a_k \succ a_i$. In this latter case we get the cycle $a_i \succ a_j \succ a_k \succ a_i$.

The full treatment of cases like these it outside of the scope of the present paper. For a more deep analysis and some tentative solutions of similar cases we refer to [4]. In the present paper we make some more comments and propose a tentative solution in section VI.

In both cases we have a failure of transitivity. It is easily seen how similar considerations hold also in some other possible cases some of which are illustrated in the examples that we present in section V. The loss of transitivity is a consequence of the pairwise comparisons among the alternatives according to independent criteria ([6], [8] and [9]).

From the definition of our binary relation › we can moreover state that:

(1) it is *irreflexive* since we have $a_i \sim_k a_i$ for all the $c_k \in \boldsymbol{C}$ from which we have x = y = 0 and z = c so that there is no link between an alternative and itself;

(2) it fails also *negative transitivity* since we have that asymmetry and negative transitivity imply transitivity but transitivity is violated whereas asymmetry is satisfied and so (since a true statement cannot imply a false one) also negative transitivity must be violated.

The fact that › is *irreflexive* means that every alternative is seen as incomparable with itself but this is of no harm since our aim is to compare each alternative with all the others and not with itself in order to get a ranking of the alternatives.

D. Some further comments

Once the particular order has been defined and its main properties verified some further comments are in order.

(1) The binary relation › has been defined under the assumption that the criteria of the set $\boldsymbol{C}$ have the same weight or importance. If this is not true it is easy to modify the evaluation of the numerical indexes such as x in order to account for such differences in weight. The basic procedure assumes as equal to 1 the weights with the lowest value and scales accordingly all the others so to use these new weights as multiplying factors in the evaluation of the foregoing numerical indexes x and y.

(2) Situations such as (o_1) and (o_2) allow us to identify the preferred alternative from each pair whereas situations like (o_3) introduce particular conditions of incomparability since the number of criteria that favor an alternative against another is equal to the number of those that give the opposite ranking.

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(3) Since the set ***C*** has a fixed cardinality we can easily verify the occurrence of one of the foregoing situations for any pair of alternatives of the set ***A***.

V. Two Toy Examples

At this point, in order to make things more clear, we present two toy examples from [4] and [7] in order to show both the particular order and the multideciders multicriteria method at work.

These examples can be used both to verify what we have stated about the properties of our binary relation and to see how the method is robust also in presence of some differences among the deciders, as we discuss in greater detail in [4].

A. The first example

In the first example we assume that a first decider $\boldsymbol{d_1}$ considers four criteria and four alternatives so that he has $\boldsymbol{A}=\{a_1, a_2, a_3, a_4\}$ and $\boldsymbol{C}=\{c_1, c_2, c_3, c_4\}$. According to this decider the four criteria may define over the four alternatives the following total orderings with possible ties:

(to_1) $a_1 \sim_1 a_2 >_1 a_3 >_1 a_4$

(to_2) $a_2 \sim_2 a_3 >_2 a_4 >_2 a_1$

(to_3) $a_3 \sim_3 a_4 >_3 a_1 >_3 a_2$

(to_4) $a_1 >_4 a_3 >_4 a_2 >_4 a_4$

These total orderings are represented as linear graphs in Fig. 1 (in the order from left to right and from top to bottom) where the alternatives are represented only with their index, an undirected arc represents an indifference relation and a directed arc represents a strict preference relation.

At this point the decider can perform the six pairwise comparisons among the four alternatives by using all the four criteria. If, for instance, he considers the pair (a_1, a_2) he gets $x > y$ or $a_1 \succ a_2$ and so a directed arc from a_1 to a_2. If now he considers the pair (a_2, a_3) he gets $x < y$ or $a_3 \succ a_2$ and so a directed arc from a_3 to a_2. If, however, he considers the pair (a_1, a_3) he finds $x = y$ so he cannot define any ordering and so any arc between such alternatives. In this case we have a failure of the transitivity since its being satisfied would have required, for instance, $a_1 \succ a_3$.

If, moreover, he considers the pair (a_1, a_4) he finds $x = y$ so, again, he cannot define any ordering and so any arc between such alternatives. Also in this case we have a failure of transitivity since, on the other hand, we have both $a_1 \succ a_2$ and $a_2 \succ a_4$.

By performing all the six pairwise comparisons the decider can perform similar considerations so to derive the directed graph of Fig. 2.

From Fig. 2 it is easily understood how the ordering is partial since the alternatives a_1 and a_3 are incomparable (see section VIII) whereas the presence of a directed path together with the lack of a directed arc between the alternatives a_1 and a_4 denotes a further incomparability condition (again see section VIII) together with a failure of transitivity.

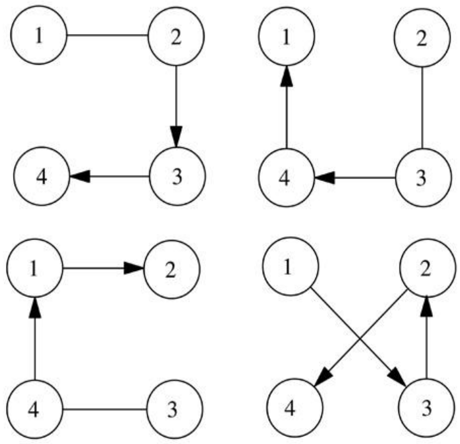

Figure 1. Four linear graphs for four total orderings, first example

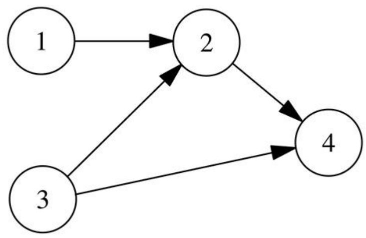

Figure 2. The resulting graph in the first example

From Fig. 2, moreover, it is easily seen how this decider considers a_4 as the worst alternative and the alternatives a_1 and a_3 as the best alternatives (so we have $\boldsymbol{A_b}=\{a_1, a_3\}$) however with a small prevalence of a_3 owing to its being preferred to two other alternatives rather than to only one.

Using a sort of *backward induction* ([10] and [11]) termed *backward pruning* it is possible to remove the worst alternative a_4 and so the corresponding node labeled as 4 (since it has no outgoing arcs) and its incoming arcs from the graph of Fig. 2. After this removal the alternative a_2 becomes, on its turn, the worst alternative and can be discarded together with the corresponding node with its incoming arcs. In this way we obtain two isolated nodes among which the final selection must be performed. We underline, indeed, that backward pruning has no effect on nodes without incoming arcs.

The dual method of *forward pruning* is based *forward induction* ([10] and [11]) and removes nodes without incoming arcs together with their outgoing arcs until it finds isolated nodes when it stops since it has no effect on isolated nodes.

In the case of Fig. 2 through the application of forward pruning we have the successive removals of the the nodes 1, 3 and 2 so that we remain with node 4 that represents the worst alternative, in this simple case.

B. The second example

In the second example we assume that another decider $\boldsymbol{d_2}$ considers only the first three criteria but the same four alternatives so that he has $\boldsymbol{A}=\{a_1, a_2, a_3, a_4\}$ and $\boldsymbol{C}=\{c_1, c_2, c_3\}$. According to this decider the three criteria define over the four alternatives the following three total orderings with possible ties:

(to_1) $a_1 >_1 a_2 >_1 a_3 \sim_1 a_4$

(to_2) $a_2 \sim_2 a_3 >_2 a_4 >_2 a_1$

(to_3) $a_3 >_3 a_4 \sim_3 a_1 >_3 a_2$

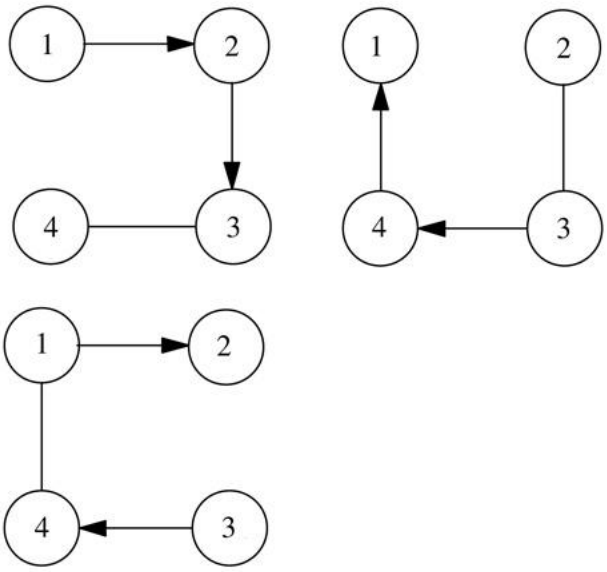

Figure 3. Three total orderings, second example

These three total orderings are represented as linear graphs in Fig. 3, again from left to right and from top to bottom.

By performing also in this case the six pairwise comparisons between the four alternatives and then evaluating, for each pair, the proper numerical parameters *x* and *y* to be pairwise compared this decider obtains the following strict preference conditions: a_1 › a_2, a_3 › a_1, a_2 › a_4 and a_3 › a_4.

Such relations reflect the fact that:

(1) for alternatives a_1 and a_2 we have $x > y$ and so a_1 › a_2

(2) for alternatives a_1 and a_3 we have $x < y$ and so a_3 › a_1

(3) for alternatives a_2 and a_4 we have $x > y$ and so a_2 › a_4

(4) for alternatives a_3 and a_4 we have $x > y$ and so a_3 › a_4

On the other hand the alternatives (a_2, a_3) and (a_1, a_4) are incomparable for him, according to the rules that we have given for the definition of the › binary relation. The situation, after the merging of the three total orders in a single partial order from this decider, is represented in Fig. 4.

According to the opinion of this second decider, therefore, alternative a_4 is the worst alternative whereas the best alternative, notwithstanding the presence of an incomparability condition on the pair (a_2, a_3), seems to be alternative a_3. In this case the use of *backward pruning* would leave the decider only with the node 3 and, correspondingly, with the best alternative a_3 whereas the use of *forward pruning* would leave him with node 4 and so with the worst alternative a_4. We recall how the best alternatives correspond to nodes with no incoming arcs but with outgoing arcs whereas the worst alternatives correspond to nodes with no outgoing arcs but with incoming arcs.

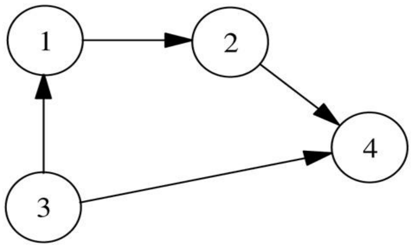

Figure 4. The resulting graph in the second example

C. *Comments on the properties of the particular order*

At this point we can use the two foregoing examples in order to make some more comments about the properties of the binary relation ›.

From Fig. 2 we derive a failure of the transitivity on the three alternatives (a_1, a_2, a_4) whereas this property is satisfied by the three alternatives (a_2, a_3, a_4) as it is shown by the presence of the directed arcs between the three pairs of alternatives. In the former case we have a_1 › a_2 and a_2 › a_4 but our criteria prevent us from stating a preference relation between a_1 and a_4. Since › is not transitive we could even have a_4 › a_1.

From Fig. 4 we derive a failure of the transitivity on the three alternatives (a_1, a_2, a_3) and (a_1, a_2, a_4) as it is shown by the absence of the third arc between a pair of distinct alternatives of each group. This absence prevents us from stating, on the ground of the current set ***C***, a preference relation between the alternatives that are at the ends of each chain of three preferences.

From these considerations we derive that transitivity fails as a global property for each graph whereas it is locally satisfied by some of the possible groups of three alternatives. In this case we speak of *local transitivity* whereas if this property were satisfied by all the possible groups of three alternatives we would speak of *[global] transitivity*. In [4] (as well as in section VI) we show how local transitivity can be used by a decider in order to get an acyclic graph from a cyclic one.

In both figures the property of asymmetry prevents the forming of local cycles that involve two nodes and, together with the failure of transitivity, may prevent the formation of simple cycles involving more than two nodes although this in not true in general how it is discussed in detail both in section VI and in [4].

VI. Isolated Nodes and Cycles in a Graph G_i

At this point every decider $\boldsymbol{d_i}$ has defined his own directed graph $\boldsymbol{G_i}$. In the best case none of such graphs has either isolated nodes or cycles so that they can be directly merged in a single directed multigraph ***MG*** as we discuss in section VII. It is worth noting, however, as also in this seemingly perfect case in ***MG*** there can occur cycles of any length owing to the presence of possibly opposite rankings of the alternatives from the various deciders.

In this section of the paper we discuss in some detail how we can tackle the presence of both *isolated nodes* and *cycles* of length greater than two (that cannot exist owing to the asymmetry of our particular order) in a directed graph $\boldsymbol{G_i}$. The proposed solutions extend those we presented in [4].

In a directed graph $\boldsymbol{G_i}$ a node $a_j \in \boldsymbol{A}$ is an *isolated node* if, for every other alternative $a_k \in \boldsymbol{A}$ with $k \neq j$, we have $x = y$.

As we have seen in subsection B of section IV we must consider two sub-cases. In the former sub-case we have $x = y = 0$ and $z = c$ so that we have $a_j \sim_i a_k$ for every $c_i \in \boldsymbol{C}$. In this case from $a_j \sim_i a_k$ and $a_j \sim_i a_h$ (with $k \neq h$) and from the reflexivity and transitivity of $\sim_i$ we get also $a_h \sim_i a_k$ for every $c_i \in \boldsymbol{C}$ so that there is no link also between the alternatives a_h and a_k. From this we derive that the graph $\boldsymbol{G_i}$ is made of isolated nodes so that we can discard it since the alternatives of the set $\boldsymbol{A}$ are seen as incomparable among themselves for the decider $\boldsymbol{d_i}$.

In the latter sub-case we have $x = y \neq 0$ and $z < c$ for at least some pairs of alternatives a_j and $a_k \in \boldsymbol{A}$. In this case we can use a partly modified version of the Borda voting method ([8] and [9]). According to this modified Borda method, for each criterion, we assign to an alternative a value that counts the number of strictly worse alternatives and then we sum such values over all the criteria. Lastly we use such totals to rank all the pairs of alternatives so to define a link only from the more ranked to the less ranked alternative of each pair. This allows us to get a graph without isolated nodes.

In order to show how this approach can help us in solving this kind of problems we present the following toy example with $\boldsymbol{A}=\{a_1, a_2, a_3, a_4\}$ and $\boldsymbol{C}=\{c_1, c_2\}$ and with the following total orderings among the alternatives:

(to_1) $a_1 >_1 a_2 >_1 a_3 >_1 a_4$

(to_2) $a_2 >_2 a_3 >_2 a_4 >_2 a_1$

By using the defining rules of our particular order we get the graph (a) of Fig. 5 in which we have an isolated node.

We can use our version of the Borda voting method in this case so that we get the assignment of points to each alternative that we show in table 1.

From the values of table 1 we get the graph (b) of Fig. 5 in which the node 1, corresponding to the alternative a_1, is no longer an isolated node.

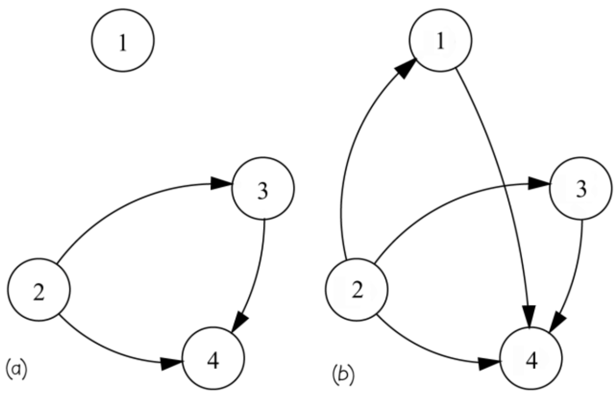

Figure 5. A possible way to deal with isolated nodes

TABLE I. MODIFIED BORDA FOR THE CASE OF FIGURE 5

a_1	a_2	a_3	a_4
3	5	3	1

The method works also in presence of ties between the alternatives as it occurs with the following total orderings among the alternatives:

(to_1) $a_1 >_1 a_2 \sim_1 a_3 >_1 a_4$

(to_2) $a_2 >_2 a_3 \sim_2 a_4 >_2 a_1$

Also in this case we get the graph (a) of Fig. 5 but the application of the modified Borda method gives the following assignment of points to each alternative:

TABLE II. MODIFIED BORDA, AGAIN FOR THE CASE OF FIGURE 5

a_1	a_2	a_3	a_4
2	3	2	1

From the above values we again get the graph (b) of Fig. 5 in which the node 1 corresponding to the alternative a_1 is again no longer an isolated node.

As we have already anticipated in section II the presence of the *cycles* in a graph $\boldsymbol{G_i}$ can be tackled according to either an ex-ante or an ex-post approach.

According to the ex-ante approach we impose to each decider to produce an acyclic graph whereas, according to the ex-post approach, we deal with the presence of cycles at the level of the multigraph $\boldsymbol{MG}$.

The ex-post approach will be examined in section XI whereas in the closing part of this section we describe briefly, with an example taken from [4], the ex-ante approach.

As we have already seen a decider $\boldsymbol{d_i}$ builds his graph $\boldsymbol{G_i}$ by merging his $\boldsymbol{c}$ linear ordering so that there is no guarantee that the resulting graph does not contain any cycle. Let us assume to have a decider with $\boldsymbol{A}=\{a_1, a_2, a_3, a_4\}$ and $\boldsymbol{C}=\{c_1, c_2, c_3\}$ and with the following total orderings among the alternatives:

(to_1) $a_1 >_1 a_2 >_1 a_3 >_1 a_4$

(to_2) $a_2 >_2 a_3 >_2 a_4 >_2 a_1$

(to_2) $a_3 >_3 a_4 >_3 a_1 >_3 a_2$

It is easy to see, by applying the rules of our particular order ›, how from such total orderings we get the cyclic graph of Fig. 6 (a) ([8] and [9]). In such a graph we have the following cycles (see Fig. 6 (a)):

(b) $a_1 › a_2 › a_3 › a_4 › a_1$

(c) $a_1 › a_2 › a_3 › a_1$

(d) $a_1 › a_2 › a_4 › a_1$

but also the following cases where local transitivity is satisfied:

(e) $a_2 › a_3 › a_4$ and $a_2 › a_4$

(f) $a_3 › a_4 › a_1$ and $a_3 › a_1$

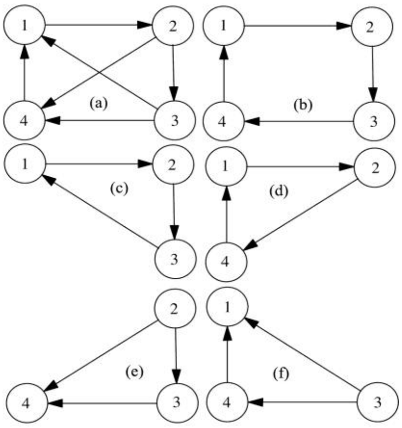

Figure 6. Graph with cycles and sub-graphs

In order to make acyclic his own graph this decider can make use of the local transitivity conditions in order to identify the minimal set of arcs whose removal satisfies this requirement. In the current case this minimal set is composed of the arc 1 ⟶ 2. The removal of this arc from the graph of Fig. 6 (a) can be motivated as follows:

a_2 is preferred to a_3 and a_4;

a_3 is preferred to a_1 and $a_{4;}$

a_4 is preferred to a_1;

a_1 is preferred only to a_2.

In this way the local transitivity of the groups of three alternatives that we see in Fig. 6 (e) and (f) allow the decider to relax his preferences on the alternative a_1 so to let him remove the arc 1 ⟶ 2.

As we argue in [4] this removal is, indeed, equivalent to the turning of a minimal number of strict preference relations $a_1 >_i a_2$ into indifference relations $a_1 \sim_i a_2$ so to make these two alternatives incomparable, according to the definition of the particular order ›.

The outcome of this procedure is shown in Fig. 7 where we have a_2 as the best alternative and a_1 as the worst one. This removal procedure was easily applied in this case but in more complex cases it may prove inapplicable so that the only solution is to adopt the ex-post approach and deal with the cycles at the level of the multigraph ***MG***.

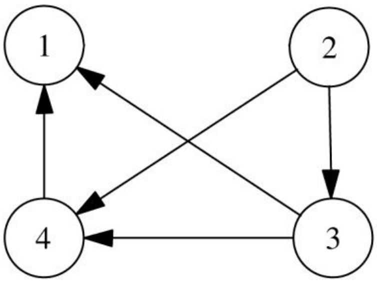

Figure 7. The made acyclic version of the graph of Fig. 6 (a)

VII. Merging the Graphs in a Multigraph

A. *The merging procedure*

Up to this point we have assumed that the deciders of the set ***D*** have been acting in isolation, each one of them producing his own partial order graph and using it as a decision aiding tool for the selection of the best alternative from the set ***A*** according to (possibly some of) the criteria of the set ***C*** ([1], [2], [3] and [4]). The fact that each decider can use a personal set of criteria as well as a personal set of alternatives, though with some constraints, is a proof of the robustness of the proposed tools, as it is discussed more thoroughly in [4].

We can imagine, however, that the deciders $\boldsymbol{d_i}$ are cooperating in a collective decision effort so that they have to merge their individual oriented graphs $\boldsymbol{G_i}$ in a single multigraph ([15]) ***MG*** on which they would like to apply the final selection procedure that we present in section X.

By definition ([15]) in a multigraph between two nodes we can have more than one directed arc, as it is denoted by the multiplicity index that we put on each arc. The real value of such a multiplicity index derives from the structure of the composing graphs $\boldsymbol{G_i}$ since whenever between two nodes of one of such graphs we find an arc with the same orientation we increase of one the value of this multiplicity index.

The merging is easily performed through the application of a mechanical procedure based on the following steps:

(1) we draw the nodes corresponding to the alternatives of the set ***A***;

(2) for each pair of alternatives $a_i, a_j \in \boldsymbol{A}$ we examine the graphs $\boldsymbol{G_i}$ and count the number of arcs that exist from a_i to a_j so to draw an arc with the same orientation in ***MG*** and to associate to this arc, as a multiplicity index, the number that coincides with the total number of these arcs;

(3) we repeat the same steps for the arcs from a_j to a_i;

(4) when all the pairs of alternatives have been examined the merging procedure is over.

With reference to Figs. 2, 4 and 8 we may say that:

(1) if we consider the nodes 1 and 2 we have a directed arc from 1 to 2 in Fig. 2 and another arc with the same orientation in Fig. 4 so in the resulting ***MG*** of Fig. 8 we have a link from node 1 to node 2 with multiplicity equal to 2;

(2) if we consider the nodes 3 and 4 we have a directed arc from 3 to 4 in Fig. 2 and another arc with the same orientation in Fig. 4 so in the resulting ***MG*** of Fig. 8 we have a link from node 3 to node 4 with multiplicity equal to 2;

(3) similar considerations hold in all the remaining cases so that at the end of the merging procedure we get the result of Fig. 8.

We underline that if, during the merging procedure, we find that between any pair of nodes we have two arcs with opposite orientations we neither cancel them out nor collapse them in an

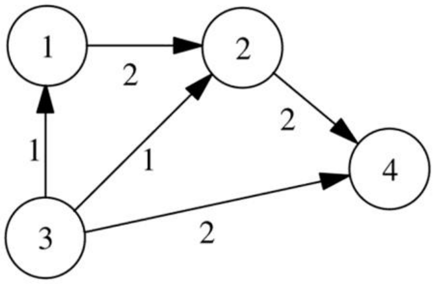

Figure 8. The resulting multigraph for the toy examples

undirected arc but keep both of them as forming a cycle of length equal to two in the resulting multigraph. The treatment of cycles of length equal to two as well as of cycles of greater length in ***MG*** is postponed until section IX. The presence of cycles of the former type can be justified by the fact that the criteria (and the corresponding orderings) are seen as incommensurable so that they cannot give rise to cancellations (or compensations) between conflicting criteria or criteria that rank two alternatives in opposite ways whereas cycles of greater length depend on the interactions among the composing graphs $\boldsymbol{G_i}$. For a deeper treatment of these issues we refer also to [4].

As we have seen in the graphs of Figs. 2 and 4 also in the resulting multigraph ***MG*** we may have situations of incomparability between pair of alternatives but such situations usually do not prevent the deciders from identifying the best and worst alternatives.

If we consider, for instance, the multigraph ***MG*** of Fig. 8 we have an incomparability situation on the pair (a_1, a_4) but this situation does not prevent the deciders from saying that the best alternative (as it is enforced also by an application of *backward pruning* technique) is the alternative a_3 (that corresponds to the node 3 or a node without incoming arcs) whereas (as it is enforced also by an application of *forward pruning* technique) alternative a_4 (that corresponds to node 4 or a node without outgoing arcs) is the worst one.

From this perspective, the multigraph is, therefore, the end product of a multideciders multicriteria procedure and must be seen as a decision aiding tool according to the general approach of the multicriteria methods ([2], [3], [4], [13] and [15]).

B. Some further comments on the toy examples

From the two toy examples that we presented in section V it should be evident how the lack of transitivity as a general property of the binary relation › derives from the fact that the single total orderings are produced by using independent criteria, from the fact that the alternatives are compared pairwise for each criterion and from the definition of the › binary relation that forces the deciders to produce partial orders ([1], [5] and [6]).

Notwithstanding their simplicity such examples show how our binary relation can be used with any set of alternatives ***A*** and criteria ***C*** by a set of deciders (some of which may even use a subset of the sets ***A*** and ***C***) with the aim of identifying at least the set $\boldsymbol{A_b}$ of the best alternatives.

We note how the final selection from the set $\boldsymbol{A_b}$ (if it contains more than one alternative) both in the single decider and in the multideciders case represents, in many cases, a political choice that must be performed by using methods distinct from those based on the criteria of the set ***C*** but, in any case, starting from a sound set of potentially best alternatives ([8], [9], [13] and [14]).

VIII. Decidability and Incomparability

From the resulting graphs $\boldsymbol{G_i}$ and ***MG*** of our toy examples we derive the possible existence of both *decidable* and *incomparable* pairs of alternatives.

We say that two alternatives are *decidable* if there is a directed arc between them and so if we can state a preference between such alternatives. If all the pairs of alternatives are decidable then, in absence of cycles (but possibly also in presence of cycles, see further on), we have a single best alternative and a single worst alternative either for the single decider or for the whole set of the deciders. As it is shown in Fig. 8 the same may hold also if not all the pairs of alternatives are decidable.

The presence of cycles represents a reason of failure of the Condorcet voting method ([5], [8], [9], [13] and [14]) and represents also a weakness of the proposed method though in the sections IV and IX (see also [4]) we discuss some possible solutions.

If, on the other hand, we have no arc between two alternatives we say that such alternatives are *incomparable*. The presence of incomparable alternatives may widen the sets of the best and the worst alternatives.

With reference to our toy examples we have that:

(1) in the first toy example the pairs of alternatives (a_1, a_3) and (a_1, a_4) are incomparable whereas all the other pairs are decidable;

(2) in the second toy example the pairs of alternatives (a_1, a_4) and (a_2, a_3) are incomparable whereas all the other pairs are decidable;

(3) in the multigraph of Fig. 8 the pairs of incomparable alternatives are those that are common to all the composing graphs (and so only the pair (a_1, a_4)) whereas all the other pairs are decidable.

The presence of incomparable alternatives is a sufficient condition for making the ordering a partial order and is also the main reason for the failure of the transitivity. Moreover the incomparable alternatives are the best candidates for being either the best or the worst alternatives of our decision problem.

IX. Cycles in the Multigraph MG

Once the multigraph ***MG*** has been defined, since we assume that the composing graphs $\boldsymbol{G_i}$ do not contain isolated nodes (possibly thanks to the application of the method we showed in section VI), we can have the following cases:

(1) the multigraph ***MG*** is acyclic;

(2) the multigraph ***MG*** contains some cycles.

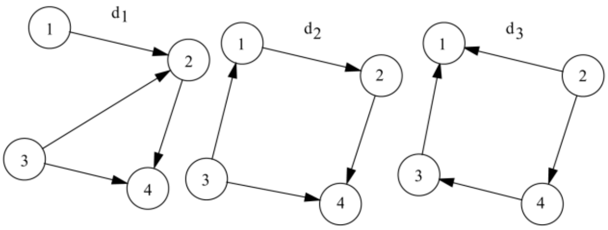

Figure 9. Three directed graphs of three deciders

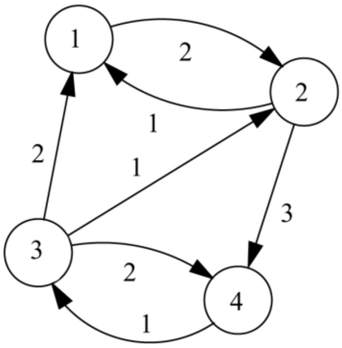

Figure 10. The resulting multigraph of the graphs of Fig. 9

In the former case the next step that the deciders have to carry out is the application of the final selection procedure ([4]) that we describe in some detail in section X and that is based, essentially, on a lexicographic ordering involving the nodes of the set A_b and their outgoing arcs.

In the latter case the cycles derive from the interactions among the graphs G_i of the deciders and can cause troubles only if their presence prevents the deciders from identifying the alternatives of the set A_b otherwise, as we show in section X, their presence can be tolerated even in cases where this presence does not allow the identification of the worst alternatives (through the use of the forward pruning procedure) or the application of the backward pruning procedure.

In order to present a problematic case together with a possible solution we can therefore assume that our three deciders have produced, as their individual effort, the graphs of Fig. 9. From such graphs, by applying the rules of our particular order, we get the multigraph ***MG*** of Fig. 10.

In the multigraph of Fig. 10 we have cycles of length equal to 2, 3 and 4 but the main problem is represented by the fact that there is no node without incoming arcs so that no node can be identified as a god candidate for being one of the best alternatives. Moreover in that graph there is no node without outgoing arcs so that no node can be identified as a good candidate for being one of the worst alternatives although this is decidedly a minor problem.

Such features, in this case, prevent us also from applying both the forward pruning and the backward pruning procedures. In order to solve this problem we can use a modified version of the Condorcet voting method where:

(1) we make pairwise comparisons among the alternatives that are connected at least by one directed arc;

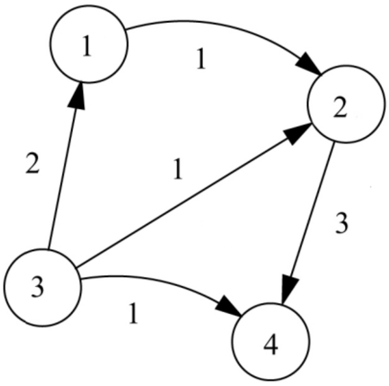

Figure 11. The reduced multigraph of the multigraph of Fig. 10

(2) we consider the multiplicity indexes as the votes of one alternative of each pair against the other so that the higher value of the index identifies the winning alternative of each pair.

The only modification we make to the Condorcet voting method is represented by the fact that in the Condorcet voting method all the alternatives are pairwise compared between themselves whereas in our version we compare between themselves only decidable alternatives.

If we apply our method to the case of Fig. 10 we have that:

(1) alternative a_1 wins against alternative a_2 for two votes against one;

(2) alternative a_2 wins against alternative a_4 for three votes against zero;

(3) alternative a_3 wins against alternative a_1 for two votes against zero;

(4) alternative a_3 wins against alternative a_2 for one vote against zero;

(5) alternative a_3 wins against alternative a_4 for two votes against one.

From such pairwise comparisons we derive the reduced multigraph ***RMG*** of Fig. 11 where the values of the multiplicity indexes represent the differences of the votes for each pair of alternatives.

The use of the method in this case allowed us to solve the problem since from the reduced multigraph ***RMG*** of Fig. 11 it is easy to see how:

(1) the alternative a_3 is the best alternative so that we have $A_b = \{a_3\}$,

(2) the alternative a_4 is the worst alternative

as it is easily enforced also by the application of both the forward and the backward pruning procedures.

Unfortunately for us the things do not go so smoothly all the times as it can be easily shown, for instance, if we add one more decider d_4 (whose graph is shown on the right side of Fig. 12) to the set $D=\{d_1, d_2, d_3\}$ of those that produce the graphs of Fig. 9 and modify the graph of the third decider d_3 as it is

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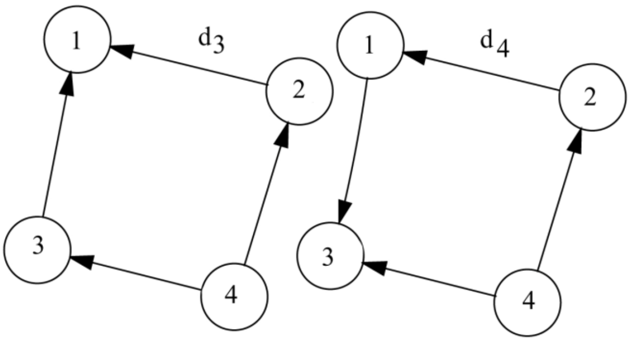

Figure 12. A modified graph and an added graph

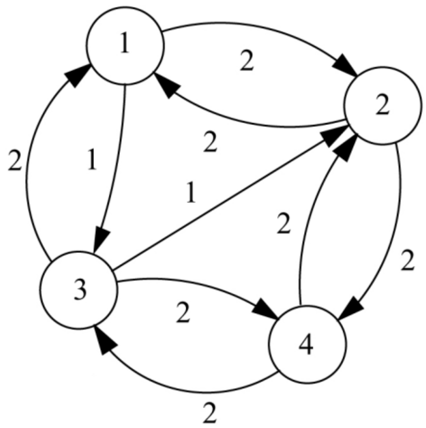

Figure 13. The resulting multigraph

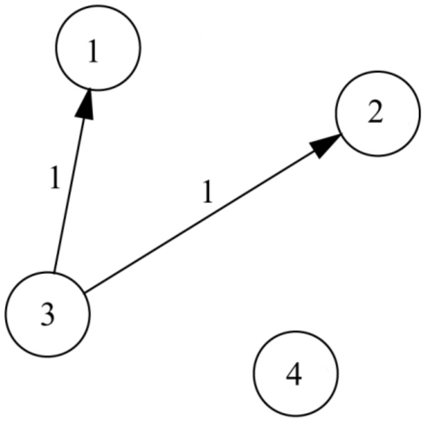

Figure 14. The final reduced multigraph

shown on the left side of Fig. 12. If we merge the four graphs G_i in this case we get the multigraph ***MG*** of Fig. 13.

If we apply in this case our modified Condorcet voting method we get the reduced multigraph ***RMG*** of Fig. 14 since we recall how in case of a tie we consider the alternatives (and the corresponding nodes) as incomparable.

In this case the final reduced multigraph contains node 4 as an isolated node but, unlike what we have seen in section VI, we have no possibility to use a set of total orders (based on relations of strict preference and indifference) as the basis for the application of a modified Borda method so to relink the isolated node 4 to some of the other nodes as we have shown in the toy example of section VI.

Things moreover can go even worse. If, for instance, in Fig. 13 we increase by one the multiplicity index on the arc $1 \rightarrow 3$ we get an ***RMG*** where also node 1 is an isolated node and if we add an arc $2 \rightarrow 3$ with a multiplicity index of 1 we transform also node 2 (in the ***RMG***) in an isolated node so that the resulting ***RMG*** is made only of isolated nodes to which there correspond incomparable alternatives.

There is also no guarantee that a reduced multigraph does not contain cycles of the disturbing type or that prevent the identification of a set $A_b \neq \emptyset$.

If, indeed, we increase by one the values of the multiplicity indexes of the arcs $1 \rightarrow 2$, $2 \rightarrow 4$ and $4 \rightarrow 3$ in the multigraph of Fig. 13 in the corresponding reduced multigraph we get the cycle

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$$

(and so a failure of transitivity) so that in this new reduced multigraph we cannot identify neither a best nor a worst alternative.

In all the cases where we get a reduced multigraph that contains either isolated nodes or cycles such that we have $A_b = \emptyset$ we are in a situation where the proposed method fails.

The possibility of failures should be of no surprise since it is well known from the literature (see, for instance, [5], [8], [9], [12] and [13]) that every voting or decision method has its own weaknesses, cases where it fails and properties it violates.

In our case, however, the efforts made by the deciders in order to produce their graphs G_i does not get lost since it allows them to gain a deeper understanding of their ranking and selection problem so that they can use this understanding in order to perform a selection of one of the available alternatives as a political choice possibly through the adoption of some voting procedure. In any case the treatment of this step is outside the scope of both the present paper and the multicriteria method itself.

The Final Selection

The deciders can use the final selection procedure that we are going to describe in this section in the following cases:

(1) if the multigraph ***MG*** has no cycles;

(2) if the multigraph ***MG*** has cycles that do not prevent the identification of the set $A_b \neq \emptyset$.

In both cases the deciders need to apply the final selection procedure only if the condition $|A_b| > 1$ holds since, in the case $|A_b| = 1$, there are no doubts about the identity of the best alternative.

We underline how:

(1) the method can be applied also to the ***RMG*** under the above specified conditions;

(2) the results of the procedure can be enforced also by the application of the forward pruning procedure and (in all the cases where this is possible) also of the backward pruning procedure.

The core part of the procedure is the *lexicographic preference procedure* that we presented in [4]. By using this procedure we impose a lexicographic ordering on the elements of the set $\boldsymbol{A_b}$ (under the condition $|\boldsymbol{A_b}|>1$) so that it is possible to select one of its elements as the best or most preferred alternative.

In order to define this lexicographic procedure we proceed as follows. We associate to each alternative $a_i \in \boldsymbol{A_b}$ a pair of integer values (y_i, x_i) where y_i counts the number of the nodes that are the end points of the arcs going out of node a_i whereas x_i counts the total multiplicity of these arcs. With these values we define a binary preference relation $\supset$ for each pair of alternatives a_i, $a_j \in \boldsymbol{A_b}$ as follows:

(1) we say that $a_i \supset a_j$ or that the former alternative is strictly preferred to the latter if one of the following conditions holds

(a) $y_i > y_j$

(b) $y_i = y_j$ and $x_i > x_j$

(2) we say that such alternatives are equivalent or tied if we have both $x_i = x_j$ and $y_i = y_j$

(3) otherwise we have $a_j \supset a_i$.

Once the alternatives have been lexicographically ordered we can select the top listed alternative or one of the top listed tied alternatives as the best or most preferred alternative.

In order to show how the procedure works in practical cases we use two toy examples from [4], examples that we reproduce in Fig. 15.

In both multigraphs of Fig. 15 we have that:

(1) we cannot use our pruning procedures owing to the structure of both graphs;

(2) we have a cycle that however do not prevent us from identifying the set $\boldsymbol{A_b} = \{a_1, a_3\}$.

It is easy to see how the multigraphs of Fig. 15 cannot be reduced multigraphs owing to the presence in both of a cycle of length equal to two.

In the case (a), by applying our lexicographic ordering, we have:

$$y_3 = 2 = y_1 \text{ and } x_3 = 6 > 4 = x_1$$

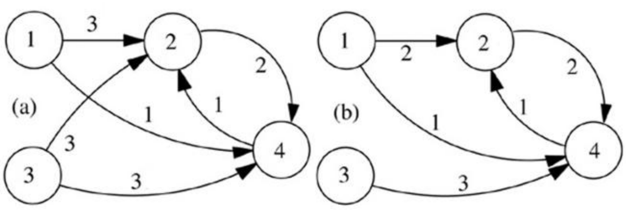

Figure 15. Two examples for the final selection

so that we have $a_3 \supset a_1$ whereas in the case (b) we have:

$$y_1 = 2 > 1 = y_3$$

so that we have $a_1 \supset a_3$.

Such results seem appropriate and sensible since in the former case the procedure privileges the alternative of $\boldsymbol{A_b}$ that has the higher number of votes against the other alternatives of $\boldsymbol{A}$ whereas in the latter it privileges the alternative of $\boldsymbol{A_b}$ that is preferred to the higher number of other alternatives of $\boldsymbol{A}$.

In both cases we have a lexicographic order without ties over the alternatives of the set $\boldsymbol{A_b}$ so that we can select alternative a_3 in the former case and alternative a_1 in the latter.

This is not always the case as it can be easily seen if we increase by two the multiplicity index of the arc $1 \rightarrow 4$ in the graph (a) of Fig. 15. In this case indeed we have

$$y_3 = 2 = y_1 \text{ and } x_3 = 6 = x_1$$

so that, according to our lexicographic ordering, the alternatives a_1 and a_3 are seen as tied or equivalent.

In this case the final selection of one of such two alternatives is again the outcome of a political decision from the deciders whose description is, also in this case, outside the scope of the paper but also of the method itself.

X. Concluding Remarks

In this paper we have presented the general structure of a decision aiding tool that can be used by a set $\boldsymbol{D}$ of deciders in order to rank the alternatives of a set $\boldsymbol{A}$ according to the criteria of a set $\boldsymbol{C}$.

The tool is based on a binary relation that defines a particular order ([1]) and has been firstly presented in [7] then it has been developed in [2] and, more recently, in [4].

In short the tool can be described as made of two consecutive steps.

The first independent step uses the particular order and can be used individually by the deciders so that each decider $\boldsymbol{d_i}$ can produce a directed graph $\boldsymbol{G_i}$ and can use it as an individual decision aiding tool for the ranking and selection of the alternatives.

At the second ancillary step the deciders merge their individual graph $\boldsymbol{G_i}$ in a single multigraph $\boldsymbol{MG}$ and use it as a collective decision aiding tool for the same main aim.

The tool makes use of classical concepts (such as the Borda and Condorcet voting methods and a lexicographic ordering method) but adapts these concepts within a coherent and goal directed framework with the aim of solving all the problems that can arise both during the individual and the collective step.

In the paper we indeed present both the normal and free of problems procedures but also present and examine some of the potential problems and possible failures for which we propose also some tentative solutions.

The solutions we propose, unfortunately, are not definitive so that some problematic situations are left over. In such cases the deciders are left in a situation of undecidability but with the acquired knowledge gained from both the individual and the collective efforts of ranking the alternatives according to the given set of the criteria. They can use this acquired knowledge in order to perform the desired selection on a more sound basis possibly through the use of voting methods even distinct from those we listed (the Borda and Condorcet voting methods) but that, how it is well known from the literature, are also themselvers absolutely not exempt from problems.

The presence of reasons of failure for the proposed tool is the main guide for the pursuing of more refined solutions although we are well aware that every solution we may propose opens, in its turn, new problems that beg for a solution.

Last but not least, other developments include the application of the proposed decision aiding tool to more concrete problems possibly using well specified criteria with their real rankings of real alternatives.

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