{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-03-18
Accepted: 2020-04-20
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.1-9

Original Paper

Evaluation of municipalities' competence in the building sector in selected regions of the SR: Case Study

Ivana Lichnerová*, Eleonóra Marišová

Slovak University of Agriculture in Nitra, Faulty of European Studies and Regional Development, Department of Law, Nitra, Slovakia

ABSTRACT

The original competence of state authorities - competence in the building order sector, was transferred by Act No. 416/2001 Coll. to municipalities (effective from 1 January 2003). Each municipality became a building office. The conditions, staffing and range of the state's participation in the financing of the delegated competence were strictly determined by the legislation in force, the basis of which still applies today. The aim of the article was to evaluate the performance of building competencies by quantitative and qualitative research methods in selected regions of the Slovak Republic (SR), using available data from a sample of building offices in Nitra (NR) and Košice (KE) Regions. The quantitative analysis was applied to joint building offices [JBOs] and single building offices [SBOs]) within the NR and KE Regions, which are an administrative umbrella of 745 municipalities of the SR. In the analysis, there are data for 2014, 2015 and 2016 (jointly and separately for the NR and KE regions), examined. We analyzed that the building offices were able to perform 69 902 operations/acts in examined years. The amount of allocated state transfer for the municipalities (745), depending on the number of inhabitants, was on average 1,114,363.82 EUR (own processing) per year for the performance of building competence. The number of acts of building offices of the NR and KE Region increased by 15.35 % (in 2016 compared to 2014). The qualitative analysis was carried out through controlled interviews with representatives of 58 JBOs and SBO s in NR and KE Regions. We identified opinions on the implementation of competence in the field of building order, its financing and investment activities in territorial districts of selected sample of building offices in NR and KE regions. From the point of view of the top employees of the building offices, it seems that the financing of this delegated power is financially undersized and investment activities are progressing.

KEYWORDS: transfer of competencies performance of delegated competencies, municipality, Building Code, qualitative and quantitative research

JEL CLASSIFICATION: D20, D40, M10

INTRODUCTION

Fragmentation of the settlement structure [6], which is associated with a very large variability in the size of municipalities in Slovakia [12], can be solved by merging municipalities or

* Corresponding autor: Ivana Lichnerová, Slovak University of Agriculture, Faculty of European Studies and Regional Development, 949 76 Nitra, Slovakia, e-mail: xlichnerovai@is.uniag.sk

inter-municipal cooperation [8]. Forms of inter-municipal cooperation are a natural part of the functioning of self-governments, which aim is to more effectively secure the competencies of municipalities in different areas of everyday life [3].

Inter-municipal cooperation in the territory of the Slovak Republic (SR) began to expand due to the decentralization of public administration, which emphasized the democratic values of society. The Constitution of the Slovak Republic established powers that passed from state authorities to self-government. The municipalities had to ensure the competencies entrusted to them by law and for which funding the state has guaranteed [10].

In 2003, the process of decentralization of public administration caused that the competence in the field of building sector were transferred to municipality. The transfer of this competence from the state to self-government was preceded by the effort of the Slovak Republic to reform the public administration. Specifically, Act No. 416/2001 Coll. on the Transfer of Some Competencies from State Administration Bodies to Municipalities and Higher Territorial Units, was adopted. By this Act, in 2002, 63 [11] competencies and later more than 300 competencies have been transferred from the state authorities to the self-government [7].

Within the meaning of the above Act (which is an amendment to the Building Act – Act, No. 50/1976 Coll.), each municipality of the Slovak Republic became a building authority.

The Slovak Republic is relatively a small but highly fragmented state – for example as the Czech Republic. The Czech Republic, that faced a large number of municipalities, who were obliged to provide delegated powers from state authorities, solved this situation by categorization of municipalities [5].

In the territory of the Slovak Republic, there was no categorization of municipalities, although in the past, Klimovský [5] outlined this possibility for Slovakia as of 2018/12/31, in the territory of the Slovak Republic 2,927 municipalities [1] were registered.

The problem is that due to the objective reasons (size of municipality, number of population, qualified employees, budget of financial resources, material and technical equipment of offices), especially the small municipalities are not able to ensure fully competence for their inhabitants in the building section. Municipality's needs are financed from its own funds and from state subsidies. The principal task of a municipality in the exercise of self-government is to provide care and general development of its territory and its population's needs [9].

On the basis of the mentioned above, municipalities started to use the possibilities to their competencies in the building section by establishment of joint building offices. The Joint Building Offices are created, according to the §20 Act No. 369/1990 Coll. as amended. However, in the Slovak Republic not all municipalities are part of the Joint Building Offices (2 649) but they are Single Building Offices (278).

Within the research project VEGA 1/0190/17, we tried to identify, by means of a quantitative survey of a selected sample of building authorities located in the Nitra (NR) and Košice (KE) Regions, whether the competence in the building section are performed more efficiently by Joint Building Offices or by Single Building Offices. As a part of the research, we also focused on identifying of the amount of the state transfer, which is every year allocated from the state budget to the building offices.

The quantitative outputs/conclusions of this paper are supplemented by information obtained through qualitative research executed in a selected sample of building offices of the NR and KE Region and related to the building competencies performance. The paper points out the possible solution of future provision of competence by municipalities in the area of building sector.

MATERIAL AND METHODS

With the financial support of the VEGA 1/0190/17 - research project, in 2014-2016 we were able to collect quantitative data on the number of issued: building permits, occupancy permits, zoning decisions, additional decisions and a group of other operations sample of building offices located in Nitra (NR) and Košice (KE) region. In a sample of the mentioned circle of building authorities, we realized controlled interviews with the top representatives employers of building offices.

In article, we analyze aggregate data (but also concrete data about acts of building offices in NR a KE Regions) on acts of building offices and answers from managed interviews within the sample files of building offices located in the NR and KE Regions.

Analysis of quantitative data is realized by: descriptive statistics – specifying by determination of percentage ratio of operations/acts (1.1) (percentage increase/decrease (1.2).

$$b = 100 \times \left(\frac{n}{\sum n}\right) \tag{1.1}$$

where: b = percentage, n = number of municipalities sample files, Σn = total number of municipalities (aggregate data) SR/ respectively NR and KE Region

$$h = 100 \times \left(\frac{b-a}{a}\right) \tag{1.2}$$

where: h = percentage increase/decrease, b = new measured value, a = original value,

and average value (1.3)

$$Average\ value = \frac{p^1+p^2+p^3+\cdots+p^n}{\sum n} \tag{1.3}$$

where: p = number of acts of individuals building offices/amount of state transfer for building sector for individual building offices, n = number of analyses sample files of building offices.

Analysis of quantitative data is realized by acquired data, where data are processed through the multidimensional - complex, non-parametric DEA analysis (Charnes – Cooper – Rhodes, 1978 in [2], [4]).

The next, through qualitative analysis we identify opinions of the top representative employees of the building offices in the Nitra and Košice Regions for performance of competence in the building sector.

The amount of state transfer, which was in the reference period of years 2014-2016, allocated for building offices in the NR and KE Regions, we expressed by the next formula (1.4):

$$Amount\ of\ state\ transfer = p \times 0{,}93\ EUR \tag{1.4}$$

where: p = number of population with permanent residence in the territory of municipality (Building Offices) as 12/31 of the previous calendar year (according to data by Slovak Statistical Office).

Question, whether the sample of building offices we have analyzed, can be considered as a quota sample, was verified by online sample size calculator. Expression Sample size (1.5):

$$Sample\ size = \frac{\frac{z^2 \times p(1-p)}{e^2}}{1+(\frac{z^2 \times p(1-p)}{e^2 N})} \tag{1.5}$$

where: N = population size, e = margin of error (percentage in decimal form), z = z-score (The z-score is the number of standard deviations a given proportion is away from the mean.)

We worked with the sample of building offices from which we have data on the activities of building offices (745) in relation to the total number of municipalities NR and KE region (815) in quantitative research.

The qualitative research was analyzed by structured interviews with top representative employees of building offices, which was a concrete sample of the top representative employees from 58 joint and single building offices located in the NR and KE regions.

RESULTS AND DISCUSSION

In the Nitra (NR) Region, there are 354 municipalities located, which represent 12.09 % of the municipalities of the Slovak Republic. These municipalities are located in the cadastral territories of 7 districts. In the NR Region, 96.89 % of the total number of municipalities took advantage of the possibility of contractual cooperation in the building sector and became part of 26 joint building offices (JBOs), but on the other hand 3.11% municipalities (from the NR Region) perform the competencies in the building sector single-handedly, municipalities are single building offices (SBO).

Košice (KE) Region, including 22 city districts has a total 461 municipalities – it is 15.74 % of the municipalities of the Slovak Republic. These municipalities are located in the cadastral territories of 11 districts. Total number of municipalities, without city districts in the KE Region, is 439. In the KE Region, from the total number of municipalities (439), 89.06 % municipalities took the possibility of contractual cooperation in the building section and became part of 28 joint building offices (JBOs). Other number of municipalities (48), what is 10.94 % – from the total number of municipalities (without city districts) in the KE Region, are single building offices (SBO) – they don´t cooperate with other municipalities in the building sector. The transferred building competence, in the 22 city districts of the Košice is performed by employees of the building offices in 4 building offices in city of Košice.

From the total number of municipalities of the Nitra and Košice Regions (815) we have quantitative data for 745 municipalities, which represents 91.41 %. As of 31 December, 2016, 22.04 % of the total population of the Slovak Republic (5 435 343 inhabitants) lived in selected municipalities, which were subjected to quantitative research as of 31 December

2016 [4]. Quantitative data represent the number of issued: building permits, house building permits, zoning decisions, additional decisions and a group of other acts of building authorities.

For correct conclusion, based on the analysis of performance of competencies in the building sector on 95 % confidence level, it is important to have quantitative data for 262 building offices (from 815 building offices) together for the NR a KE Regions. This criterion is met in our analysis, because we have already mentioned that we have data for 745 building offices located in the Nitra and Košice Regions.

The total number of acts of building offices of the NR and KE Regions (745 building offices) is shown in the following table 1.

Table 1 The number of acts of the building offices together for NR and KE Regions in the period 2014 – 2016

Year	Number of building permits	Number of occupancy permit decisions	Number of zoning decisions	Number of additional decisions	Number of additional operations	Total (Aggregate)
2014	5 182	4 608	922	784	10 439	21 935
2015	5 554	4 017	1 078	791	11 222	22 662
2016	6 374	4 550	1 128	901	12 352	25 305
Total (Aggregate)	17 110	13 175	3 128	2 476	34 013	**69 902**

Source: Own processing (2019)

In the period from 2014 to 2016, the number of acts of building offices (745 building offices) in the sample of building offices of the NR and KE Regions, slightly increased. In 2014, there was an average of 29.44 acts per building office, in 2015 it was 30.66 act and in 2016 exactly 33.96 act. On the basis of the above, the average percentage of increase in the activities of building offices NR and KE Regions in 2016 is compared to 2014 – 15.35 %.

As already mentioned, Table 1 shows the total number of acts performed by a sample of building offices within NR and KE in the region, regardless of whether these acts were carried out on building offices belonging to the joint building offices or the single building offices. However, the paper also points to a more specific analysis of building offices. The building offices are specifically analyzed within the NR and KE regions and we also specify whether the building offices perform the competence in the building sector together (JBOs) or single-handedly (SBOs).

In terms of average absolute outputs (analyzed building offices – separately in NR and separately in KE Regions) there are considerable differences between building offices located in NR and KE Regions. In the NR Region on average, there are 497 acts per 1/JBO and 326 acts in the KE Region. The opposite ratio was found in SBO 119 to 277 acts in favor of KE Region. However, the results of the multidimensional DEA analysis show that there is no statistically significant difference in the effectiveness of building offices in the NR (whether they are single or joint building offices). As the size of the served population increases, overall efficiency decreases, but in this case the decrease is not statistically significant [4].

Total, number of acts of the sample of building offices located in the NR and KE regions in the monitored period of 2014 – 2016, increased. Through qualitative research, we performed questions-driven interviews with employees of the designated building offices (33 building offices in the NR Region, from this – 24 JBOs and 9 SBOs and 25 building offices in the KE Regions and from this 9 JBOs and 16 SBOs) and identified the reasons for increased activity of building authorities. The operations mainly concerned the development of investment activities, namely:

The qualitative research shows that in the NR region 17 JBOs and 5 SBOs agreed with the fact that their territorial district is investing in the construction of new buildings and civil engineering constructions and 7 JBOs and 4 SBOs did not record any investment activity in their territorial districts. Remarkable was the note obtained in the survey in the NR region, which implies that in the NR region the investment activity is mainly focused on expanding the premises of the original buildings.

In the KE region, investment activity was recorded in 6 JBOs and 11 SBOs, while in their territorial districts there are mostly low-standard apartment buildings, apartment buildings up to 20 flats, sewerage system, water supply system and other buildings. No investment activity was recorded in 3 JBOs and 5 SBOs.

Despite the fact that the number of acts of building offices wat increased in the period under review, the financial amount of state transfer for transferred competencies from the state-government to municipalities (competence in the building sector) remained unchanged. According to the Decree of the Ministry of Transport, Construction and Regional Development of the Slovak Republic of 21 December 2010 No. 20786/2010-SRVS / z. 54145-M on the provision of subsidies from the state budget to municipalities to cover the costs of the transferred performance of state administration in the section of the Building Code and Housing was 0.93 EUR / inhabitant. This financial amount was applied from 2011 till 1st April 2018.

The following table 2 expresses the amount of state transfer, which was collectively allocated to the analyzed sample of building offices in the NR and KE Regions (745 building offices) in the range of 2014-2016 depending on the number of residents residing in the municipality as of 31 December of the previous calendar year (it is average amount of state transfer).

Table 2 The amount of state transfer determined for a sample (analyzed) set of building offices NR and KE Region

	Year 2014		Year 2015		Year 2016	
Number of populations	2013/12/31	1 198 963	2014/12/31	1 197 652	2015/12/31	1 198 107
Amount of state transfer	1 115 035.59 EUR		1 113 816.36 EUR		1 114 239.51 EUR	

Source: Own processing, data of the Slovak Statistical Office [13] and Decree of MT Cand RD[3] SR (2013-2015)

In 2018, namely with effect from 1st April 2018, the amount of the state transfer, in the meaning of Decree of the Ministry of Transport, Construction and Regional Development of the Slovak Republic of 21 December 2010 No. 20786/2010-SRVS / z. 54145-M was changed by the Notification of the Ministry of Transport and Construction of the Slovak Republic No. 84/2018 Coll.) and increased to 1.11 EUR / inhabitant.

Despite the fact that the amount of the state transfer was in comparison with the original wording of Decree no. 20786/2010-SRVS / of 544145-M increased by 0.18 EUR / inhabitant and its new measure under Notice No. 84/2018 Coll. which, in the meantime, this notification has been amended by Notice of the Ministry of Transport, Construction, No. 392/2019 Coll. with effect from 1 December 2019 (it does not stipulate the specific financial amount of the state transfer per inhabitant, but verbally expresses the amount of the transferred money for performance of state administration in the sector) from the state budget. Based on the qualitative research, we can state that the amount of state subsidy is insufficient for building offices.

The employees of the building offices agreed that the amount of the state transfer doesn´t take into account all the actual expenditures of municipalities (building offices) for the exercise of competence. Municipalities have to complement the state financial transfer from their own resources (municipal budgets), which ultimately means a violation of the Act on Municipal Establishment No. 369/1990 Coll. as amended.

At the beginning of this paper, we indicate that each municipality of the Slovak republic is a building office according to the § 117 of Act no. 50/1976 Coll. - Building Act, as amended. As we have inferred the qualitative research, based on the responses of the employees of the building offices, especially small municipalities have a particular problem with ensuring competence in part of the building sector. These municipalities solve this problem by inter-municipal cooperation in accordance with § 20 of Act No. 369/1990 Coll. on Municipal Establishment as amended. The reason is insufficient financial coverage of the costs of operating building offices. However, in NR and KE Region as well as the whole territory of the Slovak Republic, the joint building offices are created chaotically [11] (even without the establishment of a registered office according to the legislation).

To eliminate this chaotic inter-municipal cooperation, we propose to determine the seat of building offices in exact municipalities by a special law [8]. Similarly nominal, Registry offices perform the registry competence transferred from state. This regulation was done by Act No. 154/1994 Coll. as amended and this act namely stipulates all the seats of the registry offices. In our opinion, new statement of the seats of the building offices would be more organized and transparent, which could ultimately have a more effective impact on the performance of the building competence.

CONCLUSIONS

Purpose of this paper was to examine the performance of building competencies by quantitative and qualitative research methods in selected regions of the Slovak Republic (SR). We made this research in the period of 2014 - 2016 using available data from a sample of building offices in Nitra (NR) and Košice (KE) Regions.

Since 1st January, 2003 the original competence of state authorities in the building sector was transferred by Act No. 416/2001 Coll. as amended to the scope of self-government. This way each municipality of the Slovak Republic (2,927 municipalities) became a building authority.

Given that especially small municipalities in the Slovak Republic failed to provide services in the building order, they began to use the regulation of amendments to the Act on Municipal Establishment No. 369/1990 Coll. as amended - the possibility to conclude

contracts among themselves for the purpose of implementing the issuance of building, occupancy permits and zoning decisions and started to create joint building offices (JBOs).

There are 815 municipalities located in the territory of the NR and KE regions. We tried to assess the performance of the competency in the building section in a quota sample (together 745 JBOs and SBOs).

The qualitative research was analyzed by structured interviews with top representative employees of building offices, which was a concrete sample of the top representative employees from 58 joint and single building offices located in the NR and KE regions.

Financial support by the Slovak state budget relating to the transferred competencies to municipalities is regulated by §6 par. 2 of Act No. 523/2004 Coll. on budgetary rules of public administration, however, these financial resources are not sufficient, which leads municipalities at the level of the building competencies to merge into the joint building offices serving for numerous municipalities (JBO).

In the period from 2014 to 2016, the number of acts of building offices (745 building offices) in the sample of building offices of the NR and KE Regions, increased by 15.35% (2016 compared to 2014).

In spite of the increasing number of building offices activities, the state has remained unchanged the state subsidy in the monitored period. The increase of the financial subsidy has been gassed by the factor of increasing the number of residents with permanent residence in the territorial districts of individual building offices (municipalities) in the NR and KE regions.

By qualitative research, we have analyzed the mentioned investment activity and identified the dissatisfaction of the building offices with insufficient allocation of financial resources transferred to the municipalities.

As the current creation of joint construction offices is possible in the territory of the Slovak Republic only by voluntary inter-municipal contractual cooperation according to the §20 of Act No. 369/1990 Coll. on Municipal Establishment as amended, nowadays the joint building offices are created chaotically and uncontrollably. Controversially, in case of the registry offices, a precise legislation stipulates their seats and districts. The number and seats of registry offices are determined by Act No. 154/1994 Coll. - on registers, as amended.

In this paper, we have shown the possibility of building competencies problem solution by adoption of a new law that should define the seats of the building offices. The voluntary cooperation of building offices should be substituted by new legislation regulating all the seats and territories of building offices. We suppose that this way their activities could affect the future more efficient building competencies execution.

ACKNOWLEDGEMENTS

The paper has been prepared with the support of the VEGA grant no. 1/0190/17 – "Analysis of models of public services delivery in the field of construction order performance from the aspect of technical, size and spatial allocation efficiency of local governments."

REFERENCES

[1] Bačík, V. (2019). *The Slovak Republic – summary data*. [online]. Retrieved 2019-01-15 from http://www.sodbtn.sk/obce/index_kraje.php

[2] Charnes, A., Cooper, W.W. & Rhodes, E. (1978). Measuring the Efficiency of Decision Making Units. *European Journal of Operational Research*, vol. 2, 429–444, 1978.

[3] Daško, M. (2018). Current State and Aspects of Inter-municipal Cooperation. *Municipal self-government in Slovakia from the perspective of public policy*. Trnava: UCM in Trnava, pp. 59-70.

[4] Fandel, P., Marišová, E., Malatinec, T. & Lichnerová, I. (2019). Decentralization policies in public administration in Slovakia and the Czech Republic, and their impact on building offices' scale efficiency. *Administrative Sciences*, Vol. 9, 4(2019), art. number 89 [15 p.]. DOI: https://doi.org/10.3390/admsci9040089

[5] Klimovský, D. (2010). Self-governments in EU – Czech Republic. Košice: TU. *Local Self-government*, pp. 32-37 [online]. Retrieved 2019-01-15 from https://archiv.vlada.gov.sk/krajina/data/att/24361_subor.pdf

[6] Klobučník, M., Bačík, V., Matiašová, M. & Fila, R. (2018). Negative aspects of municipalities mergers within the framework of Communal Reforms - perspective on the example of the Slovak Republic. *Sociológia*, Vol. 50, No 4, 448-481. [online]. Retrieved 2020-02-03 from https://doi.org/10.31577/sociologia.2018.50.4.18

[7] Leško, D. (2015). Formation of regional level of self-government in the process of transformation in Slovakia. *Journal for universal excellence*, September 2015, Vol. 4, Issue 3, pp. A1-A11. [online]. Retrieved 2020-01-14 from http://www.fos-unm.si/media/pdf/RUO/2015%203/8.pdf

[8] Lichnerová, I. (2020). Competencies and municipal cooperation in the building order sector in Slovakia and the Czech Republic. *Theory and practice of public administration*. Reviewed proceedings of the 5th scientific conference of PhD students. Košice: UPJŠ, pp. 19-33 (e-publication).

[9] Marišová, E. et al. (2010). *European Union public administration and development policies and variations in V- 4 countries*. Nitra: Slovak University of Agriculture, 221 p.

[10] Papcunová, V., Urbaníková, M. & Korenková, M. (2016). Financing of Transferred Execution of the State Administration in the Field of Registry Offices on the Local Self-Government in the Slovak Republic. *Proceedings of XIX. International Colloquium on Regional Sciences*, Čejkovce, 15.-17.6.2016, pp. 788-795. [online]. DOI: https://doi.org/10.5817/CZ.MUNI.P210-8273-2016-101

[11] Slavík, V., Grác, R. & Klobučník, M. (2010). Reflection on the functioning of common municipal offices in the Slovak Republic. *GEOGRAPHIA CASSOVIENSIS IV*, 2/2010, 193-200. [online]. Retrieved 2020-02-04 from http://www.humannageografia.sk/clanky/slavik_grac_klobucnik_2010.pdf

[12] Sopkuliak, A. (2016). Expenditure Structure of Local-Government in Slovakia 2012-2014. *Proceedings of XIX. International Colloquium on Regional Sciences*, Čejkovce, 15.-17.6.2016, pp. 774-780. [online]. DOI: https://doi.org/10.5817/CZ.MUNI.P210-8273-2016-99

[13] *Statistical Office of the Slovak Republic*. (2017). Number of population. [online]. Retrieved 2019-01-23 from http://statdat.statistics.sk/cognosext/cgi-bin/cognos.cgi?b_action=cognosViewer&ui.action=run&ui.object=storeID%28%22i362DCE4D88EC4E13A9EE8526B286D18B%22%29&ui.name=Po%C4%8Det%20obyvate%C4%BEov%20pod%C4%BEa%20pohlavia%20-%20SR%2C%20oblasti%2C%20kraje%2C%20okresy%2C%20mesto%2C%20vidiek%20%28ro%C4%8Dne%29%20%5Bom7102rr%5D&run.outputFormat=&run.prompt=true&cv.header=false&ui.backURL=%2Fcognosext%2Fcps4%2Fportlets%2Fcommon%2Fclose.html&run.outputLocale=sk. Last data update: 12.03.2020.

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-01-10
Accepted: 2020-04-22
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.10-17

Original Paper

The sum of the telescoping series formed by reciprocals of the cubic polynomials with three different negative integer roots

Radovan Potůček*

Department of Mathematics and Physics, Faculty of Military Technology, University of Defence in Brno, Czech Republic

ABSTRACT

This paper deals with the sum of a special telescoping series and is a free follow-up to author's preceding paper. The terms of this series are reciprocals of the cubic polynomial with three different negative integer roots. The main result of the paper is to derive a formula for the sum of this series. This formula uses the limit of the sequence of the partial sums and is expressed by harmonic numbers. After that the main result is verified by some examples using the basic programming language of the computer algebra system Maple 19.

KEYWORDS: sum of the series, sequence of partial sums, telescoping series, harmonic number, computer algebra system Maple

JEL CLASSIFICATION: I30

INTRODUCTION

Let us recall some basic terms. The series

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots$$

converges to a limit s if and only if the sequence of partial sums

$$s_n = a_1 + a_2 + \cdots + a_n$$

converges to s, i.e.

$$\lim_{n\to\infty} s_n = s.$$

We say that the series has a *sum* s and write

* Corresponding author: Radovan Potůček, Kounicova 65, 662 10 Brno, Czech Republic, e-mail: Radovan.Potucek@unob.cz

$$\sum_{k=1}^{\infty} a_k = s.$$

The *sum of the reciprocals* of some positive integers is generally the sum of unit fractions.

The nth *harmonic number* is the sum of the reciprocals of the first n natural numbers:

$$H(n) = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n},$$

$H(0)$ being defined as 0. Basic and as well interesting information about harmonic numbers can be found in [1], [2]. For $n = 1,2, \ldots ,10$ we get the following table:

n	1	2	3	4	5	6	7	8	9	10
$H(n)$	1	$\frac{3}{2}$	$\frac{11}{6}$	$\frac{25}{12}$	$\frac{137}{60}$	$\frac{49}{20}$	$\frac{363}{140}$	$\frac{761}{280}$	$\frac{7129}{2520}$	$\frac{7381}{2520}$

Table 1: First ten harmonic numbers [source: own modelling in Maple 19]

The *telescoping series* is any series where nearly every term cancels with a preceding or following term, so its partial sums eventually only have a fixed number of terms after cancellation. Interesting facts about telescoping series can be found in [3]. For example, the series

$$\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)(k+4)}$$

has the general kth term, after partial fraction decomposition, in a form

$$a_k = \frac{1}{(k+1)(k+2)(k+4)} = \frac{1}{6}\left(\frac{2}{k+1} - \frac{3}{k+2} + \frac{1}{k+4}\right).$$

After that we arrange the terms of the nth partial sum $s_n = a_1 + a_2 + \cdots + a_n$ in a form where can be seen what is cancelling. Then we find the limit $\lim_{n\to\infty} s_n$ of the sequence of the partial sums s_n in order to find the sum s of the infinite telescoping series. In our case we get

$$s_n = \frac{1}{6}\left[\left(\frac{2}{2} - \frac{3}{3} + \frac{1}{5}\right) + \left(\frac{2}{3} - \frac{3}{4} + \frac{1}{6}\right) + \left(\frac{2}{4} - \frac{3}{5} + \frac{1}{7}\right) + \left(\frac{2}{5} - \frac{3}{6} + \frac{1}{8}\right) + \left(\frac{2}{6} - \frac{3}{7} + \frac{1}{9}\right) + \cdots\right.$$
$$\cdots + \left(\frac{2}{n-3} - \frac{3}{n-2} + \frac{1}{n}\right) + \left(\frac{2}{n-2} - \frac{3}{n-1} + \frac{1}{n+1}\right) + \left(\frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+2}\right) +$$
$$\left. + \left(\frac{2}{n} - \frac{3}{n+1} + \frac{1}{n+3}\right) + \left(\frac{2}{n+1} - \frac{3}{n+2} + \frac{1}{n+4}\right)\right] =$$
$$= \frac{1}{6}\left(\frac{2}{2} - \frac{1}{3} - \frac{1}{4} - \frac{2}{n+2} + \frac{1}{n+3} + \frac{1}{n+4}\right).$$

So we have

$$s = \lim_{n\to\infty} \frac{1}{6}\left(1 - \frac{1}{3} - \frac{1}{4} - \frac{2}{n+2} + \frac{1}{n+3} + \frac{1}{n+4}\right) = \frac{1}{6}\left(1 - \frac{1}{3} - \frac{1}{4}\right) = \frac{5}{72} = 0.069\overline{4}.$$

PARTICULAR EXAMPLE

Let us consider a particular example – to determine the sum of the telescoping series formed by reciprocals of the cubic polynomial with three different specific negative integer roots.

Example 1 Using the nth partial sum calculate the sum $s(-2,-6,-9)$ of the series

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+6)(k+9)} \tag{1}$$

representing the telescoping series formed by reciprocals of the cubic polynomials with negative roots $k=-2$, $k=-6$, and $k=-9$.

By means of the CAS Maple 19 we get that the partial fraction decomposition of the kth term

$$a_k = \frac{1}{(k+2)(k+6)(k+9)}$$

has the form

$$a_k = \frac{1}{28(k+2)} - \frac{1}{12(k+6)} + \frac{1}{21(k+9)} = \frac{1}{84}\left(\frac{3}{k+2} - \frac{7}{k+6} + \frac{4}{k+9}\right).$$

The expression in parentheses we now express as the reciprocal of the natural numbers:

$$a_k = \frac{1}{84}\left(\frac{7-4}{k+2} - \frac{7}{k+6} + \frac{4}{k+9}\right) = \frac{7}{84}\left(\frac{1}{k+2} - \frac{1}{k+6}\right) - \frac{4}{84}\left(\frac{1}{k+2} - \frac{1}{k+9}\right),$$

i.e.

$$a_k = \frac{1}{12}\left(\frac{1}{k+2} - \frac{1}{k+6}\right) - \frac{1}{21}\left(\frac{1}{k+2} - \frac{1}{k+9}\right).$$

The nth partial sum $s_n(-2,-6,-9)$ of the infinite series (1) is

$$s_n(-2,-6,-9) = \frac{1}{12}\sum_{k=1}^{n}\left(\frac{1}{k+2} - \frac{1}{k+6}\right) - \frac{1}{21}\sum_{k=1}^{n}\left(\frac{1}{k+2} - \frac{1}{k+9}\right)$$
$$= \frac{1}{12}s_n(-2,-6) - \frac{1}{21}s_n(-2,-9).$$

Because

$$s_n(-2,-6) = \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\cdots+\frac{1}{n+1}+\frac{1}{n+2} -$$
$$-\left(\frac{1}{7}+\frac{1}{8}+\cdots+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\frac{1}{n+4}+\frac{1}{n+5}+\frac{1}{n+6}\right) =$$
$$= \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{n+3}-\frac{1}{n+4}-\frac{1}{n+5}-\frac{1}{n+6}$$

and

$$s_n(-2,-9) = \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\cdots+\frac{1}{n+1}+\frac{1}{n+2} -$$

$$-\left(\frac{1}{10}+\frac{1}{11}+\cdots+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{n+8}+\frac{1}{n+9}\right)=$$

$$=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}-\frac{1}{n+3}-\frac{1}{n+4}-\frac{1}{n+5}-\frac{1}{n+6}-\frac{1}{n+7}-\frac{1}{n+8}+\frac{1}{n+9},$$

we get

$$s_n(-2,-6,-9)=\frac{1}{12}\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{n+3}-\frac{1}{n+4}-\frac{1}{n+5}-\frac{1}{n+6}\right)-$$

$$-\frac{1}{21}\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}-\frac{1}{n+3}-\frac{1}{n+4}-\cdots-\frac{1}{n+8}-\frac{1}{n+9}\right).$$

Since for arbitrary integer a it holds

$$\lim_{n\to\infty}\frac{1}{n+a}=0\,,$$

we have

$$s(-2,-6,-9)=\lim_{n\to\infty}s_n(-2,-6,-9)\,,$$

so

$$s(-2,-6,-9)=\frac{1}{12}\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)-\frac{1}{21}\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}\right)=$$

$$=\frac{1}{12}[H(6)-H(2)]-\frac{1}{21}[H(9)-H(2)]=\frac{1}{12}\left(\frac{49}{20}-\frac{3}{2}\right)-\frac{1}{21}\left(\frac{7129}{2520}-\frac{3}{2}\right)=$$

$$=\frac{1}{12}\cdot\frac{19}{20}-\frac{1}{21}\cdot\frac{3349}{2520}=\frac{19}{240}-\frac{3349}{52920}=\frac{1681}{105840}\doteq 0.015882\,.$$

THE SUM OF THE TELESCOPING SERIES FORMED BY RECIPROCALS OF THE CUBIC POLYNOMIAL WITH THREE DIFFERENT NEGATIVE INTEGER ROOTS

This paper is a free follow-up to author's paper [4] dealing with the sum of the telescoping series formed by reciprocals of the quadratic polynomials with two different positive integer roots. Before we derive the main result of this paper, we present two lemmas.

Lemma 1 Let $A<B<C$ are positive natural numbers. Then a fraction

$$\frac{1}{(k+A)(k+B)(k+C)}$$

can be rewritten in the form

$$\frac{1}{(C-B)(C-A)(B-A)}\left(\frac{C-B}{k+A}-\frac{C-A}{k+B}+\frac{B-A}{k+C}\right). \qquad (2)$$

Since $C-B=(C-A)-(B-A)$, we can (2) write in the form of the difference

$$\frac{1}{(C-B)(B-A)}\left(\frac{1}{k+A}-\frac{1}{k+B}\right)-\frac{1}{(C-B)(C-A)}\left(\frac{1}{k+A}-\frac{1}{k+C}\right). \qquad (3)$$

Proof. The proof can be done in Maple 19 using the **simplify** command applied to expressions (2) and (3).

Lemma 2 Let $A < B$ are positive natural numbers. Then it holds

$$\sum_{k=1}^{\infty}\left(\frac{1}{k+A}-\frac{1}{k+B}\right)=H(B)-H(A)\,, \tag{4}$$

where $H(n)$ is the nth harmonic number.

Proof. The sum $s(A,B)$ of the infinite series in (4) is the limit

$$\lim_{n\to\infty} s_n(A,B)$$

of the sequence of the partial sums

$$s_n(A,B)=\sum_{k=1}^{n}\left(\frac{1}{k+A}-\frac{1}{k+B}\right),$$

where

$$s_n(A,B)=\frac{1}{1+A}+\frac{1}{2+A}+\cdots+\frac{1}{B}+\frac{1}{1+B}+\frac{1}{2+B}+\cdots+\frac{1}{n+A-1}+\frac{1}{n+A}-$$
$$-\left(\frac{1}{1+B}+\frac{1}{2+B}+\cdots+\frac{1}{n+A-1}+\frac{1}{n+A}+\frac{1}{n+A+1}+\cdots+\frac{1}{n-1+B}+\frac{1}{n+B}\right)=$$
$$=\frac{1}{1+A}+\frac{1}{2+A}+\cdots+\frac{1}{B}-\frac{1}{n+A+1}-\cdots-\frac{1}{n-1+B}-\frac{1}{n+B}.$$

Hence we have

$$s(A,B)=\lim_{n\to\infty}\left(\frac{1}{1+A}+\frac{1}{2+A}+\cdots+\frac{1}{B}-\frac{1}{n+A+1}-\cdots-\frac{1}{n-1+B}-\frac{1}{n+B}\right)=$$
$$=\frac{1}{1+A}+\frac{1}{2+A}+\cdots+\frac{1}{B}=H(B)-H(A).$$

Now, let us consider the series formed by reciprocals of the cubic polynomial with three different negative integer roots $a>b>c$, i.e. the series

$$\sum_{k=1}^{\infty}\frac{1}{(k-a)(k-b)(k-c)} \tag{5}$$

and let us determine its sum $s(a,b,c)$.

Clearly, for arbitrary negative integers $a>b>c$ are $A=-a$, $B=-b$, $C=-c$ positive integers, $A<B<C$, so for $k=1,2,\ldots$ the expressions $k-a=k+A$, $k-b=k+B$, $k-c=k+C$ are positive integers, and we get the series

$$\sum_{k=1}^{\infty}\frac{1}{(k+A)(k+B)(k+C)} \tag{6}$$

and we determine its sum $s(A,B,C)$. We express the kth term a_k of this series as the sum of three partial fractions

$$a_k=\frac{1}{(k+A)(k+B)(k+C)}=\frac{P}{k+A}+\frac{Q}{k+B}+\frac{R}{k+C}\,.$$

According to Lemma 1 we can write

$$a_k = \frac{1}{(C-B)(B-A)}\left(\frac{1}{k+A}-\frac{1}{k+B}\right)-\frac{1}{(C-B)(C-A)}\left(\frac{1}{k+A}-\frac{1}{k+C}\right),$$

so the nth partial sum is

$$s_n(A,B,C) = \frac{1}{(C-B)(B-A)}\sum_{k=1}^{n}\left(\frac{1}{k+A}-\frac{1}{k+B}\right)-$$
$$-\frac{1}{(C-B)(C-A)}\sum_{k=1}^{n}\left(\frac{1}{k+A}-\frac{1}{k+C}\right).$$

According to Lemma 2 the first partial sum equals $H(B)-H(A)$ and the second one equals $H(C)-H(A)$, so the sum $s(A,B,C)$ is

$$s(A,B,C) = \frac{H(B)-H(A)}{(C-B)(B-A)}-\frac{H(C)-H(A)}{(C-B)(C-A)}.$$

We have derived the following statement:

Theorem 1 The series

$$\sum_{k=1}^{\infty}\frac{1}{(k-a)(k-b)(k-c)},$$

where $a>b>c$ are negative integers, has the sum

$$s(a,b,c) = \frac{H(-b)-H(-a)}{(b-c)(a-b)}-\frac{H(-c)-H(-a)}{(b-c)(a-c)}, \tag{7}$$

where $H(n)$ is the nth harmonic number.

NUMERICAL VERIFICATION

We solve the problem to determine the values of the sum $s(a,b,c)$ for $a=-1,-2,-3,-4$, for $b=a-1,a-2,\ldots,a-5$ and for $c=b-1,b-2,\ldots,b-5$. We use on the one hand an approximative direct evaluation of the sum

$$s(a,b,c,t) = \sum_{k=1}^{t}\frac{1}{(k-a)(k-b)(k-c)},$$

where $t=10^7$, using the basic programming language of the computer algebra system Maple 19, and on the other hand the formula (7) for evaluation the sum $s(a,b,c)$. We compare 60 pairs of these ways obtained sums $s(a,b,c,t)$ and $s(a,b,c)$ to verify the formula (7). We use the following simple procedure `tsabcneg` and the following double repetition statement:

```
> tsabcneg:=proc(a,b,c,t)
      local A,B,C,k,sabc,sabct;
      A:=-a; B:=-b; C:=-c; sabct:=0;
      sabc:=(harmonic(B)-harmonic(A))/((B-C)*(A-B))
           -(harmonic(C)-harmonic(A))/(B-C)*(A-C));
      print("s(",a,b,c")=",evalf[12](sabc));
```

```
      for k from 1 to t do
          sabct:=sabct+1/((k+A)*(k+B)*(k+C));
      end do;
      print("s(",a,b,c,t")=",evalf[12](sabct));
      print("diff=",evalf[12](abs(sabct-sabc)));
  end proc:

> for i from -1 by -1 to -4 do
      for j from i-1 by -1 to i-5 do
          for k from j-1 by -1 to j-5 do
              tsabcneg(i,j,k,10000000);
          end do;
      end do;
  end do;
```

The approximative values of the sums $s(a,b,c)$ rounded to 10 decimals obtained by this procedure are written into the table 1:

Table 1: Some approximate values of the sums $s(a,b,c)$ obtained by means

$\boldsymbol{a=-1}$	$c=-3$	$c=-4$	$c=-5$	$c=-6$	$c=-7$
$b=-2$	0.0833333333	0.0694444444	0.0597222222	0.0525000000	00469047619.
$b=-3$	×	0.0555555556	0.0479166667	0.0422222222	0.0377976190
$b=-4$	×	×	0.0402777778	0.0355555556	0.0318783069
$b=-5$	×	×	×	0.0308333333	0.0276785714
$b=-6$	×	×	×	×	0.0245238095
$\boldsymbol{a=-2}$	$c=-4$	$c=-5$	$c=-6$	$c=-7$	$c=-8$
$b=-3$	0.0416666667	0.0361111111	0.0319444444	0.0286904762	0.0260714286
$b=-4$	×	0.0305555556	0.0270833333	0.0243650794	0.0221726190
$b=-5$	×	×	0.0236111111	0.0212698413	0.0193783069
$b=-6$	×	×	×	0.0189285714	0.0172619048
$b=-7$	×	×	×	×	0.0155952381
$\boldsymbol{a=-3}$	$c=-5$	$c=-6$	$c=-7$	$c=-8$	$c=-9$
$b=-4$	0.0250000000	0.0222222222	0.0200396825	0.0182738095	0.0168121693
$b=-5$	×	0.0194444444	0.0175595238	0.0160317460	0.0147652116
$b=-6$	×	×	0.0156746032	0.0143253968	0.0132054674
$b=-7$	×	×	×	0.0129761905	0.0119708995
$b=-8$	×	×	×	×	0.0109656085
$\boldsymbol{a=-4}$	$c=-6$	$c=-7$	$c=-8$	$c=-9$	$c=-10$
$b=-5$	0.0166666667	0.0150793651	0.0137896825	0.0127182540	0.0118121693
$b=-6$	×	0.0134920635	0.0123511905	0.0114021164	0.0105985450
$b=-7$	×	×	0.0112103175	0.0103571429	0.0096340388
$b=-8$	×	×	×	0.0095039683	0.0088458995
$b=-9$	×	×	×	×	0.0081878307

Source: own modelling in Maple 19

Computation of 60 pairs of the sums $s(a,b,c)$ and $s(a,b,c,10^7)$ took about 25 minutes. The absolute errors, i.e. the differences $|s(a,b,c)-s(a,b,c,10^7)|$, are all only about $5\cdot 10^{-15}$.

CONCLUSIONS

We dealt with the sum of the telescoping series formed by reciprocals of the cubic polynomials with three different negative integer roots $a>b>c$, i.e. with the series

$$\sum_{k=1}^{\infty} \frac{1}{(k-a)(k-b)(k-c)} .$$

We derived that the sum $s(a,b,c)$ of this series is given by the formula

$$s(a,b,c) = \frac{H(-b) - H(-a)}{(b-c)(a-b)} - \frac{H(-c) - H(-a)}{(b-c)(a-c)},$$

where $H(n)$ is the nth harmonic number.

We verified this result by computing 60 sums using the computer algebra system Maple 19. This series so belong to special types of infinite series, such as geometric series, which sums are given analytically by means of a simple formula.

ACKNOWLEDGEMENTS

The work presented in this paper has been supported by the grant "Rozvoj oblastí základního a aplikovaného výzkumu dlouhodobě rozvíjených na katedrách teoretického a aplikovaného základu FVT (K215, K217)".

REFERENCES

[1] Wikipedia contributors: *Harmonic number*. Wikipedia, The Free Encyclopaedia. Retrieved 2020-01-10 from https://en.wikipedia.org/wiki/Harmonic_number

[2] Benjamin, A. T., Preston, G. O. & Quinn, J. J. (2002). A Stirling Encounter with Harmonic Numbers. *Mathematics Magazine*, 75 (2), pp. 95–103. Retrieved 2020-01-10 from https://scholarship.claremont.edu/cgi/viewcontent.cgi?article=1534&context=hmc_fac_pub

[3] Laval, P. B. (2016). *Telescoping Sums*. Kennesaw State University, 2016. Retrieved 2020-01-10 from http://ksuweb.kennesaw.edu/~plaval/math2202/telescoping.pdf

[4] Potůček, R. (2019). The Sum of One Type of the Telescoping Series. *University mathematical education as a basis for innovations in science and technology*. Nitra: Slovak University of Agriculture in Nitra, pp. 66-73. Retrieved 2020-01-10 from http://ves.uniag.sk/files/pdf/t03ori33box327nk7r38veusuqg191.pdf

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-04-17
Accepted: 2020-06-26
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.18-24

Original Paper

Influence of biodegradation process on methane formation and composting efficiency

Peter Hlinka[1*], Ingrid Karandušovská[2], Dušan Páleš[2], Štefan Mihina[2]

[1] Ponitrie Villages Local Union for Waste Separation and Management, Slovak Republic
[2] Slovak University of Agriculture in Nitra, Faculty of Engineering, Department of Building Equipment and Technology Safety, Nitra, Slovak Republic

ABSTRACT

In this study, the aim was to analyze the production of methane (CH_4) during the degradation process in three piles of biowaste (V1, V2, V3) and to determine the efficiency of the used biowaste treatment technology. The biowaste treatment technology in reference pile V1 was untreated. In the second pile V2, a biological preparation containing probiotic bacteria was applied and the sample V3 was treated by turning and watering once a week. Degassing shafts were installed in each pile to measure of methane concentrations during the degradation process. The Multigas Monitor 1312 gas analyzer with the Multipoint Sampler 1309 was used to measure of the methane production in the first, fourth and seventh week. The production of methane had an increasing trend throughout the process. In the V1 pile (34.60 mg·m^{-3}, 66.25 mg·m^{-3} and 115.85 mg·m^{-3}), but much more in the V3 pile (32.81 mg·m^{-3}, 220.97 mg·m^{-3} and 325.89 mg·m^{-3}). In the V2 pile with biodegradable preparation, the highest methane production was at fourth week (42.35 mg·m^{-3}, 116.28 mg·m^{-3} and 72.83 mg·m^{-3}). In the fourth and seventh week, statistically higher values of methane concentrations were recorded from the V3 pile, than from V1 and V2 piles ($P < 0.05$). Although the mechanically treated method resulted in the highest release of harmful gas, the most efficient way of biowaste treatment was shown in the V3 pile, where the percentage of sieving residues was only 18.44%, in the V2 pile it was 25.22% and up to 32.12% in the untreated V1 pile.

KEYWORDS: methane in compost, composting technology, composting process, temperature measuring

JEL CLASSIFICATION: Q15

INTRODUCTION

Biodegradable waste can be treated in different ways, but the easiest way is composting. This process is a way to treat biodegradable waste that is currently landfilled together with mixed municipal waste in most cases. Composting reduces the amount of waste and thus saves the municipalities' fees for landfill of municipal waste and in agricultural production reduces the costs of fertilization [3]. According to Favoino and Hogg (2008) [5], the use of compost can

[*] Corresponding author: Peter Hlinka, Ponitrie Villages Local Union for Waste Separation and Management, Slovak Republic, e-mail: peter.hlinka19@gmail.com

reduce the need for chemical fertilizers, which means reducing the greenhouse gas emissions associated with their production and use. Composting in composting plants may also contribute to the reduction of methane (CH_4) emissions, depending on the composting method [3]. Although the benefits of composting are evident, greenhouse gases can be generated and emitted to the atmosphere during this process, contributing to global warming by producing methane [11]. CH_4 is produced in large quantities in the landfills due to degradation of organic matter under anaerobic conditions [12]. Therefore, it is very important to separate biodegradable waste from mixed municipal waste and increase the efficiency of the degradation process to produce quality compost. The aim of this study was to analyze the production of methane (CH_4) in three piles of biowaste (V1, V2, V3) depending on the treatment technology used and to determine the efficiency of the used technology in terms of the final product.

MATERIAL AND METHODS

An attempt to monitor the production of methane and the efficiency of composting was carried out during eight weeks in the summer period. Three loose piles of biowaste (V1, V2, V3) were created at the same time and with the same volume of 50 m^3 (Fig. 1). The biowaste in the reference pile V1 was untreated. In the second pile V2 a biological preparation containing probiotic bacteria was used to accelerate the degradation process. The V3 pile was treated once a week by turning and watering. Degassing shafts were installed in each pile to remove landfill gas and measure of methane concentrations during the degradation process. The Multigas Monitor 1312 gas analyser, together with the Multipoint Sampler 1309 sampler was used to measure of concentrations (Innova, Denmark). The measurement system is based on Photoacoustic Infrared Detection, which delivers the ability to measure virtually any gas that is absorbed in the infrared spectrum. Measurements of methane concentrations and humidity of the composted mixture were performed in the first, fourth and seventh weeks of the degradation process. During the experiment, the temperature inside each pile was monitored daily using a Pfeuffer GT 1 injection thermometer to determine the need of turning and irrigating the waste in the V3 pile. The average temperature was determined by taking three measurements at the left, middle, and right side in each pile. Determination of compost moisture was performed in a certified laboratory. Samples were collected from left, middle, and right side of each pile at 30, 60 and 90 cm depths. These samples were combined and mixed into one composite sample. The composite samples were collected from each pile at day 7, 28 and 49. Humidity measurement was also carried out by means of an orientation test for the need of watering the V3 pile. Statistical analysis of significant differences in MATLAB environment was used to process the obtained data. We verified the hypothesis H0: there is no significant difference between the production of methane concentrations in V1, V2 and V3 heaps in weeks of composting (weeks 1, 4 and 7). The non-parametric Kruskal-Wallis test (K-W test) was used, which has less restrictive conditions [10]. In this case, the verified hypothesis is the hypothesis of equality of medians of individual samples. The efficiency of each composting process was determined after completion of the measurement. Approximately 2.5 - 3 t were taken from each pile using a loader, which was accurately weighed and then sieved on a Pezzolato L 3000 drum sieve with holes dimensions of 40 mm. The sieved material of the compost fell directly into the bucket of the loader; this was weighed on the bridge scale together with the loader. Then the drum sowing outlets were

weighed and again reweighed. The weight of the loader was calculated at each weighing, which was 8.52 t with the engineer.

Figure 1 Three loose piles of biowaste (V1, V2, V3)

RESULTS AND DISCUSSION

Figure 2 shows the average results of a 24-hour continuous measurement of CH_4 concentrations from the three piles V1, V2 and V3 observed in the first, fourth and seventh weeks of compost maturation.

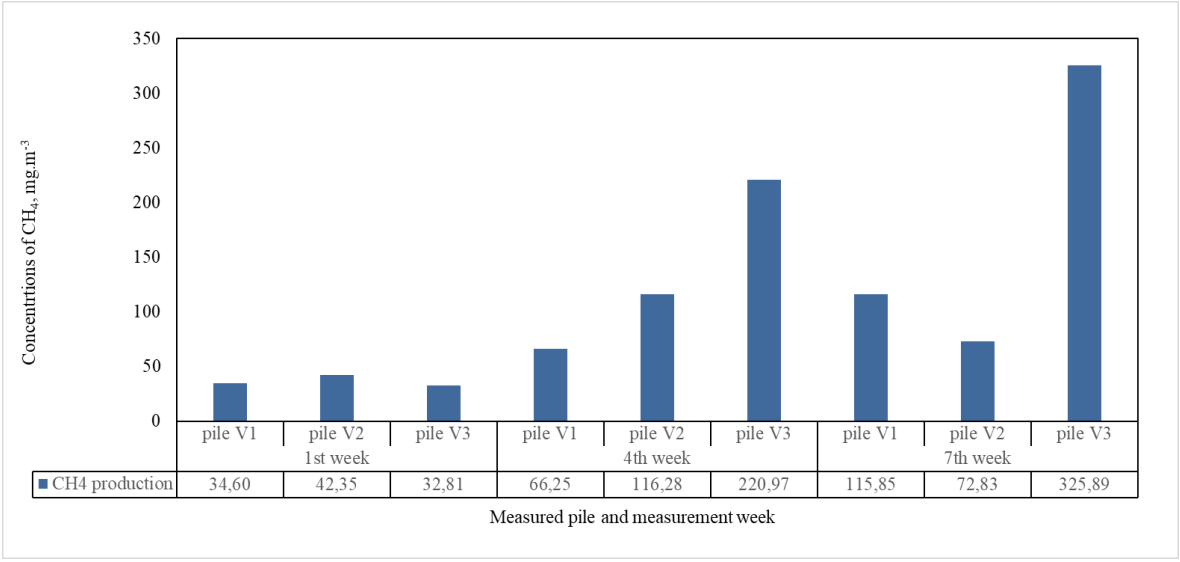


Figure 2 Average CH_4 production in V1, V2 and V3 pile in the first, fourth and seventh week after piles creation

The average CH_4 concentrations in the first week were 34.60 mg·m^{-3} (V1), 42.35 mg·m^{-3} (V2) and 32.81 mg·m^{-3} (V3). From the material samples taken were found humidity 67.42 % (h_{V1}), 67.53 % (h_{V2}) and 67.56 % (h_{V3}). These values can be considered satisfactory. In the fourth week of maturing compost, when the thermophilic phase was in progress, the mean values of CH_4 concentrations were 66.25 mg·m^{-3} (V1), 116.28 mg·m^{-3}(V2) and 220.97 mg·m^{-3}(V3).

Humidity in each pile decreased to 45.98 % (h_{V1}), 46.78 % (h_{V2}) and 48.34 % (h_{V3}). At week 7th, in the compost maturation phase, the mean values of CH_4 concentrations in the piles were 115.85 mg·m^{-3} (V1), 72.83 mg·m^{-3}(V2) and 325.89 mg·m^{-3} (V3). From samples of material from each pile, lower humidity 33.1 % (h_{V1}), 40.45 % (h_{V2}) and 43.34 % (h_{V3}) were found.

In the first week, CH_4 production did not meet the assumption of equality of medians of each sample. The amount of CH_4 produced in the V2 pile was statistically significantly higher ($P < 0.05$) (Tab. 1). As reported by Tiqua et al. (2000) [14] the windrows can be exposed to several external environmental variables, and this may create variability in windrows with the same composting material at the beginning of the process, even in windrows with the same treatment. In our experiment, a biological preparation containing probiotic bacteria was applied to this V2 pile at the beginning of experiment to accelerate the degradation process, which could cause higher CH_4 production.

Table 1 CH_4 production from piles V1, V2, V3 in the first, fourth and seventh week

Meas. in week	Meas. piles	Number of meas.	Average + stand. dev.	Median	P-value		
1	V1	43	34.60 ± 9.76	34.36 [a]	(12) 0.0171	(23) 0.0035	(13) 0.4319
	V2	43	42.35 ± 14.77	41.82 [b]			
	V3	43	32.81 ± 7.77	32.62 [a]			
4	V1	43	66.25 ± 9.02	63.69[a]	(12) 0.0000	(23) 0.0000	(13) 0.0000
	V2	43	116.28 ± 14.41	119.87[b]			
	V3	43	220,97 ± 56.56	217.30[c]			
7	V1	36	115.85 ± 19.94	112.18[a]	(12) 0.0000	(23) 0.0000	(13) 0.0000
	V2	36	72.83 ± 17.10	68.20[b]			
	V3	36	325.89 ± 123.24	306.90[c]			

[abc] different upper indices indicate significant difference in CH_4 concentrations in V1, V2 and V3 at $P < 0.05$

In the fourth week from the V3 pile, which was turned, and irrigated, statistically significantly higher CH_4 production values were recorded than from a pile of V1 and V2 ($P < 0.05$). Methane production was 3.3 times higher from V3 pile than V1 and 2 times higher than V2 pile (Tab.1). In the seventh week it was statistically proven that from the V3 pile statistically significantly higher values of CH_4 production ($P < 0.05$) were recorded, it was almost 3 times more than V1, and nearly 5 times more than V2.

Production of CH_4 concentrations in both the untreated V1 pile and the turned and watering V3 pile had a growing trend throughout the process. Conversely, in the V2 pile with the addition of the biopreparation, it was increased shortly after the pile formation and the highest CH_4 concertation's production was already in the fourth week, which was in accordance with rapid decomposition of compost materials because rapid aerobic decomposition leads to suitable anaerobic conditions for CH_4 emission production [12]. In the seventh week, readily available carbon compounds were depleted, which reduced the activity of microbes in the

composting material, thereby reducing the production of CH_4. This model was similar to research of Fukumoto et al. (2003) [6] and Szanto et al. (2007) [13].

As reported Michel, et al. (2013) [9], Illmer and Schinner (1997) [7] windrow turning is one of the composting strategies that affect the degree of decomposition, and quality of the composted product. Szanto et al. (2007) [13] observed lower CH_4 emissions in turned piles than in static systems. Several authors reported that even in well-aerated process CH_4 was emitted [2] while [1] observed a rapid decrease when the oxygen supply was increased.

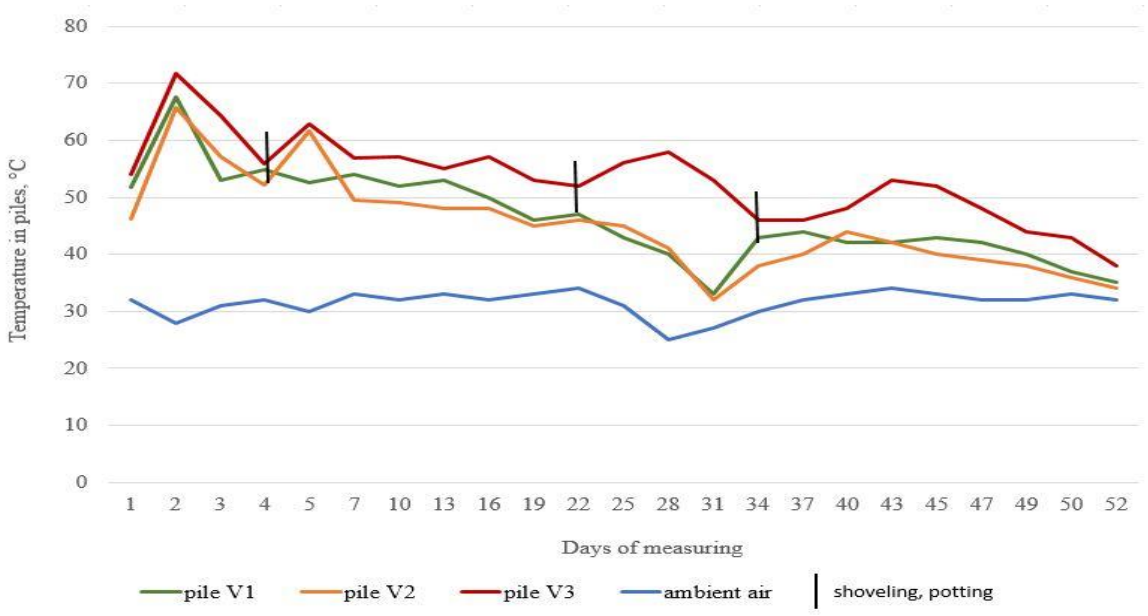


Fig. 3 Temperature profile in pile V1, V2, V3 and ambient air during the composting process

The temperature of the compost in each pile increased rapidly after the start of the experiment (Fig. 3), the maximum temperatures (68 °C, 66 °C and 72 °C) were observed on day 2. Then the temperature decreased gradually during the process between 40 °C and 60 °C and the degradation process took place in the thermophilic phase. The thermophilic phase of all treatments was long enough to satisfy the requirement for sanitation effect. Watering was carried out whenever it was evident that the temperature would fall below the desired 45 °C. Since the piles were of a smaller size, their temperature was also influenced by the outside temperature, as can be seen in Figure 3. At the inverted V3 pile, the highest temperatures were observed even when the ambient air temperature dropped. After 50 days, which is enough time for the entire composting process to take place, the temperature began to drop and reached below 40 °C. At the end of the degradation process, each pile was sieved and weighed to determine the efficiency of the processing technology used. It was found that turning and irrigation in the V3 pile had a significant effect on the amount of compost produced. The proportion of sieving residue in V3 was the smallest and only 18.44% and the remaining 81.56 % was quality compost. In the case of V2, the proportion of sieving was up to 25.22 % and in the case of V1, it was 32.12 %. The compost was only 74.78 % in V2 and 67.88 % in V1. From the given results it is evident that by treatment we achieved the highest rate of waste degradation and reached the highest percentage of the final product in the form of compost. In unturned piles, aerobic conditions prevail mostly at the outer surface of the

piles, while anaerobic conditions dominate inside [8]. According to the results, it is clear that controlled composting process is financially efficient because it reduces the time degradation process, as well as space composting facilities, thereby reducing the overall operating costs of the composting plant.

CONCLUSIONS

Based on the measured and evaluated data in the first week, the hypothesis H0 can´t be accepted, CH_4 production did not meet the assumption of median equality. The amount of CH_4 produced in the V2 pile was statistically significantly higher ($P < 0.05$). Statistically significantly higher values of the methane production were recorded from the V3 pile, which was turned and irrigated, than from the unturned piles V1 and V2 in the fourth week. Analysis of significant differences in the amount of concentrations in the seventh week showed that statistically significantly higher values of methane production were recorded from the V3 pile than from V1 and V2 ($P < 0.05$). Turning affected not only the temperature but also the resulting amount of product. Understanding the effects of different composting strategies is important because the composted product will eventually be used as a supplement to the soil. Based on the results from the measurement, it was shown that turning and irrigating resulted to the highest release of CH_4 into the air, but also faster decomposition of microorganisms, which can reduce the time required to achieve a stable compost product and increase the efficiency of the composting plant. These results of measurements can be used in other research activities that will deal with the formation of gases during landfilling or composting. From a life-cycle assessment perspective, it is necessary to have experimental data both on greenhouse gas emissions and process efficiency to have a fair evaluation of the environmental impacts of composting.

ACKNOWLEDGEMENTS

This publication was created thanks to the support under the Operational Program Research and Innovation for the project: Support of research activities at the Slovak University of Technology in the field of energy, 313011X029, co-financed by the European Regional Development Fund.

REFERENCES

[1] Beck-Friis, B., Pell, M., Sonesson, U., Jönsson, H. & Kirchmann, H. (2000). Formation and emission of N_2O and CH_4 from compost heaps of organic household waste. *Environmental Monitoring and Assessment*, vol. 62, pp. 317-331.

[2] Clemens, J. & Cuhls, C. (2003). Greenhouse gas emissions from mechanical and biological waste treatment of municipal waste. *Environmental Technology*, vol. 24, no. 6, pp. 745-754. doi: https://doi.org/10.1080/09593330309385611

[3] Colón, J., Cadena, E., Pognani, M., Barrena, R., Sánchez, A., Font X. & Artola, A. (2012). Determination of the energy and environmental burdens associated to the biological treatment of source-separated municipal solid wastes. *Energy & Environmental Science*, vol. 5, no. 2, pp. 5731–5741. doi: https://doi.org/10.1039/C2EE01085B

[4] Colón, J., Martinéz-Blanco, J., Gabarell, X., Artola, A., Sánchez, A., Rieradevall, J. & Font, X. (2010). Enviromental assessment of home composting. *Resources, Conservation and Recycling*, v. 54, n. 11, p. 893-904. doi: http://dx.doi.org/10.1016/j.resconrec.2010.01.008

[5] Favoino, E. & Hogg, D. (2008). The potential role of compost in reducing greenhouse gases. *Waste Management & Research*, vol. 26, no.1, pp. 61-69. doi: https://doi.org/10.1177/0734242X08088584

[6] Fukumoto, Y., Osada, T., Hanajima, D. & Haga, K. (2003). Patterns and quantities of NH_3, N_2O and CH_4 emissions during swine manure composting without forced aeration- effect of compost pile scale. *Bioresource Technology*, vol. 89, no. 2, pp. 109-114. doi: https://doi.org/10.1016/S0960-8524(03)00060-9

[7] Illmer, P. & Schinner, F. (1997). Compost turning a central factor for rapid and high quality degradation in household composting. *Bioresource Technology*, vol. 59, no. 2-3, pp. 157-162. doi: https://doi.org/10.1016/S0960-8524(96)00156-3

[8] Jiang, T., Schuchardt, F., Li, G., Guo, R. & Zhao, Y. (2011). Effect of C/N ratio, aeration rate and moisture content on ammonia and greenhouse gas emission during the composting. *Journal of Environmental Sciences*, vol. 23, no.10, pp. 1754-1760. doi: https://doi.org/10.1016/S1001-0742(10)60591-8

[9] Michel, F.C. JR., Forney, L.J., Huang, A.J.F., Drew, S., Czuprenzski, M., Lindeberg, J.D. & Reddy, A. (2013). Effects of turning frequency, leaves to grass mix ratio and windrow vs. pile configuration on the composting of yard trimmings. *Compost Science & Utilization*, vol. 4, no. 1, pp. 26-43. doi: https://doi.org/10.1080/1065657X.1996.10701816

[10] Ostertágová, E., Ostertág, O. & Kováč, J. 2014. Methodology and application of the Kruskal-Wallis Test. *Applied Mechanics and Materials*, vol. 611, pp. 115-120. doi: https://doi.org/10.4028/www.scientific.net/AMM.611.115

[11] Sánchez, A., Artola, A., Font, X., Gea, T., Barrena, R., Gabriel, D., Sánchez Monedero, M.A., Roig, A., Cayuela, M.L. & Mondini, C. (2015). Greenhouse gas emissions from organic waste composting. *Environmental Chemistry Letters*, vol. 13, pp. 223–238. doi: https://doi.org/10.1007/s10311-015-0507-5

[12] Seo, Y. & Kim, S. (2013). Estimation of greenhouse gas emissions from road traffic: a case study in Korea. *Renewable and Sustainable Energy Reviews*, vol. 28, pp. 777-787. doi: https://doi.org/10.1016/j.rser.2013.08.016

[13] Szanto, G.L., Hanekers, H.V.M., Rulkens, W.H. & Veeken, A.H.M. (2007). NH_3, N_2O and CH_4 emissions during passively aerated composting of straw-rich pig manure. *Bioresource Technology*, vol. 98, no. 14, pp. 2659-2670. doi: https://doi.org/10.1016/j.biortech.2006.09.021

[14] Tiquia, S. M., Richard, T. L. & Honeyman, M. S. (2000). Effect of windrow turning and seasonal temperatures on composting of hog manure from hoop structures. *Environmental Technology (United Kingdom)*, vol. *21*, *no.* 9, pp. 1037-1046. doi: https://doi.org/10.1080/09593332108618048

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-03-31
Accepted: 2020-06-26
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.25-30

Original Paper

Production function estimation and economics of fast-growing plants cultivation for bioenergy purposes. Case study of Kolíňany

Martin Mariš*

Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Slovakia

ABSTRACT
The paper explores the economic opportunities of growing fast-growing plants for bioenergy production and use. Based on primary data of the above-ground dry biomass of the *Miscanthus* × *giganteus*, from the experimental field of Kolíňany, the average yield during the life cycle of the crop was 33.31 t · ha $^{-1}$ (stand. dev. 7.07). The next step was assembling the economic model of gross financial yield based on using the biomass for the production of bioenergy. Using the Discounted Cash-flow model, the gross financial yield, neglecting the costs, was set to 1547 € · ha $^{-1}$. Finally, adjusting for the growing conditions of the Miscanthus, we set the gross financial yield as a deferred annuity to 3036.93 € · ha $^{-1}$ per one life cycle of the crop.

KEYWORDS: dry ground biomass, perpetuity, *Miscanthus Giganteus*, Gompertz curve, discounted cash-flow model, deferred annuity

JEL CLASSIFICATION: C13, C51

INTRODUCTION

The paper focuses on the economic analysis of the possibilities of using dry biomass of fast-growing trees for bioenergy purposes. Biomass, as a potential source of energy, is ranked among the so-called "alternative energy sources." This term refers to forms of energy that are outside the conventional forms of energy. It can cover both renewable and non-renewable energy sources. The importance of alternative energies has come to the forefront in the context of oil shocks in the 1970s that hit the global economy and contributed to a surge in fossil fuel prices [1]. According to the IEA, currently, around 24% of energy demand is met by renewable energy sources. Their importance is expected to continue to grow and to meet up to 30% of global energy demand in 2023. Hydro energy remains the most critical resource in meeting global energy demand, at 16%, followed by wind (6%), solar (4%) and bioenergy (3%) [2].

* Corresponding author: Martin Mariš, Slovak University of Agriculture, Faculty of European Studies and Regional Development, Slovakia, Tr. A. Hlinku 2, 949 76, Nitra, e-mail: martin.maris@uniag.sk

Renewable energy sources (RES) as alternative fuels offer some advantages. The most important reasons for promoting the development of RES [3]:

a) Contributing to the reduction of CO_2 emissions and mitigating the effects of climate change. The concentration of greenhouse gas emissions (GHG) in the air is increasing mainly due to the energy dependence of developed countries on fossil fuels. There are strong assumptions that rising greenhouse gas emissions will lead to the warming of the planet. Renewables are largely low-carbon or neutral, and increasing their levels of use can contribute to a decline in GHG concentrations over the long term.

b) Energy security. Energy security has once again become a concern for the depletion of available fossil fuel deposits at world level and the decline in production levels in England and the US; the growth of competitiveness and energy demand of third-country economies and the political instability of hydrocarbon-rich areas.

c) Increasing energy availability. It is currently estimated that at least 2 billion people do not have access to clean energy sources. The problem is even hotter in rural areas of developing countries. RES offers some benefits in this regard they reduce environmental and health damage and improves working conditions.

d) Employment opportunities. The use of RES has the potential to create jobs and increase employment thanks to a decentralized, modular technology structure.

e) Other spill-over effects. The use of RES contributes to improving macroeconomic stability by reducing dependence on hydrocarbon fuel imports and improving the current account of the country's balance of payments in international trade.

However, in spite of the clear advantages of using renewable energy sources mentioned above, these energies have currently not been able to compete against traditional energy sources such as hydrocarbon fuels and other sources. The literature has identified a number of barriers to the more intensive penetration of RES into the energy markets. Painuly [4] provided a framework for identifying and analyzing barriers. In general, these barriers can be analyzed at several levels: they can first be included in a broader category. Within each category, a certain number of barriers can be identified. At the third level, conditioning factors can be identified [4].

There are four main categories of barriers with factors within these categories. These include technology barriers (related to supply), unequal market conditions (compared to the fossil fuel market, market barriers (such as network access, regulatory measures), and non-market barriers (such as administrative measures and others [5].

In practice, the economy of RES production faces several problems. The most serious ones may include [1]:

(a) Inappropriate valuation methods. The value of electricity for end consumers varies according to the mode of use (low tariff vs. high tariff). However, the supply of energy from RES is indivisible, which causes complications in valuation with an impact on the viability of RES projects.

(b) Non-internalizing externalities. RES has some environmental advantages over fossil fuels. However, the production and use of fossil resources often bear "social" costs that reduce the social surplus and are not fully reflected in the price. On the other hand, RES are at a disadvantage.

The most important costs of RES production can be included [1]:

(a) Energy costs. These costs are generally related to the production of energy in energy installations: energy, operating, and maintenance costs.

(b) Capacity costs. These costs include installation costs and fixed operating and maintenance costs. For RES, they often represent the main cost element, often up to 50% - 80% of the total cost.

(c) Other costs. These other costs can often include a broad category of costs, depending on the nature of the renewable resource. In general, e.g., environmental costs related to adverse environmental and climate impacts, reserve capacity costs, cultivation and establishment costs, and others.

MATERIAL AND METHODS

The main sources of the research are primary sources of data on hectare crops of selected species and their varieties of fast-growing tree species (FGT), which were established at the experimental centre of biomass cultivation in Kolíňany. The object of investigation was the selected variety of Miscanthus: Miscanthus Tatai. For the given variety, experimentally measured annual yields of the yield of dry above-ground biomass directly usable for bioenergy production were obtained. In the first step of the investigation, we estimated the average yield of the dry above-ground biomass using the mathematical function within the life cycle of FGT. After several experiments, based on empirical and theoretical sources and knowledge, we chose the Gompertz function to estimate and extrapolate dry biomass yield values, which can be written as follows:

$$y_t = \beta_0 e^{-\beta_1 e^{-\beta_2 t}}, \tag{1.0}$$

where e represents the basis of the natural logarithm, β_0, β_1a β_2 represent estimates of model parameters and t represents time trend $t_i = 1,2, \dots t_n$ [8].

After determining the parameters of the chosen mathematical model, we were able to determine the average height of dry above-ground biomass for variety within of investigated species. In the next part of the study, we continued with the Miscanthus Tatai species, because of the availability of data on biogas content after biomass gasification and its calorific value as an energetic parameter. The experimentally determined data were converted to a unit of energy consumption measured in kWh, and the unit price for kWh consumption for the end consumers of electricity from the energy supplier in Slovakia was determined. On the basis of the above data, it was possible to construct a relatively simple economic model of the present value of bioenergy from FGT grown for experimental purposes in the experimental field in Kolíňany. In general, we identified the first potential income considering the relatively infinite cultivation time of the investigated FGT as perpetuity.

$$PV = \frac{CF_t}{r}, \tag{1.1}$$

where CF_t represents cash-flow received in time $t_i = 1,2, \dots t_n$, r represents the chosen discount rate. Respecting the life cycle of the individual SRC as well as the crop cycle, we used the standard discounted cash-flow model (DCF) and the delayed annuity maturity model for further estimates of Present Value.

$$PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n} \tag{1.2}$$

$$A = P \cdot \frac{[1-(1+r)^{-n}]}{(1+r)^{t} \cdot r} \tag{1.3}$$

where *A* represents the annuity, *P* is the payment, *n* is the number of periods, and *t* is the number of deferred periods. The discount rate r represents the required rate of return and, in general, can be written as

$$r = r_f + r_m\,, \tag{1.4}$$

where r_f is the risk-free interest rate and r_m is the risk premium.

RESULTS AND DISCUSSION

In the first step of the study, we made individual estimates of the yield of dry above-ground biomass of selected species of Miscanthus considering the twenty-one-year life cycle.

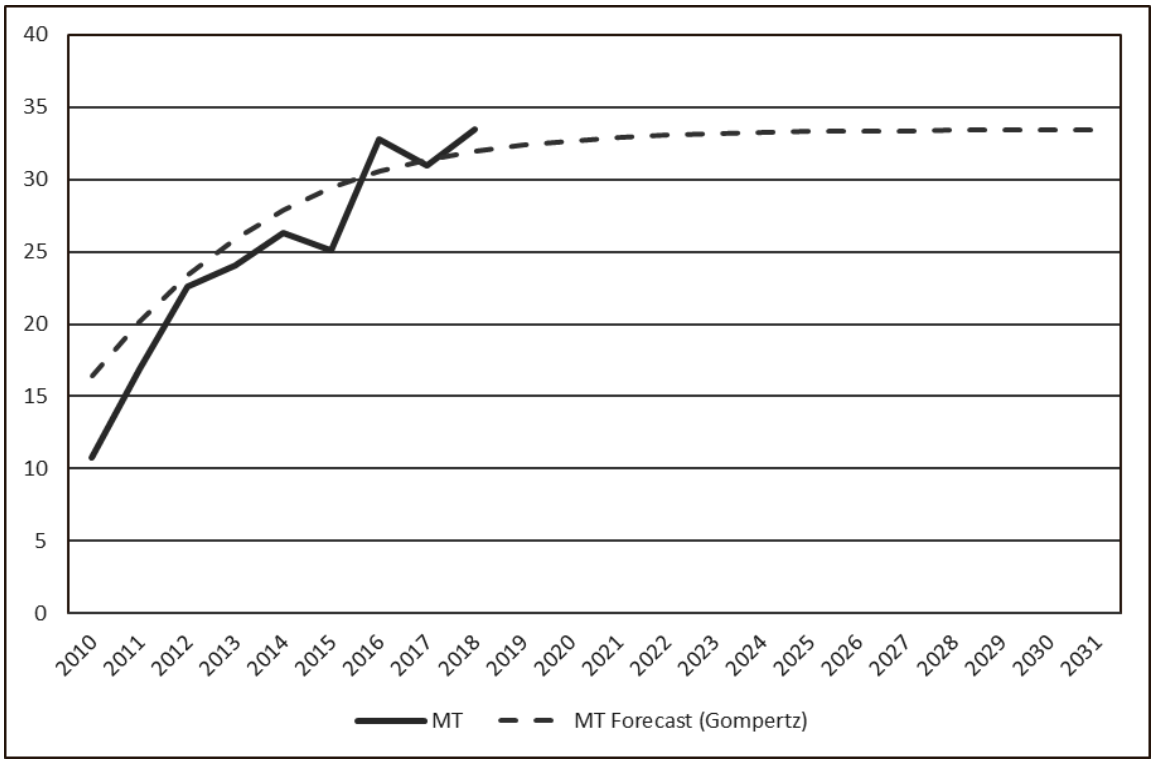


Fig. 1 Average dry above-ground biomass of *Miscanthus Tatai*, extrapolation of the trend
Source: own calculations

Tab. 1 Historical yields of dry above-ground biomass of *Miscanthus Tatai* [$t \cdot ha^{-1}$] using Gompertz curve

Variety/year	2010	2011	2012	2013	2014	2015	2016	2017	2018
MT	10.8	16.9	22.6	24.1	26.3	25.1	32.82	30.94	33.45

Source: primary data, Kolíňany 2019

Tab. 2 Prognosis of the trend of the yield of dry above-ground biomass of Miscanthus Tatai [$t \cdot ha^{-1}$] using Gompertz curve

Prognosis: Gompertz curve extrapolation	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031
	32.4	32.7	32.9	33.1	33.2	33.3	33.3	33.4	33.4	33.4	33.4	33.4	33.4

Source: own calculations

The average yield of the dry above-ground biomass was estimated at 30.23 $t \cdot ha^{-1}$, with the standard deviation of the estimate (SD) being 4.72. The confidence interval of the estimate was set at [28.26; 32.5] at the significance level $\alpha = 0.05$.

In the next part of the study, we will derive an economic model of biomass profit for bioenergy purposes. Gasification in the biogas plant was chosen as the primary technology for obtaining bioenergy. A useful component of this process is precisely biogas, which is then converted into another type of energy (such as electricity, heat). Biogas yield of the grass feed is about 140 $[m^3 \cdot t^{-1}]$ [6], while the experimentally determined value at the biogas station at the Slovak University of Agriculture in Kolíňany was 113 $m^3.t^{-1}$. In terms of combustion parameters of gases, in the case of biogas, the net calorific value is 6 $kWh.m^{-3}$ [6]. On average, the price of 1 kWh for the end consumer, taking into account the electricity used for cooking, lighting, and heating in combined modes (NT - low tariff and VT - high tariff) was empirically determined at € 0.0685 including VAT.

Based on input data on the average dry biomass yield (Miscanthus) and biogas calorific value, and the unit price of kWh, we were able to determine the cash flow from 1 ha/year at € 1547. Furthermore, we have determined the potential gross yield assuming infinite perpetuity at a level, considering a discount rate of 10%:

$$GI = \frac{CF}{r} = \frac{1547}{0.1} = 15470 € \cdot ha^{-1}.$$

In the next step, we determined the life cycle of the Miscanthus stand, the maturity, and the harvest cycle period. The determination of these parameters was based on experimentally determined life cycle data – 21 years, grain maturation – 2 to 4 years and collection cycle period – 3 years [7]. Thus, 7 crops can be expected during the plant life cycle. Thus, using the DCF model, we determined the Current Value of Financial Return in the 3rd year of the collection cycle as follows:

$$PVGI_3 = \frac{CF}{(1+r)^3} = 1162.28 € \cdot ha^{-1}].$$

Then we determined the present value of the total gross financial income, consisting of 7 collections over the entire life cycle of the crop, by estimating an annuity with deferred maturity, abstracting the costs of cultivation and energy conversion.

$$PVGI = \sum_{n=1}^{7} \frac{CF}{\{a_n\}^7} = 3036.93 € \cdot ha^{-1},$$

where $a_n = (1+r)^{3n}$.

CONCLUSIONS

The main aim of the article was to point out the possibilities of FGT cultivation for the production and obtaining bioenergy. The aim of the paper was to derive a financial model of the profit generated by the sale of energy produced to end customers, abstracting the costs of cultivation and conversion to energy. The amount of gross financial income was determined on the basis of data from the primary collection (experimental area Kolíňany) and the secondary data from theoretical and empirical sources (volume of biogas in a tone of dry biomass, the calorific value of biogas and price relations of electricity). The financial model was developed, taking into account the real investment-business conditions when deciding on the allocation of investment. The amount of gross financial income was determined for the species *Miscanthus*, variety *Tatai*, on the basis of the mass of dry above-ground biomass in $t \cdot ha^{-1}$ with extrapolation of the production curve over the entire life cycle of the crop. The value parameters of the model can be made more realistic by clearing the revenue from the costs of growing, converting, and supplying energy to the network for end customers, including any subsidies to promote the use of RES, managing FGP cultivation to ensure annual crop harvesting, etc.

REFERENCES

[1] Bhattacharyya, C.S. (2011). *Energy Economics: Concepts, Issues, Markets and Governance*. London: Springer.

[2] IEA. *Renewables 2018*. (2018). IEA.

[3] Goldemberg, J. (2004). The case for renewable energies. Thematic Background paper 1, *International conference on Renewable Energies*, Bonn. Retrieved 2020-03-24 from http://www.renewables2004.de/pdf/tbp/TBP01-rationale.pdf

[4] Painuly, J. (2001). Barriers to renewable energy penetration: a framework for analysis. *Renew Energy*, 24(1):73 – 89.

[5] Neuhoff, K. (2005). Large-scale deployment of renewables for electricity generation. *Oxf RevEcon Policy*, 21(1): 88 -10.

[6] Čičmanec, S. (2008). Biogas - a suitable supplement to natural gas. *Alternative fuels*. Retrieved 2020-03-18 from http://spnz.sk/stara_stranka/Casopis/08_05/08_05_12.pdf

[7] Demo, M. & Tóthová, M. et al. (2013). *Different methods of planting fast-growing energy trees of the genus Salix and their influence on rooting, growth dynamics and biomass yield*. Nitra: Slovak University of Agriculture in Nitra (in Slovak).

[8] Rublíková, E. (2007). *Time series analysis*. Bratislava: Iura Edition (in Slovak).

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-05-10
Accepted: 2020-06-29
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.31-39

Original Paper

Evaluation of the effectiveness of EU Intellectual property course at SAU in Nitra

Jarmila Lazíková[1*], Ľubica Rumanovská[2], Ivan Takáč[2]

[1] Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Department of Law, Nitra, Slovak Republic
[2] Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Department of European Policies, Nitra, Slovak Republic

ABSTRACT

Education in the field of intellectual property rights can have a positive effect on their ability to be employed and can provide relevant information considering the student choice and indirectly measure the quality of higher education. We decided to supply the course EU Intellectual Property for students at Slovak University of Agriculture (SUA) in Nitra. We had more than 100 graduated students during the two years of the course. Nowadays, we need feedback on the effectiveness of the course based on the exam results. We found out that the most of students are interesting in the issues of EU Intellectual Property and are able to do their exam with the mark A or B. There are no statistically significant differences between the students base on the study programmes; however, the statistical significant differences were proved among the students by the form of study. The full-time form appears to be the most appropriate form of study. Moreover, creativity of students evaluated by the seminar papers lags behind their theoretical knowledge proved by tests. It is an important fact mainly at the course of the EU Intellectual Property where the creativity is one of the most important elements. The article can be used as an example how to measure the quality of higher education by statistical methods.

KEYWORDS: EU Intellectual Property, European Union, new course, exam results, parametric and non-parametric statistical tests

JEL CLASSIFICATION: C12, K11, O34

INTRODUCTION

Innovation is a key component of the growth strategy adopted by the European Union and characterised by the creation of a more competitive economy with higher employment. The achievement of this aim depends on different factors and one of them is undoubtedly an

[*] Corresponding author: Jarmila Lazíková, Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Department of Law, Nitra, Slovak Republic, E-mail: Jarmila.Lazikova@uniag.sk

efficient system of intellectual property rights [11]. The system of EU Intellectual Property includes copyrights, patents, utility models, designs, topography of semiconductor, breeder's right, trademarks, geographical indications and traditional specialities guaranteed. This system of Intellectual Property Rights has been gaining considerably in importance for businesses in many key industries to ensure economic competitiveness [20]. Intellectual Property Rights - intensive industries generated 27.8% of all jobs in the EU during the period 2011-2013, they pay significantly higher wages than other industries and have proved most resilient to the economic crisis [8]. On the other hand, the mere filing of an Intellectual Property right is not sufficient to trigger growth, but it can signal a firm's stronger ability to sustain growth through the creation, protection and exploitation of intellectual assets [12].

Intellectual property plays a vital role in promoting innovation as it provides those who invest time, effort and money in innovation with a mechanism to protect and benefit from it [9]. Intellectual property management is a key element in improving the competitiveness of any company. Unfortunately, SMEs do not benefit from open innovation or from patenting in the same way as larger firms in spite of their importance for economic welfare and innovation in Europe [4]. Reasons why SMEs do not take any measures to protect their innovation are not seeing any benefit in protecting innovations (35%), lack of knowledge on how to protect innovations (13%) and the cost of procedures (10%). There were the top three reasons why SMEs do not protect their innovation [7]. Three years later, the EUIPO repeated its research. The results reveal that the main reason, for all sizes of SMEs, is a lack of sufficient knowledge that is, what Intellectual property is and how do we register it (38%) [9]. Many other researchers confirm that resource shortages and lack of Intellectual Property management practices are some of the barriers faced by SMEs [6; 14; 15; 16].

Moreover, according to the study of EUIPO [7] just as in 2013, the youngest generation reports the lowest level of 'good' understanding in comparison with other age groups, and this level of understanding is decreasing: 64 % of the Europeans surveyed aged 15 to 24 report an overall good understanding of Intellectual Property, compared with 68 % in 2013. In addition, 21% of 15 to 24 year olds say they intentionally use illegal sources of digital content and 13% say they have intentionally bought counterfeits in the past 12 months [10]. Today, too many graduates start their first job without ever having studied anything about Intellectual Property Rights. A combination of lecture modules, practical cases and a compulsory Intellectual Property Rights part in all Master's projects would build Intellectual Property Rights awareness and competence for the future graduates [18]. However, there are few higher education programmes dedicated to Intellectual Property Rights management [18].

The courses oriented on the intellectual property were missing also at SUA. The issues linked to the intellectual property were mentioned also within other courses as marginal matter (e.g. Commercial law, EU Business Law, etc.). Therefore the Department of Law within the cooperation of the Department of European Policies at the Faculty of European Studies and Regional Development of SUA in Nitra filled the application of Jean Monnet Module project focused on the EU intellectual Property. The project was approved and the new course EU Intellectual Property was introduced as optional course in master programmes at the Faculty of European Studies and Regional Development. During two years there were 130 students who graduated in the new course. The aim of this paper is to find out which group of students is the most successful at this course by the various criteria such as year of study, sex, form of study, study programmes. The result of the exam consists of two parts: results of seminar papers and results of tests. Therefore we looked for if there are statistically significant

differences among the results of seminar papers that declare higher creative potential than results of tests created by theoretical questions. For this purpose we use the parametric and non-parametric tests of statistical induction described below.

MATERIAL AND METHODS

The new course EU Intellectual property has been taught at SUA for two years within the project of Jean Monnet Module EU Intellectual Property no. 599683-EPP-1-2018-1-SK-EPPJMO-MODULE. The course was graduated by 130 students of master study programmes at Faculty of European Studies and Regional Development; of it 57 students in study programme Rural Development and Development of Rural Tourism and 73 students in study programme Regional Development and Policies of the EU. In the first year (2018), there were 60 students and in the second year (2019), there were 70 students; of it, 99 females and 31 males. Most of the students were enrolled in the present form of study (86 students), only 22 students visited the external form of study and 22 students visited individual form of study. To achieve the aim of this paper we looked for the statistically significant differences in the exam results among the students according to the above mentioned criteria such as sex, form of study, study programmes. We use the parametric tests of statistical inductions, z-test.

The statistical value is as follows:

$$u = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{n_2\sigma_1^2 + n_1\sigma_2^2}}\sqrt{n_1 n_2}$$

The critical value is stipulated by excel function NORMSINV, which returns the inverse of the standard normal cumulative distribution [19].

The statistical significant differences were observed also among the three forms of studies; however there were only 22 students in external and individual form of studies, since the measured values did not have a normal distribution; therefore we used the non-parametric Kruskal – Wallis test expressed as follows by the relation:

$$H = \left(\frac{12}{N(N+1)} . \sum_{j=1}^{k} \frac{R_j^2}{n_j}\right) - 3.(N+1)$$

H – Test statistics of Kruskal -Wallis test,

N – Number form of studies,

R_j – Order for each form of study,

n_j – Number of observations in each form of studies,

k – Number of forms of studies.

The potential statistically significant differences are analyzed by the tests of contrasts available in program Statgraphics, mainly Fischer LSD test which confirms the statistically significant differences between two average values of objects included into a particular group expressed as follows:

$$|\bar{y}_i - \bar{y}_j| \geq t_{\alpha,m(n-1)}\sqrt{\frac{2s_r^2}{n}}$$

where α, $m(n-1)$ is the critical value of *t*-distribution with $m\cdot(n-1)$ degrees of freedom.

Moreover, we compared also the partial results of the exam; i.e. the results from the seminar papers and the results from the written test. We used the t- test paired two sample for means available in excel expressed as follows:

$$t = \frac{\bar{d}}{s_d}$$

where

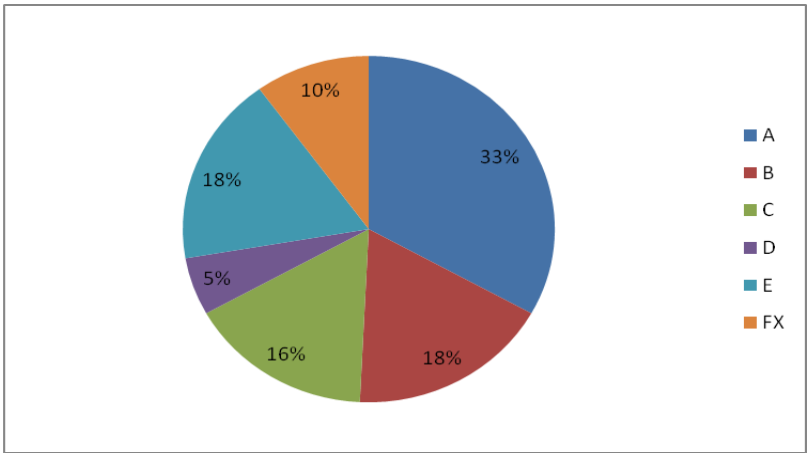
$$\bar{d} = \sum_{i=1}^{n} d_i \quad , \quad d_i = x_{2i} - x_{1i} \, , \quad s_d = \sqrt{\sum \frac{(d_i - \bar{d})^2}{n(n-1)}} \, .$$

The critical value is expressed by function TINV which calculates the inverse of the two-tailed Student's t-distribution, which is a continuous probability distribution t at the alfa level 0.05 and $(n-1)$ degrees of freedom [19].

RESULTS AND DISCUSSION

The course EU Intellectual Property visited 130 students in the year 2018 and 2019. The results of exams are documented in figure 1. The most of students received mark A; despite the fact, that mark A is the most often appeared mark among the students, it received only 33% of all students. The average value of all marks is 1.985 (C) but 50% of all students received mark A or B. The standard deviation is 0.986; it means that the most of marks range from A to E, what indicates that the marks are spread out over a wider range.

Figure 1 Results of exam received by all students in percentage



Source: own calculations

We looked for the statistically significant differences among the males and females. The table 1 represents the distribution of students by sex.

Table 1 Exam results received by males and females

Mark	All students	Males		Females	
		number	percentage	number	percentage
A	43	7	16.28	36	83.72
B	23	7	30.43	16	69.57
C	21	3	14.29	18	85.71
D	7	2	28.57	5	71.43
E	23	7	30.43	16	69.57
FX	13	5	38.46	8	61.54

Source: own calculations

To find out the statistically significant differences we used z-test.

According to the results the value of test statistics $z = 1.57 < z_{crit} = 1.96$ with p-value $= 0.12 \geq \alpha = 0.05$; null hypothesis (the hypothesis that, there are no statistically significant differences between the observed variables) we do not reject, that means that we can conclude that there are no statistically significant differences between the exam results for men and women. The average mark for men is 2.24 and for women it is 1.90.

Secondly, we compared the exam results of students who graduated the course EU Intellectual property in 2018 with the students who graduated in 2019. The table 2 represents their distribution.

Table 2 Exam results for students in 2018 and 2019

Mark	All students	2018		2019	
		number	percentage	number	percentage
A	43	21	48.84	22	51.16
B	23	6	26.09	17	73.91
C	21	8	38.10	13	61.90
D	7	2	28.57	5	71.43
E	23	19	82.61	4	17.39
FX	13	4	30.77	9	69.23

Source: own calculations

To find out the statistically significant differences we used z-test. According to the results the value of test statistics $z = 0.88 < z_{crit} = 1.96$ with p-value $= 0.38$. Whereas the probability value $p \geq \alpha = 0.05$; therefore, we do not reject the tested hypothesis and we can conclude that there are no statistically significant differences between the exam results for students who

graduated in 2018 and 2019. The average mark for who students graduated in 2018 is 2.07 and for students graduated in 2019 is 1.91.

Thirdly, we compared the exam results of students by the study programmes. The students were enrolled in master study programme Rural Development and Development of Rural Tourism (hereinafter as RDDRT) and master study programme Regional Development and Policies of the EU (hereinafter as RDPEU). The table 3 represents their distribution.

Table 3 Exam results for students of different master study programmes

Mark	All students	RDDRT		RDPEU	
		number	percentage	number	percentage
A	43	27	62.79	16	37.21
B	23	5	21.74	18	78.26
C	21	10	47.62	11	52.38
D	7	1	14.29	6	85.71
E	23	8	34.78	15	65.22
FX	13	6	46.15	7	53.85

Source: own calculations

To find out the statistically significant differences we used z-test.
According to the results the value of test statistics $z = -1.45 < z_{crit} = 1.96$ with p-value $= 0.15 \geq \alpha = 0.05$; therefore we can state that there are no statistically significant differences between the exam results for students enrolled in different study programmes. The average mark for students of RDDRT is 1.84 and for students of RDPEU is 2.10.

Fourthly, we compared the exam results of students by the form of study (full time, external and individual form of study). The table 4 represents their distribution.

Table 4 Exam results for students of different form of study

Marks	All students	Full time form		External form		Individual form	
		number	percentage	number	percentage	number	percentage
A	43	42	97.67	0	0.00	1	2.33
B	23	16	69.57	0	0.00	7	30.43
C	21	13	61.90	3	14.29	5	23.81
D	7	4	57.14	0	0.00	3	42.86
E	23	4	17.39	18	78.26	1	4.35
FX	13	7	53.85	1	7.69	5	38.46

Source: own calculations

Since the measured values did not have a normal distribution; due to only 22 students in external and individual form of study, therefore we used the non-parametric Kruskal – Wallis

test to find out the statistically significant differences. To calculate the KW statistics we used the programme Statgraphics plus. According to the results KW statistics $H = 38.86 > z_{crit} = = 5.99$ with p-value = $3.64x10^{-9} < \alpha = 0.05$; whereas the probability value p is less than 0.05, we reject the null hypothesis and we can conclude that there are statistically significant differences among the exam results for students of different form of study. The average mark for full-time students is 1.65; for external students 2.91 and for students with individual form of study is 2.36. The statistically significant differences were confirmed by LSD test between all pairs of form of study. The students of external or individual forms of study do not usually have enough time to prepare more precise for the exams due to their job or family duties.

Moreover, we are interested in the form of exam which is more acceptable for students. The result of exam consists of two parts: results of two seminar papers and results of two tests. The seminar papers are related to the copyright licence agreement and application of trade mark at the particular state body with the obligation to prepare a short project for trademark. The tests consisted of open questions which asked for short but precise answers related to the theory of EU intellectual property. Therefore we looked for if there are statistically significant differences among the results from seminar papers that declare higher creative potential than results from tests with theoretical questions. We tested 58 students who graduated the course in 2019. We excluded the students with FX and students which repeated the exam. The results are documented in table 5.

Table 5 Number of students with the partial exam results

Mark	Partial results from tests	Partial results from seminar papers
A	17	15
B	10	7
C	14	16
D	13	0
E	4	9
FX	0	11

Source: own calculations

The measured values fulfilled the conditions for the use of parametric test since the measured values had a normal distribution. To find out the statistically significant differences we used the parametric t- test paired two samples for means available in excel. By using the t-test we have calculated the value of test statistics. According to the results $t = 2.80 > t_{crit} = 2.00$ with a probability value p-value = $0.007 < \alpha = 0.05$. According to the results KW statistics $H = 38.86 > z_{crit} = 5.99$ with p-value = $3.64x10^{-9} < \alpha = 0.05$; whereas the probability value p is less than 0.05, we reject the null hypothesis and we can conclude that there are statistically significant differences between the partial exam results from seminar paper and from tests. The average mark from seminar papers is 2.22 and from tests is 1.80. The results proved that the students on average are able to accept the theoretical knowledge; however their effort to use it in practice is quite small. It is a very important finding though not very satisfactory that the students' creativity lags behind their theoretical knowledge.

CONCLUSIONS

Intellectual Property refers to unique, value-adding creations of the human intellect that result from human ingenuity, creativity and inventiveness [15]. The EU Intellectual Property has a strong impact on market leadership and the overall performance of a company [2] and is an important instrument for profiting from innovation [1]. Knowledge and IP management competencies are crucial to implementing and executing the open strategies successfully [3]. On the other hand, there is only a limited defensibility of such rights in juridical disputes because of high costs and time investments [4]. Therefore, to advance the cause of the rights and wrongs of the laws that promote and protect intellectual property at the national and international levels, education in intellectual property is required and must be advocated [17]. Training and education are a crucial component of a well-functioning and balanced IP system [5]. Typically, IP education is available only in law school (courses are limited there as well) and for librarians, teachers, and others who deal directly with potential IP litigation in the course of their jobs. Even basic education for the lay person about the field of IP is not always available [17]. Therefore we decided to supply the course EU Intellectual Property to students at Slovak University of Agriculture to receive at least basic knowledge on EU Intellectual Property and legal measures how to protect it. We found out that the most of students are interesting in the issues of EU Intellectual Property and are able to do their exam with the mark A or B. There are no statistically significant differences between the students by the sex, study programme or year of course graduation. However, the statistically significant differences were proved among the students by the form of study. The full-time form appears to be the most appropriate form of study. Moreover, the creativity of students evaluated by the seminar papers lags behind their theoretical knowledge proved by tests. It is an important fact mainly at the course of EU Intellectual Property where creativity is one of the most important elements. We should put a higher accent on the creative tasks on the lessons. The use of mathematical-statistical methods significantly contributed to the evaluation of the subject.

ACKNOWLEDGEMENTS

This work was supported by the Jean Monnet Module no. 599683-EPP-1-2018-1-SK-EPPJMO-MODULE, "EU Intellectual Property."

REFERENCES

[1] Anton, J.J. & Yao, D.A. (2004). Little patents and big secrets: managing intellectual property. *The RAND Journal of Economics*, vol. 35, no. 1, pp. 1–22.

[2] Bollen, L., Vergauwen, P. & Schnieders, S. (2005). Linking intellectual capital and intellectual property to company performance. *Management Decision*, vol. 43, no. 9, pp. 1161-1185.

[3] Braun, A. & Müller, E. (2009). The role of trade secrets in open innovation: examples of pharmaceutical SME. Paper at the 2nd ISPIM *Innovation Symposium – Stimulating Recovery – The Role of Innovation Management*, New York, 6 – 9 December 2009.

[4] Brem, A., Nylund, P. A. & Hitchen, E. L. (2017). Open innovation and intellectual property rights: How do SMEs benefit from patents, industrial designs, trademarks and copyrights? *Management Decision*, vol. 55, no. 6, pp. 1285-1306.

[5] De Beer, J. & Oguamanam, Ch. (2010). *Intellectual Property Training and Education: A Development Perspective.* Geneva: International Centre for Trade and Sustainable Development, 2010. 58 p.

[6] Eppinger, E. & Vladova, G. (2013). Intellectual property management practices at small and medium-sized enterprises. *International Journal of Technology Management*, vol. 61, pp. 64-81.

[7] EUIPO. (2016a). *Intellectual Property (IP) SME Scoreboard 2016*. Alicante: EUIPO, 2016. 140 p.

[8] EUIPO. (2016b). *Intellectual property rights intensive industries and economic performance in the European Union*. Alicante: EUIPO, 2016. 156 p.

[9] EUIPO. (2019a). *Intellectual Property SME Scoreboard*. Alicante: EUIPO, 2019. 144 p.

[10] EUIPO. (2019b). *Intellectual Property and Youth Scoreboard*. Alicante: EUIPO, 2019. 73 p.

[11] EUIPO. (2019c). *IPR-intensive industries and economic performance in the European Union.* Munich: EUIPO, 2019. 158 p.

[12] EUIPO. (2019d). *High-growth firms and intellectual property rights*. Munich: EUIPO, 86 ps.

[13] European Union. (2019). *Your Guide to IP in Europe.* St. Ingbert: European IP Helpdesk, c/o Eurice GmbH, 2019. 31 p.

[14] Jensen, P. H. & Webster, E. (2006). Firm Size and the Use of Intellectual Property Rights. *The economic record*, vol. 82, no. 256, pp. 44–55.

[15] Kalanje, C.M. (2006). *Role of intellectual property in innovation and new product development.* World Intellectual Property Organization. Retrieved 2020-04-08 from http://www.wipo.int/sme/en/documents/pdf/ip_innovation_development.pdf

[16] Korcsmáros, E., Šeben, Z., Machová, R. & Fehér, L. (2019). Promotion of Euro Introduction in Slovakia: Financial Literacy of Generation X and Y. *Marketing and Management of Innovations*, 2019, no. 3 (2019), pp. 11-21.

[17] Lakhan, S. E. & Khurana, M. K. (2007). The state of Intellectual Property Education Worldwide. *Journal of Academis Leadership*, vol. 5, no. 2, pp. 1-11.

[18] Moulin, A. & Lie, H. T. (2005). *Intellectual Property Rights and Nordic SMEs. A study of IPR practise in the IT and Biotech sectors.* Oslo: Leogriff AS, 2005, 70 p.

[19] Pacáková V. et al. (2015). *Statistical induction for economists and managers*. Bratislava: Wolters Kluwer, 365 p. (in Slovak).

[20] Radauer, A., Dudenbostel, T. & Dintrich, A. (2018). *Feasibility study on Intellectual Property pre-diagnostic*. Final report. Brussels: European Commission, 2018. 94 p.

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2020(6), 1

Received: 2020-05-29
Accepted: 2020-07-13
Online published: 2020-07-08
DOI: https://doi.org/10.15414/meraa.2020.06.01.40-46

Original Paper

Study results comparison in Mathematics with English language of instruction

Dana Országhová *

Slovak University of Agriculture in Nitra, Faculty of Economics and Management, Department of Mathematics, Nitra, Slovak Republic

ABSTRACT

At the Slovak University of Agriculture in Nitra mathematical subjects are included in study programs, which are taught in English. The main goal of the paper is to compare the education results in the study course Mathematics with English language of instruction. We focused on the level of mathematical competencies of students in solving selected problems from linear algebra and mathematical analysis. The research sample was composed of the first-year students of English study program Regional Development and Policies of the EU. Empirical data were obtained from partial tests, exam tests and final grades in the exam in Mathematics in the academic years 2015/2016 to 2019/2020. Data analysis was performed by descriptive statistics and using non-parametric one-sample Wilcoxon test. We found that in the group taught in English, students achieve better exam grades in Mathematics than students taught in Slovak. Results of hypothesis testing have not proved the significance of differences in the gained point score of individual tasks in the analyzed study group.

KEYWORDS: Mathematics, teaching in English, exam grades, one-sample Wilcoxon test

JEL CLASSIFICATION: D40, C50, M10

INTRODUCTION

The working career of every person is conditioned by the level of education and acquired specialized competences. The training of students in economics study program is important for their employment and work in firms, trade companies, services, etc. Graduates will perform activities in various areas such as organizing, planning, leading, decision-making, and control, which are important for the success and application of the company in the competition in the labor market. The content of economic study programs also includes

* Corresponding author: Assoc. prof. RNDr. Dana Országhová, CSc., Slovak University of Agriculture, Faculty of Economics and Management, Department of Mathematics, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic, E-mail: dana.orszaghova@uniag.sk

mathematical subjects that provide exact tools and procedures for solving theoretical and application problems.

The university study in a foreign language at Slovak universities brings new possibilities for professional employment of graduates, provides students with opportunities to acquire new professional and language competencies. As Gálová et al. [3] report the internationalization extends interest of students in education at university level. The completion of studies in foreign language brings new value to the quality of education and development of competencies through mobility projects [6]. Study stays abroad and summer work activities are pre-conditioned with mastery of standard communication in a foreign language and also with understanding of scientific terms [11]. The teaching of mathematics in a foreign language is based on the general language training of students and teachers [12]. Direct experience from mathematics teaching in English confirmed that the content and method of mathematics teaching in English-speaking countries may differ from the content and didactic methods used in other European countries [9]. For teaching in English, it is important to create study materials in printed and electronic form and apply effective tools of information technology for the knowledge transfer from teachers to students [4].

Educational process is a result of many factors interaction (subjects of education, means, and conditions of education) and their analysis results in new findings. According to Khakpour [5] universities are the important institutions for the constant dissemination and pursuit of knowledge. Universities and faculties transform and transfer knowledge resources through the teaching, research, publications, operation and assessment.

In research studies, mathematical abilities and mathematical skills are often the subject of analysis. Cígler [1] deals in details with mathematical abilities and mathematical skills, stating the main differences, ways in which they arise and could be measured. Results of didactic research in different countries have confirmed that students' interest in science subjects is decreasing and students come across difficulties in STEM subjects (i.e. science, technology, engineering, and mathematics) [14]. Mathematical knowledge becomes permanent only if students sufficiently understood math terms with its logic meaning and processed them in mind adequately [10]. The important part in explanation is the visualization of new math concepts, which facilitates the main idea of concepts and could shorten the learning process itself [2]. To support learning in depth it is important to combine new mathematical concept with solving applied problems [13].

MATERIAL AND METHODS

In this paper we focused on the comparison of final exams in Mathematics, students' knowledge level and their ability to solve tasks in this compulsory study subject. Research sample consisted of students of the Faculty of the Faculty of European Studies and Regional Development, Slovak University of Agriculture (SUA) in Nitra. We got sources for this paper by studying the specialized articles and literature related to the pedagogical research. The next research material was obtained from the teaching process.

The subject Mathematics is included in the winter semester in the bachelor's study program Regional Development and Policies of the EU and the scope of teaching is 1 hour per week lecture and 3 hours seminar. During the semester, students submitted a seminar project, wrote one partial test and one final exam. In the individual academic years from 2015/2016 to

2019/2020, study groups contained a small number of students, so together we analyzed 27 partial and exam tests.

In evaluation of data we used descriptive statistics and via one-sample Wilcoxon test we tested the significance of differences between gained points in solved math problems. The null hypothesis states that distributions of characters *X*, *Y* are the same; the alternative hypothesis claims that distributions of analyzed characters *X*, *Y* are not the same. We assume that quantitative characters *X*, *Y* have continuous distribution functions. Then we determine differences $x_i - y_i$ $(i = 1,2,...,n)$ for empirical data (x_1, y_1), (x_2, y_2), ... , (x_n, y_n) and arrange them in a non-decreasing sequence, omitting the zero differences. We find the sum of the orders of positive differences (value *T*+) and the sum of the orders of negative differences (value *T*–). The value of the test statistic is defined as $T = \min(T_+, T_-)$. If the value of the test criterion is less than or equal to the critical value $w_n(\alpha)$, then we reject tested null hypothesis at the chosen level of significance α [7].

RESULTS AND DISCUSSION

Exam grades in study subject Mathematics

In this part we analyze study outputs in Mathematics of mentioned study program in years 2015/2016 to 2019/2020.

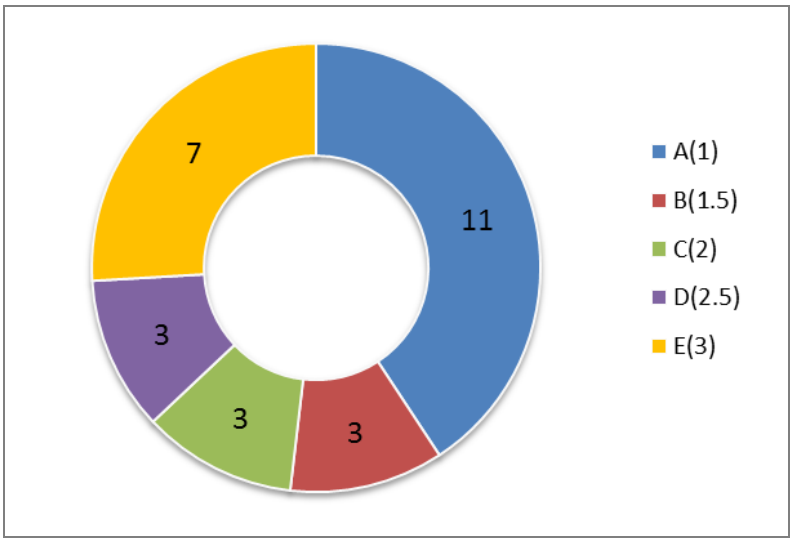


Fig. 1 Exam grades in Mathematics taught in English, years 2015/2016 – 2019/2020
Source: author

As we mentioned above, overall number of students was 27 in all groups with English language of instruction. According to Fig. 1 we see, grade A (1) was achieved by the largest number of students (11 in total), followed by grade E (3) (7 students).

In Tab. 1 we present the average grades in Mathematics within five academic years for study groups taught in English (EG) and for groups taught in Slovak (SG). In the last column we

also listed the numbers of students in each group. From data we see that approximately one tenth is represented by students in the group with English language of instruction. The values show that the study average in the EG group was better each year than the study average in the SG group, which is also valid when comparing the study averages for all years together.

Tab. 1 Comparison of average grades and numbers of students

Academic year	English group Average grade	Slovak group Average grade	Number of students EG/SG
2015/2016	2.31	2.39	8/75
2016/2017	1.75	2.25	4/55
2017/2018	1.67	2.28	3/39
2018/2019	1.42	2.25	6/55
2019/2020	1.83	1.96	6/42
Overall	**1.85**	**2.25**	**27/266**

Source: author

The calculated average grades for the English groups are demonstrated in graphical form (Fig. 2). We see that in the individual years, the average exam mark gradually improved, while it decreased again in the last year. Only in the academic year 2015/2016 the study average in mathematics (2.31) was worse than the overall average for all years (1.85). In the group, Slovak and foreign students are together, while foreign students complete one or two semesters at the selected faculty as a part of the Erasmus+ stay. These are usually students with excellent study results and their motivation to achieve good exam grades is also high so that they would obtain a scholarship after returning to their home university. Exactly these students improve the final study results of the group taught in English.

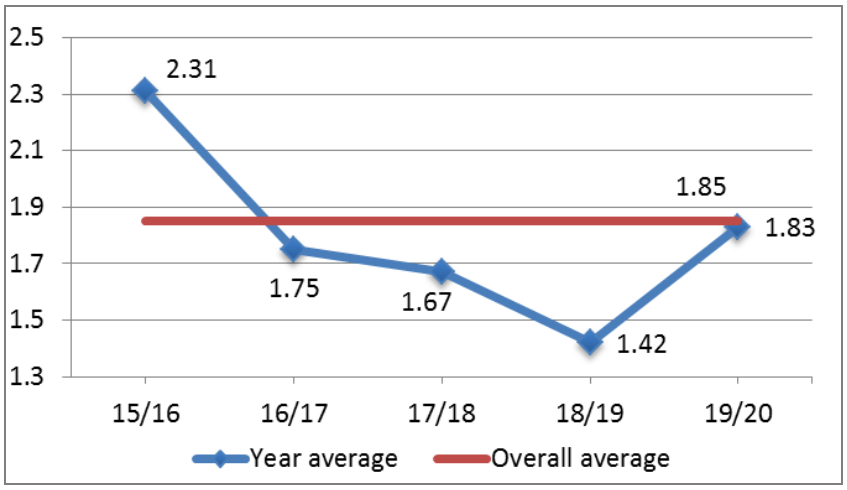


Fig. 2 Average grades in Mathematics taught in English
Source: author

Tasks analysis of mid-term and exam tests

In this part we present an analysis of students' success in solving selected problems in mathematics. From mid-term tests and exam tests, we selected two tasks from linear algebra (Task 1, Task 2) and two tasks from mathematical analysis (Task 3, Task 4):

Task 1: Solve the system of linear equations by elimination method.
Task 2: Find inverse matrix.
Task 3: Sketch graph of a function and write its properties.
Task 4: Find monotonicity and local extremes of function with usage of derivative.

We arranged the point scores from the problem solving into pairs and using the Wilcoxon one-sample test we tested the hypotheses about the distribution of the investigated characters (Tab. 2). The last analyzed pair was created from a set of tasks on linear algebra together (Task 1, Task 2) and a second set of tasks on functions together (Task 3, Task 4). The value *n* expresses the number of non-zero differences in the set of tested data, referred to in the literature as the effective sample size. Based on results we conclude that in all seven cases the calculated test statistic is greater than critical value $w_n(\alpha)$ for $\alpha = 0.05$. Therefore, we cannot refuse null hypothesis at chosen significance level which means analyzed data have the same distribution function. In the research sample, no significant differences in the point score between the individual task pairs were confirmed.

Tab. 2 Results of Wilcoxon one-sample test, $\alpha = 0.05$

X	*Y*	*n*	*T*+	*T*–	$w_n(\alpha)$	Result
Task 1	Task 2	22	176	**77**	65	$77 > w_n(\alpha)$
Task 1	Task 3	21	169.5	**61.5**	58	$61.5 > w_n(\alpha)$
Task 1	Task 4	16	98.5	**37.5**	30	$37.5 > w_n(\alpha)$
Task 2	Task 3	22	150.5	**102.5**	65	$102.5 > w_n(\alpha)$
Task 2	Task 4	22	130.5	**122.5**	65	$122.5 > w_n(\alpha)$
Task 3	Task 4	20	**94**	116	52	$94 > w_n(\alpha)$
Linear algebra	Functions	43	587.5	**358.5**	310	$358.5 > w_n(\alpha)$

Source: author

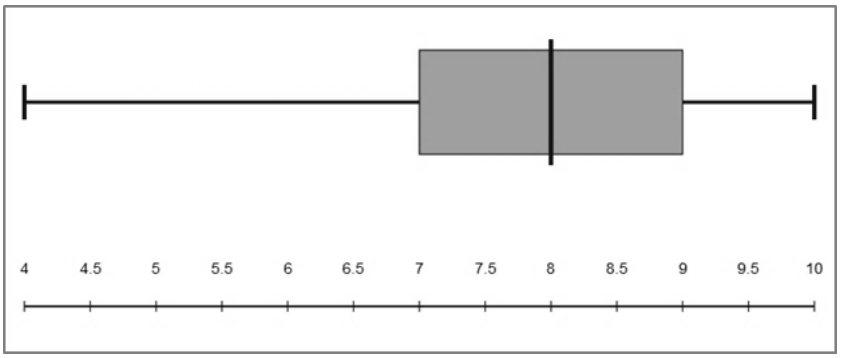


Fig. 3 Points rate for tasks from linear algebra together
Source: author

Fig. 3 shows a box plot for points obtained for problems from linear algebra together. The median of this set is 8, the mode is also 8, and the average number of points is 7.94. In Fig. 4 we present a box plot for the obtained points for the mathematical analysis problems together. The median of the file is 8, the mode is 10, and the average value of the gained points is 7.43.

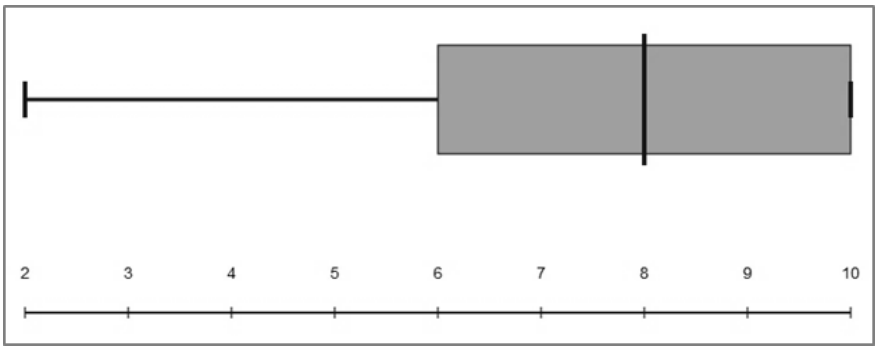


Fig. 4 Points rate for tasks from mathematical analysis together
Source: author

Some problematic factors in the study of mathematics have been identified directly from teaching. Linear algebra is included in the content of analyzed subject at the beginning of the semester, i.e. during the preparation for the exam students study this part better and longer. Functions and derivatives of function are included in the second part of the course; therefore in case of poor study organization for the exam, students will not be able to handle this part just as well. Negative factors include lower knowledge of high school mathematics, frequent numerical errors, misunderstanding of the assignment and students' inattention during tests.

The deterioration of study outcomes in mathematics in tertiary education is also claimed by Matušek and Hornyák Gregáňová [8]. University studies are based on working with knowledge that is most often accessible in the text form. As Průcha [15] states, the correct interaction between the text and its reader is essential for understanding the information in the text. Understanding a mathematical text is also associated with the logical thinking of an educated person and with a sufficient number of hours to practice new topics.

CONCLUSIONS

The main aim of tertiary education is to equip students with knowledge and skills important for real life and their future professions. Rapid changes in the work process and the digitization of work activities must be reflected in the new competencies of university graduates and in modern methods of education, which also include the offer of study programs in English.

In this paper we presented the analysis of study outcomes in the subject Mathematics, taught in English at the Faculty of European Studies and Regional Development SUA in Nitra. The results demonstrate that students successfully complete the study of mathematics in English. We evaluated mathematics exam grades, where analyzed study group achieved a better grades average in comparison with study groups taught in the Slovak language. When comparing the level of point scores in problems from linear algebra and mathematical analysis, statistically significant differences were not confirmed using the one-sample Wilcoxon test.

ACKNOWLEDGEMENTS

The paper was supported by the project KEGA No. 029SPU-4/2018 *Digital educational applications in mathematics*.

REFERENCES

[1] Cígler, H. (2018). *Mathematical skills: theoretical overview and their measurement.* 1st ed. Brno: Masaryk University, 199 p. (in Czech).

[2] Drábeková, J. & Čeryová, D. (2016). Geogebra and several economic applications of learning curve. Proceedings of the conference *Didmattech 2016*. Retrieved 2020-05-20 from http://didmattech.inf.elte.hu/wp-content/uploads/2016/08/DJ_CD_Geogebra.pdf

[3] Gálová, J., Ubrežiová, I., Krasnodębski, A. & Horská, E. (2016). Internationalization as a way of adding value and attractiveness to university education. *The agri-food value chain: challenges for natural resources management and society: International scientific days 2016*. Proceedings. Nitra: Slovak University of Agriculture, pp. 585-591. doi: http://dx.doi.org/10.15414/isd2016.s8.02

[4] Jedličková, Ľ. & Šeben Zaťková, T. (2015). Information literacy and communication skills of university teacher. *R&E-source*. Baden: Pädagogische Hochschule Niederösterreich, 2, spec. iss., 114-122. Retrieved 2020-05-14 from http://journal.ph-noe.ac.at/index.php/resource/article/view/257/296

[5] Khakpour, A. (2015). Knowledge Management in Educational Organizations: Opportunities and Challenges. *7th International Knowledge Management Conference*. Retrieved 2019-05-20 from https://www.researchgate.net/publication/272886479_Knowledge_Management_in_Educational_Organizations_Opportunities_and_Challenges

[6] Kozelová, D., Frančáková, H., Felixová, I. & Pietriková, M. (2012). The effect of foreign language competences and student mobilities on graduates employability in the European labour market. *Acta technologica Dubnicae*, 2, issue 1 (2012), 47-52.

[7] Markechová, D., Tirpáková, A. & Stehlíková, B. (2011). *Basics of statistics for teachers*. UKF: Nitra, 2011 (in Slovak). Retrieved 2020-05-20 from http://www.km.fpv.ukf.sk/upload_publikacie/20120130_90405__1.pdf

[8] Matušek, V. & Hornyák Gregáňová, R. (2019). Comparison of exam results in Mathematics at Faculty of Economics and Management, Slovak University of Agriculture in Nitra. *Mathematics in education, research and applications*, 5(2), 78-83. doi: https://doi.org/10.15414/meraa.2019.05.02.78-83

[9] Nýdl, V. & White Baravalle Gilliam, V. (2001). Lecturing Mathematics in English. *New trends in mathematics teaching*. Nitra: Department of Mathematics, SUA, pp. 11-14.

[10] Országhová, D., Gregáňová, R. & Matušek, V. (2013). *On current mathematical competencies in higher education in economics and technical fields of study*. 1st ed. Nitra: Slovak University of Agriculture. 141 p. (in Slovak).

[11] Országhová, D. & Hornyák Gregáňová, R. (2014). Study of mathematics in a foreign language as synthesis of mathematical and language competencies of students. *Forum statisticum Slovacum*, 10, 2 (2014), 101-106.

[12] Országhová, D. & Horváthová, J. (2015). Pedagogical and statistical analysis of students' mathematical and language competences. *Mathematics in education, research and applications*, 1(1), 28-35. doi: http://dx.doi.org/10.15414/meraa.2015.01.01.28-35

[13] Pechočiak, T., Kecskés, N. & Hornyák Gregáňová, R. (2019). Differential equations in university education. *ICERI 2019, 12th Annual International Conference of Education, Research and Innovation*. Valencia: IATED, pp. 3756-3763.

[14] Porozovs, J. (2019). Latvia students' interest in different science subject topics. *ICERI 2019, 12th Annual International Conference of Education, Research and Innovation*. Valencia: IATED, pp. 2749-2758.

[15] Průcha, J. (1998). *Textbooks: theory and analysis of educational media.* Brno: Paido, 26 p. (in Czech).