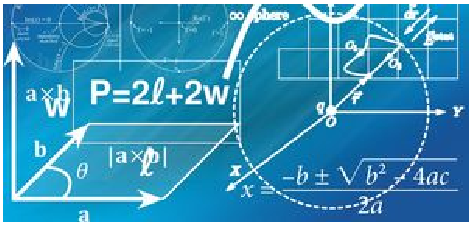
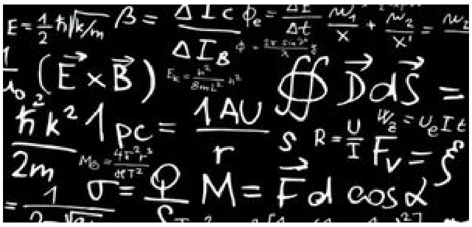
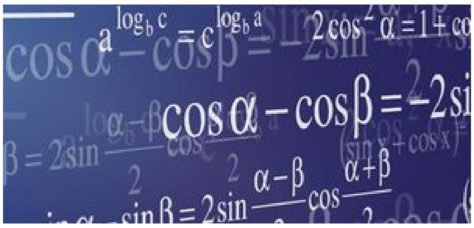

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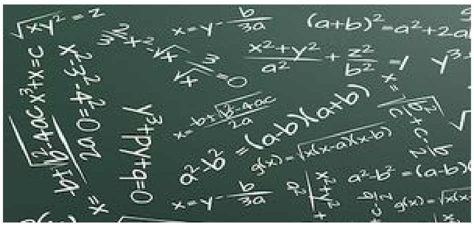
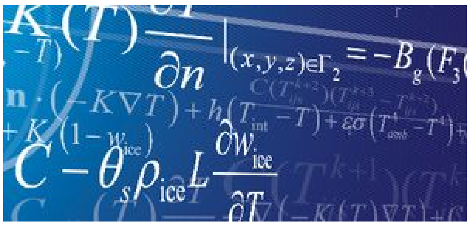
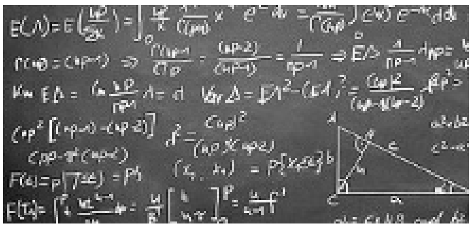

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Original Paper

Application of the logistic regression analysis to assess credibility of the farm

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ABSTRACT

Logistic regression is useful tool of statistical analysis used in various field of research, especially to classify units according their parameters, or to estimate chance of event occurrence. On the economic field this method is usually used to estimate bankruptcy and credit models, or to predict consumers' behavior. Objective of the proposed paper is to present application of the logistic regression analysis to assess credit of the farm. This paper can be used also as guide through the process of modelling, model verification and interpretation of its results. Data used to estimate logistic regression were individual farm data cover large farms from the database of the Ministry of Agriculture and Rural Development in Slovakia for the period 2009 to 2013. 4000 observations were used to estimate final model, and 427 observations were used as the sample for the model verification. Then, logistic regression model was estimated and verified. From the initial set of 13 variables were selected 7 significant variables to final model. Factor which increased probability of getting loan the most significantly was proportion of loans, on the other hand, factor which decreased this probability the most was the proportion of crop production. Quality and prediction ability of the final model according to standard indicators was fair, however there could be suggested including additional variables to improve model prediction ability, and its further testing by its application on more testing samples. Paper offers better insight into process of logistic regression application, and suggests ways of current topic further developing.

KEYWORDS: logistic regression, credit model, chance model, logit

JEL CLASSIFICATION: C01, C10, C18, B23, B26

INTRODUCTION

In the 19th century Sir Francis Galton introduced to the world very popular statistical method called regression. Method is preferred in various areas of investigation, but in the first, it was used in genetics. Galton estimated regression model for the prediction of height of the child based on the data of parents. He found, that the difference between high of child and average

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high in the population is proportional to his parent's deviation from typical people in the population.

Classical linear econometric model was not appropriate in case, when dependent variable had binary or categorical character. The main reason was, that probability does not have linear nature, and that predicted values should fall in the interval between 0 and 1. These assumptions were not met in case of linear regression model. The need for a new method, which will satisfy these two conditions led to development of the logistic regression model.

This method has been already applied in various fields of research. For example, it has been applied in healthcare research, social, geographic, ecological, physical research and in the field of economics. Presented paper is focused on application of logistic regression in the field of economics and finance, especially in assessing the credibility or bankruptcy with the use of logistic regression models. Paper shows application of this method to assess farm solvency, resp. to assess a chance, that farm will get bank loan. This method has already been used in this area of research which is described in the examples below.

In France, logistics regression was used for prediction of individual bankruptcy of enterprises. Because of lack of traditional prognostic models, Jabeur [1] developed a model that includes financial ratios as the explanatory variables, and deals with correlation and use penalizing weights for the wrong data in the matrix. This model was applied to predict business failure, which was appreciated not only by bankers but also by investors in France. Suggested model allows them to predict bankruptcy in advance and helps them to avoid bad investment.

When analyzing the financial statements of corporate entities, Nikolic et.al [2] used a model of logistic regression in the prediction of the credit score. Researchers proposed corporation credit scoring model, thanks to which they can predict probability of bankruptcy in 1 year period advance. They used the test sample to verify prediction ability of estimated models. Logistic regression was evaluated as the best predictive credit scoring model from the set of suggested solutions. Their model includes eight explanatory variables that showed the best predictive performance. Analysis was conducted in the Serbian region, so the final model could be implemented in a bank that operates in the same area, or in the region of South Eastern Europe. In other regions, it is possible to build an analogic model based on a similar technique.

Serener [3] analyzed the use of internet banking. They suggested model to estimate the probability of using Internet banking by customers. Factors considered as the explanatory variables were age, gender, income, marital status, education, occupation, experience with online shopping. Results of their analysis suggests, that clients aged 56 - 65 are less likely to use these services than respondents aged 18 - 25. Persons in marriage are less likely to use Internet banking than single respondents. With the increase in the individual's income, the likelihood of using internet banking also increased. Similarly, university graduates have a higher chance to use internet banking than lower educated people. Respondents who already had experience with online shopping showed tendency to use the internet banking. Among the professions considered in research, internet banking is most likely used by bank staff. Further application of the logistic regression can be expanded to marketing support. Appropriate sales support could be focused on the specific group of people suggested by logistic regression results.

There could be mentioned more examples of logistic regression applications. For its valuable properties and availability of software solutions it may be applied in many field of research. Presented paper is focused on the application of logistics regression in the field of finance to assess farm credibility and to determine factors which can influence it. Proposed paper describes the whole procedure of model specification, estimation, verification and interpretation of the results. Therefore, presented paper can be also used as the application instructions to logistic regression.

MATERIAL AND METHODS

Data used to present the logistic regression model were farm data over the period 2009 to 2013 divided in two groups based on the criterion, whether the farm did receive a bank loan or not. Individual farm data cover large farms from the database of the Ministry of Agriculture and Rural Development in Slovakia (Information letters of farms with double entry accounting). Dataset was divided into two parts. First part was used to estimate logistic regression model and included 4000 observations. Second part of dataset was used for verification of model prediction ability and included 427 observations. Model was estimated and verified using R Cran software package.

Model

If the Y is a binary response variable equal to 1 if attribute is present and 0 if it is not present in observation. If $x = (x_1, x_2, x_3, \ldots, x_k)$ is a set of explanatory variables which can be discrete, continuous or a combination. First, 13 variables were considered as exogenous factors in the model. After backward elimination and model selection process were left following 7 variables in the final model:
Debt - firm debt, ploan - proportion of loans, Ebitda, size - size of firm, revpha - revenue per ha, own - number of owners and pprv - proportion of crop production.

Logistic regression model presents conditional probabilities (log odds) through a linear function of the predictors expressed as:

$$\ln\left(\frac{P(y_i = 1)}{P(y_i = 0)}\right) = \beta_0 + x_i^T\beta = I_i \tag{1}$$

Where $\beta = (\beta_1, \beta_2, \ldots, \beta_k)^T$ is the estimated vector of k predictor coefficients. Vector of parameters β is estimated using maximum likelihood method. Following likelihood function is maximized:

$$\ln[L(\beta)] = \sum_{i=1}^{n}\left\{y_i \ln\left[\frac{exp(I_i)}{1+exp(I_i)}\right] + (1-y_i)\ln\left[\frac{1}{1+\exp(I_i)}\right]\right\} = \sum_{i=1}^{n}(y_i I_i - \ln[1+\exp(I_i)]) \tag{2}$$

Then predicted probability can be expressed as follows:

$$F_i(I_i) = P(y_i = 1) = \frac{\exp(I_i)}{1 + \exp(I_i)} \tag{3}$$

In case of logistic regression is no more necessary to hold the assumptions of classical linear econometric model based on ordinary least square. Linear relationship between dependent and independent variables, explained variables and error term does not need to be normally distributed. Logistic regression also does not need variances to be homoscedastic and can handle also nominal or ordinal data as explanatory variables.

Model evaluation and diagnostics

Likelihood ratio test

This method compares the likelihood of the data under the full model against the likelihood of the data under a model with fewer predictors.

Let L1 is the maximum value of the likelihood without the additional assumption (unrestricted model) and L2 the maximum value of the likelihood when the parameters are restricted (reduced model). Calculate the ratio:

$$\lambda = \frac{L_2}{L_1} \tag{4}$$

Result is always between 0 and 1. Then test statistics can be calculated:

$$\chi^2 = -2\ln\lambda \tag{5}$$

And it follows Chi-square distribution with k degrees of freedom (k-number of restriction in the second model). H_0 holds that the reduced model is appropriate, and p-value for the overall model fit statistic less than 0.05 would suggest rejecting the null hypothesis. It provides evidence in favor of current model.

Pseudo R2

Usual R2 cannot be applied in case of logistic regression, due to binary nature of dependent variable. Estimated logistic function does not fit the real observations which can take values of 0 or 1. Therefore, it was necessary to introduce indicator, which better reflect the nature of binary data. McFadden Pseudo R2 can be calculated using following equation:

$$R^2_{McFadden} = 1 - \frac{Ln(L_c)}{Ln(L_{intercept})} \tag{6}$$

Lc – refers to the maximized likelihood value from the current fitted model and Lintercept refers to likelihood value from the model with only the intercept and no covariates. If comparing two models on the same data, McFadden's would be higher for the model with the higher likelihood.

Classification Rate

Classification rate is calculated comparing predicted probabilities with real results on the control group of data. If P(Y=1|X) > 0.5 then predicted Y=1 if P(Y=1|X) < 0.5 then predicted Y is 0. In some other application could be considered different boundaries to assess the model. The higher classification rate means better model.

ROC curve

ROC curve and Area under the curve (AUC) present typical performance indicator for binary classifier. An area of 1 represents a perfect test; an area of 0.5 represents a worthless test. A rough guide for classifying the accuracy of a diagnostic test is the traditional academic point system: 0.90 - 1 = excellent, 0.80 - 0.90 = good, 0.70 - 0.80 = fair, 0.60 - 0.70 = poor, 0.50 - 0.60 = failed.

RESULTS AND DISCUSSION

First, the dataset was divided into two parts, first part including 4000 observations was used to modelling process (variables selection, model estimation and verification) and second part including 427 observations was used to test prediction ability of the final model. Then, process of variables selection was applied; from total number of 12 variables were selected 7, as factors which significantly influence the probability of getting loan. These variables were used to estimate final logistic regression model. Estimated coefficients and their statistics are shown in Table 1.

Table 1 Estimated logistic regression model

Variable	Estimate	Std. Error	z value	Pr(>|z|)	Significance
Intercept	-7.958e-01	1.163e-01	-6.845	7.62e-12	***
Debt	1.356e+00	1.443e-01	9.398	< 2e-16	***
Ploan	5.740e+00	2.611e-01	21.983	< 2e-16	***
EBITDA	-8.289e-07	1.120e-07	-7.399	1.37e-13	***
Size	4.582e-04	5.480e-05	8.362	< 2e-16	***
Revpha	-7.086e-06	2.683e-06	-2.641	0.008261	**
Own	1.057e-03	3.009e-04	3.512	0.000445	***
Pprv	-5.175e-01	1.121e-01	-4.615	3.92e-06	***

Sig. Codes *** 0.001 ** 0.01 *0.05

Source: Authors

Estimated coefficients in this case does not mean direct influence of the explanatory variables to probability of getting loan, but their influence on the log odds ratio of getting loan. Therefore, it is difficult to evaluate influence of each variable on dependent variable by coefficient value; on the other side, it is possible to evaluate significance of each variable according to their p-values in the final model. Significance is indicated in last column of table 1. Most of the factors are significant at alfa = 0.001, only variable revenues per ha is significant at the alfa = 0.01. Overall significance of the model evaluated by the probability of likelihood ratio test is close to 0, which suggest significant model. The same test was applied also to evaluate significance of individual variables. In this case was compared likelihood of model with only intercept (without explanatory variables) and model including additional explanatory variable. If the adding of explanatory variable improved prediction ability of the model, variable is denoted as significant. This procedure differs from the method used in coefficients table above.

Table 2 Evaluation of variables significance

	Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)	Significance
NULL			3999	4989.0		
Debt	1	104.05	3998	4884.9	< 2.2e-16	***
Ploan	1	873.32	3997	4011.6	< 2.2e-16	***
EBITDA	1	09.6	3996	4002.6	0.002616	**
Size	1	118.93	3995	3883.6	< 2.2e-16	***
Revpha	1	6.49	3994	3877.1	0.010839	*
Own	1	18.32	3993	3858.8	1.865e-05	***
Pprv	1	21.50	3992	3837.3	3.536e-06	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Source: Authors

Overall model quality can be evaluated also by using ROC curve and its area under the curve. The curve shows relationship between true positive rate (correctly predicted loans) and false positive rate (when model predicted loan in case when it did not occur). ROC curve is shown on the Figure 1. In this case it can be noticed, that model is slightly in favor of true positive prediction. From ROC curve is derived another important indicator AUC, which denotes Area under curve. AUC indicator of presented model is equal to 0.69. According common rules this means that model is close to fair quality.

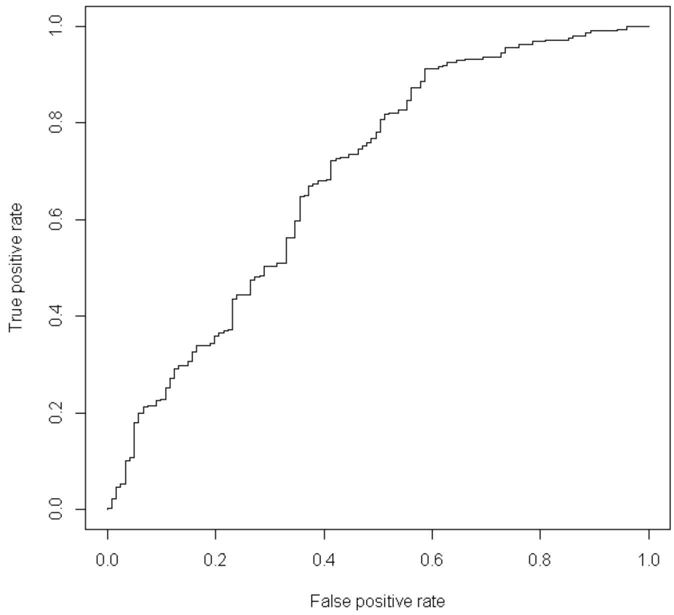

Figure 1 ROC curve

Quality of model can be assessed also using McFadden R square, which value is equal to 0.23. In case of logistic regression R square values are not as important as in case of classical linear regression model. Due to character of dependent variable, their values are usually low.

Real values of dependent variable are equal to 0 or 1; on the other side predicted values are decimal numbers varying between 0 and 1. Better indicator of model quality, which compares predicted and real values, in this case is number of correct predictions. If the predicted value is higher than 0.5, it means predicted 1 (firm will get loan), otherwise 0 (firm will not get loan). Presented model successfully predicted 60% of cases. It suggests fair quality of the model, on the other side the model should be reconsidered to increase its prediction ability. Number of correct predictions and ROC curve were estimated by application of the model on the training set of data (427 observations).

As it was already mentioned above, coefficients in estimated models does not indicate direct influence of explanatory variables on the probability of getting loan, but their influence on the log odds ratio of getting loan. Estimated function is not linear; influence of each explanatory variable on the final probability value will depend on the value of X. It will be different between low and high value of X. To describe the real influence of independent variable to probability it would be necessary to describe influence of change in low, medium and high values of this variable. Much easier is to describe constant effect on the odds ratio. Therefore, to derive influence of each variable on the probability of getting loan, it is necessary to exponentiate each coefficient value to get odds ratio for each variable. Odds ratios of explanatory variables with confidence intervals are shown in the table.

Table 3 Odds ratios with confidence intervals

Variable	OR	2.5 %	97.5 %
Debt	3.88	2.93	5.16
Ploan	310.92	188.09	523.59
EBITDA	0.999999	0.999999	0.999999
Size	1.000458	1.000352	1.000567
Revpha	0.999993	0.999987	0.999998
Own	1.001057	1.000490	1.001673
Pprv	0.60	0.48	0.74

Source: Authors

Odds ratios higher than one mean positive effect of explanatory variable on chance of getting loan, odds ratio value lower than one mean negative effect on chance of getting loan. Significantly positive or negative influence on chance of getting loan means only odds ratio without 1 in its confidence interval (1 means indifferent to positive or negative influence). Confidence intervals present range of values where the odds ratio should be with 95% probability. Highest positive influence on chance of getting loan has proportion of loans, then debt, and only slightly positive influence farm size and number of owners. For example, if the number of owners increases by 1, the chance of getting loan increases by 0.1%. The rest of the odds ratios could be interpreted analogically. On the other side, highest negative influence on chance of getting loan has proportion of crop production and slightly negative influence EBITDA and revenue per ha. In case, when it is necessary to assess credibility of farm, parameters of the individual farm can be filled into model and probability of getting loan can be estimated to assess its solvency.

CONCLUSIONS

The main objective of presented paper was to demonstrate application of logistic regression in case, when it is necessary to classify units according their attributes. In the economic field, this method is usually used for insurance and bank purpose, bonity models, bankruptcy models or in targeted marketing. Application in this paper was evaluation of farms solvency to get loan. First, from the set of 13 explanatory variables it was selected the set of 7 variables as the exogenous variables to final model. All the variables and overall model were considered significant according usual statistical procedures. Model quality was assessed by its application on test set of data and calculating McFadden R square, ROC curve and area under curve, and number of correct predictions. These indicators considered model as fair (60% of correct predictions). However, if the model should be used in practice, another explanatory variable should be considered to improve its prediction ability. It should be also applied on more testing data groups to get cross validated verification and get better insight into its prediction accuracy. When interpreting results of the model it should be noted, that estimated coefficients does not mean direct influence of explanatory variables on final probability of getting loan, but on the log odds ratio of getting loan. Probability in this kind of model is non-linear, therefore there is no constant influence of the variable, but the change in the probability depends on the specific value of explanatory variable and is different between low, medium and high values. This is the reason, why in logistic models is usually estimated and interpreted odds ratio (exponentiated value of coefficients) to assess variables influence. In presented example the highest positive influence on chance of getting farm loan had variable debt and proportion of loans, which suggest, that if the farm have already successfully got loan in the past, it probably would get also new loan. Highest negative influence on the chance of getting loan had proportion of crop production. The output of the model is probability of getting bank loan. In conclusion, producing final model would require further modification and cross validation, but it was presented that logistic regression offers efficient tool for data classification. This method can be applied in various fields of research and help to get better insight into process where outcome is categorical variable.

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Original Paper

Mathematical and econometric methods in price transmission analysis

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ABSTRACT

Price transmission analysis has gained a lot of attention in recent years. As the research in this area evolved, authors came with newer and newer methods and models with the aim to properly quantify the price transmission mechanism. Generally prices can be transmitted horizontally, across different locations, or vertically, along the vertical supply chain. In our paper, we focus on horizontal price transmission analysis. The aim of this paper is to bring a brief overview of methods applied in horizontal price transmission. By applying different methods we try to answer the question, which of the methods would be more appropriate to analyze horizontal price transmission in case of Slovak milk market. We apply the time series analysis, particularly the Engle - Granger methodology, Johansen cointegration test and threshold cointegration. Monthly data for producer prices of raw milk are used, covering the period from January 2005 to June 2017. Our results confirm that the asymmetric threshold cointegration model describes our data better than the other two models.

KEYWORDS: cointegration analysis, error correction models

JEL CLASSIFICATION: C13, C32, Q11

INTRODUCTION

The process of transmission of price changes in the agro-food sector, at the vertical or horizontal level, has become a widely studied area in the agricultural economy in the last years.

At the end of the last century attention of researchers has increasingly begun to focus on examining the price relationships within the food supply chain. Along with this, different econometric techniques started to develop. One of the first methods used to study price transmission were simple regression and correlation analysis, followed by dynamic regression models, represented by Vector Autoregression (VAR) model. VAR model formed a basis for estimating the Impulse Response Functions (IRF) as well as cointegration techniques and Vector Error Correction (VEC) model. VEC models were later improved to describe nonlinear and asymmetric patterns in price transmission and threshold cointegration

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models and asymmetric cointegration models were introduced. Meyer and von Cramon-Taubadel [19] and Frey and Manera [11] presented the comprehensive overview of estimating and testing approaches for asymmetric price transmission.

These approaches have mostly focused on the area of agriculture and food processing sector, and one of the first applications of these methods to the agriculture sector was presented by von Cramon-Taubadel [7] who examined how the price changes were transmitted along the pork meat supply chain in Germany. Many recent studies have followed von Cramon-Taubadel's approach and tested the presence of unit roots and cointegration (Zeng and Gould [27]; Acosta and Valdés [2]; Acosta et al. [3]; Bor et al. [5]; Rezitis and Reziti [23]; Rojas et al. [24]; Luoma et al. [17]; Abdulai [1]; Miller and Hayenga [20]). Their main goal is to prove the existence of asymmetries movement of prices with application of more and more advanced statistical methods.

While many studies focused on vertical price transmission along the food supply chain, prices may be transmitted horizontally across different areas (spatial price transmission) or across different commodities (cross commodity price transmission). The first type of horizontal price transmission is based on the assumption of integrated markets and the Law of One Price. The other one, cross commodity price transmission is driven by substitutability or complementarity between different commodities.

The analysis of horizontal transmission [3, 6, 15] is considered as a common tool in market integration analysis [14]. If the locations are integrated, the transmission of price shocks will be perfect and the price of a product should be freely transmitted between trading partners to attain an integrated and efficient market [4]. "Without integration, there is no mechanism by which excess demand changes may be transferred spatially so that no price shocks are shared between non-integrated locations" [18].

According to Goodwin and Piggott [13] or Muratori and Fricke [21] as basic mechanisms of integration are considered the spatial trade, arbitrage and hypothesis related to the Law of One Price (LOP). The arbitrage uses the advantage of a price difference between two or more separated markets. In an equilibrium concept, in a well-functioning market, the price shocks occurring in one market, cause responses in other markets [25].

$$P_j - P_i \leq r_{ij} \tag{1}$$

where r_{ij} are the costs of moving of products from market i to market j. These costs contain all relevant costs of arranging the transaction between two markets [13]. Also, Goodwin and Harper [12] or Lo and Zivot [16] identify the significant transaction costs in market integration.

In this study, we focus on horizontal price transmission analysis. Our aim is to bring a brief overview of methods applied in horizontal price transmission and by applying different methods we try to answer the question, which of the methods would be more appropriate to analyze horizontal price transmission in case of Slovak milk market. We apply the time series analysis, particularly the Engle - Granger methodology, Johansen cointegration test and threshold cointegration analysis. Monthly data for producer prices of raw milk of Slovakia and the European Union are used, covering the period from January 2005 to June 2017.

MATERIAL AND METHODS

Monthly price data (from 2005 till 2017) are used to test the price movements from European toward Slovak market and vice versa. The individual price series of raw milk are illustrated in Figure 1 and the descriptive statistics of examined variables are in Table 1. We applied the logarithmic transformation of variables to interpret the results in the percentage change.

Before the examining the relationship between variables, the stationarity of selected data series is needed to be tested. A simple first order autoregressive process can be written as:

$$Y_t = \mu_0 + \mu_{1t} + \alpha Y_{t-1} + \varepsilon_t \qquad (2)$$

where Y_t is the stochastic process, μ_0, μ_1, α are parameters, t is the time period, ε_t is a random error term (with white noise properties of zero mean, constant variance and the zero covariance). To test the stationarity of time series we used the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test [22]. PP test was conducted because the ADF test loses its power for sufficiently large values of k, the number of lags. It includes an automatic correction to the Dickey-Fuller process for autocorrelated residuals. The number of lagged difference terms to include is often determined empirically; the idea is to include enough terms so that the error term in the test is serially uncorrelated. The number of lags of a dependent variable is determined by the Akaike Information Criterion (AIC).

Engle - Granger test of cointegration and error correction model (ECM)

The procedure to test co-integration was developed by Engle and Granger [10]. It involves the estimation static cointegrating regression, using OLS, and applying unit root tests (the ADF and Phillips-Perron to the estimated residuals) in order to test the null hypothesis of no cointegration.

$$y_t = \alpha + \beta x_t + \upsilon_t \qquad (3)$$

if y and x are integrated of the first order I(1) then the residual t from the regression of those series would also be I(1), unless they are cointegrated. Thus if the residuals are distributed I(1) we accept the null hypothesis of no cointegration, but if the residuals are I(0) then we reject the null and accept that y and x are cointegrated.

Error correction model

If the null of absence of cointegration is rejected in the Engle and Granger procedure [10], the adjustment to the long-run equilibrium can be modelled through an error correction model (ECM) specification, such as:

$$\Delta P_{1t} = \alpha + \sum_{j=1}^{k} \beta_j \Delta P_{2t-j+1} + \gamma v_{t-1} + e_t \qquad (4)$$

where P_{1t} and P_{2t} are two related prices, indicator Δ is the difference indicator, β and γ are the estimated coefficients and υ are the deviations from the long run equilibrium [26].

Johansen cointegration test

As a second test of cointegration we employ the Johansen approach to test for cointegration. The Johansen approach is based on a vector autoregressive model and reformulates it into a vector error correction model:

$$Z_t = A_1 Z_{t-1} + \ldots + A_k Z_{t-k} + \varepsilon_t \quad (5)$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-k} + \varepsilon_t \quad (6)$$

where Z_t is a vector of non-stationary variables, A are different matrices of parameters, t is time subscript, k is the number of lags and ε_t is the error term assumed to follow i.i.d. process with a zero mean and normally distributed $N(0, \sigma 2)$ error structure. The estimates of Γ_i measure the short-run adjustment to changes in the endogenous variables, while Π contains information on the long-run cointegrating relationships between variables in the model.

Threshold cointegration approach

The above cointegration tests assume symmetric price transmission. In order to capture asymmetric movements in the residuals, Enders and Granger [8] and Enders and Siklos [9] propose to use threshold cointegration approach. Assuming the long run relationship between two nonstationary variables X and Y:

$$Y_t = \lambda_0 + \lambda_1 X_t + \mu_t \quad (7)$$

where μ is the error term. Engle and Granger (1987) show, that cointegration exists if the null hypothesis $\rho = 0$ is rejected in:

$$\Delta \mu_t = \rho \mu_{t-1} + \xi_t \quad (8)$$

where ξ is the error term for the residuals. Adjustment of the series of residuals expressed in $\rho \mu_{t-1}$ would be symmetric. To capture the assymmetry in adjustment process, a two-regime threshold cointegration approach should be used:

$$\Delta \mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t) \rho_2 \mu_{t-1} + \xi_t \quad (9)$$

where I_t is the Heaviside indicator $I_t = 1$ *if* $\mu_{t-1} \geq \tau$ or $I_t = 0$ *if* $\mu_{t-1} < \tau$. If μ_{t-1} is bigger than the threshold τ, then adjustment is at the rate ρ_1. If μ_{t-1} is smaller than the threshold τ, adjustment is shown in ρ_2. If $\rho_1 = \rho_2$, then the adjustment process is symmetric.

If the null hypothesis $\rho_1 = \rho_2 = 0$ is rejected, then X and Y are cointegrated and the following TAR model is estimated:

$$\Delta Y_t = \theta_Y + \delta_Y^+ E_{t-1}^+ + \delta_Y^- E_{t-1}^- + \sum_{j=1}^{J} \alpha_{Yj}^+ \Delta Y_{t-j}^+ + \sum_{j=1}^{J} \alpha_{Yj}^- \Delta Y_{t-j}^- + \sum_{j=1}^{J} \beta_{Yj}^+ \Delta X_{t-j}^+ + \sum_{j=1}^{J} \beta_{Yj}^- \Delta X_{t-j}^- + \upsilon_{Yt} \quad (10)$$

where ΔY_t and ΔX_t are dependent and independent variables in their first differences, E is the error correction term, δ represents the speed of adjustment coefficients of ΔY_t if Y_{t-1} is above and below its long-run equilibrium, θ, δ, α and β are coefficients and υ is the error term, t is time subscript and j is the number of lags.

Two error correction terms are defined as:

$$E_{t-1}^{+} = I_t \mu_{t-1} \quad (11)$$

$$E_{t-1}^{-} = (1 - I_t)\mu_{t-1} \quad (12)$$

Enders and Granger [8] and Enders and Siklos [9] proposed also a model for cointegration, known as a momentum threshold autoregressive model. The term "momentum" describes the rate of acceleration of prices and takes into account steep variations in the residuals; it is especially valuable when the adjustment is believed to exhibit more momentum in one direction than the other. Heaviside Indicator in this case is $I_t = 1$ *if* $\Delta\mu_{t-1} \geq \tau$ or $I_t = 0$ *if* $\Delta\mu_{t-1} < \tau$. To summarize, four asymmetric models are considered in our study. They are threshold autoregression model with threshold value equal to zero; threshold autoregression model with threshold value estimated (consistent threshold autoregression model); momentum threshold autoregression model with threshold value equal to zero; and consistent momentum threshold autoregression model with threshold value estimated. A model with the lowest AIC and BIC (Bayesian information criterion) will be used.

RESULTS AND DISCUSSION

The development of raw milk prices in Slovakia follows similar trend and patterns as the EU prices, suggesting there may be a long-term relationship present among the prices (Figure 1). The most significant increases in prices are recorded over the period 2007 - 2008 and in the year 2014. In 2008 - 2009 milk prices have fallen from their historical maximum by approximately 40 - 50%. This development was caused by the reduction in demand for dairy products due to economic recession and surplus of supply in international markets. As a result, stocks of some milk products as butter and milk powder increased, particularly in the US and EU countries. This situation had very damaging effects on the dairy sector, with some countries starting to rethink their long-term strategies. These strategies and also mitigation of economic crises caused the milk prices to increase and climb up to their maximum in 2013. High milk prices in 2013 reflected in lower demand and increased milk production in China together with the embargo of the Russian Federation led to the decrease in milk prices again.

As seen from the Figure 1, the development of price series suggests there may exist a long run relationship between the EU and Slovak price series. Therefore we examined the extent to which increases in the international price of raw milk in the European Union have been transmitted to domestic Slovak prices.

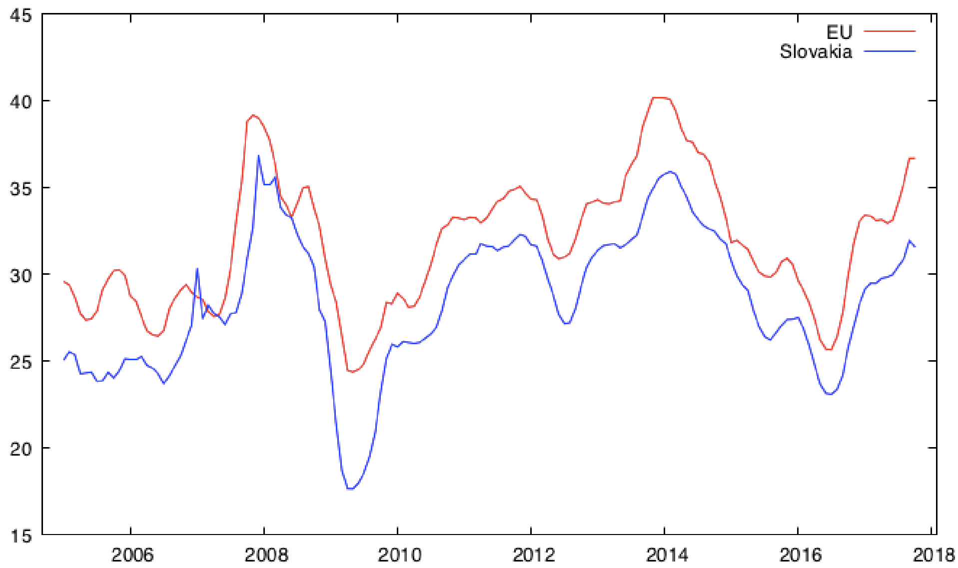

Figure 1 Historical Price Series of Cow's Raw Milk in EURO/100 kg
Source: Milk Market Observatory, European Commission

Table 1 Descriptive statistics

	Observations	**Mean**	**Median**	**S.D.**	**Min.**	**Max.**
Slovakia	154	28.50	28.27	4.062	17.67	36.88
European Union	154	31.87	31.86	3.85	24.39	40.21

Source: own calculations

Before examining the cointegration relationship between the variables, it was essential to test them for unit root and identify the order of stationarity. To check the stationarity we used the augmented Dickey-Fuller and Phillips-Perron tests (Table 2). Each variable was found to be non-stationary and also integrated of the first order, I(1). In the other words, the variables follow a random walk, but the first difference is stationary, I(0).

Table 2 Dickey-Fuller and Philips-Perron tests

Variables	**ADF test**		**PP test**	
	level form	**1st diff.**	**level form**	**1st diff**
European Union	0.37	0.00***	0.29	0.00***
	(-2.42)	(-6.69)	(-2.57)	(-5.16)
Slovakia	0.16	0.01***	0.26	0.00***
	(-2.92)	(-4.05)	(-2.65)	(-6.90)

Source: own calculations. Note: *, **, *** refers to the significance at 10%, 5%, 1% level

Engle - Granger cointegration test consists of testing the stationary of residuals form cointegration regression. The results (in Table 3) indicate that there is a cointegration relationship present between the producer prices of cow's milk in Slovakia and the European Union in the long-run. In the next step, the error correction model was set up (Table 3). It allows estimating parameters, determining the speed of adjustment to deviations from the long-term equilibrium (error correction term). Error correction term indicates the rate of adaptation (speed) of domestic prices to potential price shocks. The value needs to be negative (to ensure the variable leads to restore back to equilibrium) and significant. Since the data in the analysis are used in the logarithm form, the coefficient of error correction model also indicates the approximate impact of price change of this commodity in EU to Slovak (the short elasticity of price transmission).

Table 3 Engle – Granger cointegration test and error correction model

	Coefficient	T-Stat	p-value
Cointegration test	1.139	-4.301	0.000
Error correction model (EU → SK)			
Error correction term	-0.144	-3.796	0.000
Long Run Elasticities	0.737	9.092	0.000

Source: own calculations. Note: Crit. Values: -4.3266***, -3.7809**, -3.4959*
(*, **, *** refers to the significance at 10%, 5%, 1% level)

Table 4 Johansen cointegration test and vector error correction model

Cointegration test	Trace Stat	Max Stat
Maximum rank 0	20.289	19.883
Maximum rank 1	0.405***	0.405***
VEC Model (EU → SK)		
Error correction term	-0.086**	
Long Run Elasticities	1.302***	

Source: own calculations. Note: *, **, *** refers to the significance at 10%, 5%, 1% level

Similar results of cointegration were obtained by Johansen cointegration test. The test results indicated the presence of cointegrating relationship between the variables, the error correction term in the VEC model is negative and statistically significant. As the last step, we set up a threshold cointegration model that allows asymmetric price transmission (APT) of Slovak prices to price shocks in the EU.

Table 5 Consistent momentum threshold autoregression model (cMTAR)

Model	Threshold	Lags	H_0: no cointegration	H_0: no APT
cMTAR	0.017	3	19.399*** [0.000]	7.408*** [0.007]
	Positive		Negative	
Error correction term	-0.376***		-0.096*	
Long run elasticities	0.426***		0.191***	

Source: own calculations. Note: *, **, *** refers to the significance at 10%, 5%, 1% level

The theory does not guide us in the exact model specification and therefore in this paper, we used four different threshold models: threshold autoregression model, consistent threshold autoregression model, momentum threshold autoregression model, and consistent momentum threshold autoregression model. We report the results for consistent momentum threshold autoregression model with the lowest AIC and BIC. Estimated models show, that the prices are cointegrated with threshold adjustment.

From the results (Table 5) it follows that there is a strong evidence of asymmetry for Slovak and the EU price of milk. In other words, Slovak prices react differently to rise and decline in the EU prices. The results also indicate that the asymmetric threshold cointegration model describes our data better than the other two models.

CONCLUSIONS

This study analyses the relationship between raw cow's milk markets in Slovakia and in the European Union. We aim to bring a brief overview of methods applied in horizontal price transmission and to assess the linkage and patterns between the prices of raw cow's milk. To clarify the relationships between the prices in the markets we apply different methods starting with the unit root tests, Engle – Granger cointegration test, Johansen cointegration test and the test for asymmetric price transmission and we set up appropriate error correction models. We applied monthly data covering the period from January 2005 to June 2017. All applied cointegration tests confirmed the price pairs are cointegrated and there exists a long-run relationship between variables. Our results also indicate that the prices are cointegrated with threshold adjustment and there is a strong evidence of asymmetry for Slovak and the EU price of milk. This brings us to the conclusion that the asymmetric threshold cointegration model describes our data better than the other two models.

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Original Paper

Opportunities of using the forecasting model on an example of time series analysis

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ABSTRACT

Time series analysis is an integral part of every empirical investigation which aims at describing and modeling the evolution over time of a variable or a set of variables in a statistically coherent way. In the paper we have used Brown triple exponential smoothing model for estimating the trend as well as a forecast of a given time series of the share of renewable energy sources on gross inland energy consumption on a example of the Slovakia and EU average. According to the model, Slovak Republic might reach the share of the renewables on gross inland energy consumption till to 2020 with anticipated share of 16.25% (90%CI: 13.98: 18.93) and in case of the EU-28 with anticipated share of 15.94% (90% CI: 13.08: 18.08) failing short of expected 20% share set as a overall target.

KEYWORDS: Brown model, renewable energy sources, gross inland consumption, energy policy

JEL CLASSIFICATION: C53

INTRODUCTION

One of the biggest challenges of energy sector nowadays is to provide energy supply in a sustainable and eco-friendly way in the long-term perspective. In order to assure sustainability and durability of our energy sources and keeping in mind the environmental impact of energy production, it is necessary to divert our interest from energy sources that will eventually be depleted (such as fossil fuels) and concentrate on renewable energy sources (RES).

The European Union is one of the largest energy consumers, as well as one of the largest greenhouse gas (GHG) emitters in the world, which calls for a common strategy in the energy sector. The Europe 2020 is the key strategy of the EU aiming at enhancing the economic growth of the EU over the years 2010-2020. This strategy involved energy and climate policy including the so called 20/20/20 targets, namely reduction of greenhouse gas emissions (by 20%), the increase of RES share (to 20%) and the increase of energy efficiency, thus, saving up to 20% in the energy consumption [4]. The strategy 2020 was preceded by Communication

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"An Energy Policy for Europe", which outlines developments in the energy sector by 2010 as well as the 2020 targets. Subsequently, in April 2009, the European Council adopted the Directive 2009/28/ES , which is an important element of climate change commitments:

- reduce greenhouse gas emissions by 20% by 2020 compared to 1990,
- increasing the share of renewable energy sources to 20% by 2020,
- achieving a share of 10% of renewable energy in transport by 2020,
- achieving 20% energy savings compared to projection by 2020 [2].

The Directive 2009/28/EC set mandatory national RES targets for Member states. Taking into consideration the RES potential of each Member State and respecting its energy mix, it was decided that the goal could vary, apart from the 10% for transport that was set as an equal target for all. According to the Directive, each Member State would have to adopt a national Renewable Energy (RE) Action Plan, including national targets for the share of RES consumed in transport, electricity, heating and cooling until 2020, notifying their national RE Action Plans to the Commission by 30 June 2010 [5]. Most of the Member states are on the track to meeting, their RES criteria set out by their national plans [2], and almost half of the Member States (Austria, Bulgaria, Czech Republic, Denmark, Germany, Greece, Spain, France, Lithuania, Malta, Netherlands, Slovenia and Sweden) even planned to exceed their own targets [4]. The 2020 targets vary from 10% for Malta up to 49% for Sweden [2]. According to Eurostat, the leaders in RES share are Sweden (53,9%), Finland (39,3%) and Latvia (37.6%).

The 2020 Energy Strategy defines the EU's energy priorities between 2010 and 2020. It aims to:

- reduce greenhouse gases by at least 20%,
- increase the share of renewable energy in the EU's energy mix to at least 20% of consumption,
- improve energy efficiency by at least 20%.

EU countries have agreed that the following objectives should be met by 2030:

- a binding EU target of at least a 40% reduction in greenhouse gas emissions by 2030, compared to 1990
- a binding target of at least 27% of renewable energy in the EU
- an energy efficiency increase of at least 27%, to be reviewed by 2020 with the potential to raise the target to 30% by 2030
- the completion of the internal energy market by reaching an electricity interconnection target of 15% between EU countries by 2030, and pushing forward important infrastructure projects [3].

Slovak Republic had joined to common European energy policy. In own program document Slovakia adopted priorities on the field of energy security, efficiency, competitiveness and sustainable development. Regarding the RES, Slovak Republic adopted framework goal to raise the use of RES relatively to gross consumption of energy from 6.7% in 2005 at least on 14% in 2020 [7].

MATERIAL AND METHODS

The main objective of the paper is to show on the possibilities of developing a forecasting model based on underlying non-linear trend for a given variable in time series analysis. For this purpose we decided to use *Brown triple exponential smoothing model* in estimating of the point forecast ex ante, determined in time $t = T$ for time $t + h$ for $h = 1,2, \dots, H$. As a underlying variable we choose an evaluation of the development of the use of the RES in Slovakia compared to the EU-28 average and assessment. Thus we would like to consider if national strategic goals regarding the share of the RES on gross inland energy consumption[1] are achievable till 2020.

The research sample consists time series-data about the share of the RES of gross inland energy consumption in percentage for Slovakia in the respective time period 1990 – 2015, thus $N = 26$. The development of the share of the RES gross inland energy consumption, we compare with the EU-28 average. Furthermore, we make a short-term forecast of the RES share till 2020 in the case of the Slovakia and also for the EU-28 average via using statistical extrapolation methods. The time-series data are obtained from Eurostat.

After consideration of various methods of extrapolation [8], [6] we have opted for *Brown triple exponential smoothing model*. For our research we use the *second order polynomial (quadratic trend).* We use adaptive estimation of the three parameters of local quadratic trends in time t, generally defined:

$$y_t = \beta_0 + \beta_1 t + \frac{1}{2}\beta_2 t^2 + \varepsilon_t \tag{1}$$

Furthermore, triple exponential smoothing is derived from the use of three exponential averages

$$S_t^{(1)} = \alpha y_t + (1-\alpha)S_{t-1} \tag{2}$$

$$S_t^{(2)} = \alpha S_t^{(1)} + (1-\alpha)S_{t-1}^{(2)} \tag{3}$$

$$S_t^{(3)} = \alpha S_t^{(1)} + (1-\alpha)S_{t-1}^{(3)} \tag{4}$$

Each parameter we estimate by use the:

$$\hat{\beta}_{0,t} = 3S_t^{(1)} - 3S_t^{(2)} + S_t^{(3)} \tag{5}$$

$$\hat{\beta}_{1,t} = \frac{\alpha}{2(1-\alpha)^2}\left[(6-5\alpha)S_t^{(1)} - (10-8\alpha)S_t^{(2)} + (4-3\alpha)S_t^{(3)}\right] \tag{6}$$

$$\hat{\beta}_{2,t} = \frac{\alpha^2}{(1-\alpha)^2}\left(S_t^{(1)} - 2S_t^{(2)} + S_t^{(3)}\right) \tag{7}$$

Where $\alpha \in \langle 0; 1 \rangle$ is smoothed constant.

Next we make the point estimation of the forecast ex ante for time $t = T$ on time $t = T + h$ for $h = 1,2, \dots, H$:

$$\hat{y}_{T+h} = \hat{\beta}_{0,T} + h\hat{\beta}_{1,T} + \frac{1}{2}h^2\hat{\beta}_{2,T} \tag{8}$$

Next, the crucial issue to determine the smoothing constant of α. According the literature, under the subjective choice we should apply the rule: if slope parameter of the time series is unstable (higher variability), than $\alpha \to 1$, if the time series is stable (lower variability) we should choose „lower“ level of α [8].

Further we should determine the confidence interval for y_{T+h} on [8], [1].

$$\hat{y}_{T+h}(T) \pm z_{1-\frac{\alpha}{2}} * d_h * MAE_T \tag{9}$$

[1] Note: **Gross inland energy consumption**, sometimes abbreviated as **gross inland consumption**, is the total energy demand of a country or region. It represents the quantity of energy necessary to satisfy inland consumption of the geographical entity under consideration

Where $\hat{y}_{T+h}(T) = \hat{\beta}_{0,T} + h\hat{\beta}_{1,T} + \frac{1}{2}h^2\hat{\beta}_{2,T}$ is point estimation in the time $t = T$ for time $t = T + h$; $z_{1-\frac{\alpha}{2}}$ is % quantil of the normal distribution $N(0; 1)$, d_h we compute following

$$d_h = 1.25 * \sqrt{\frac{1+\frac{1-\alpha}{(1+\alpha)^3}[(1+4\alpha+5\alpha^2)+2(1-\alpha)(1+3\alpha)h+2(1-\alpha)^2h^2]}{1+\frac{1-\alpha}{(1+\alpha)^3}[(1+4\alpha+5\alpha^2)+2(1-\alpha)(1+3\alpha)+2(1-\alpha)^2]}} \tag{10}$$

$$MAE_T = \sum_{t=1}^{T} \frac{|y_t - \hat{y}_{t-1}(1)|}{T} \tag{11}$$

Where α is smoothed constant.
Finaly we verify the practical convenience of the model via using the Theil decomposition of the mean square error (MSE) as following:

$$MSE = \left(\bar{\hat{y}} - \bar{y}\right)^2 + \left(s_{\hat{y}} - rs_y\right)^2 + (1 - r^2)s_{\hat{y}}^2 \tag{12}$$

For interpretation purpose we divide both sides of the equation of the MSE

$$1 = \frac{\left(\bar{\hat{y}} - \bar{y}\right)^2}{MSE} + \frac{\left(s_{\hat{y}} - rs_y\right)^2}{MSE} + \frac{(1-r^2)s_{\hat{y}}^2}{MSE} = U_m + U_R + U_D \tag{13}$$

where U_m measures the part of the MSE error caused by the distortion of the reality by the model itself. U_R measures part of the MSE error caused by the deviation of population regression function between the values y and $\hat{y}$. U_D is the result of imperfect correlation between the y and $\hat{y}$, thus it is the error with non-systematic nature [8].

For ideal forecasting model it is valid:

$$U_m = 0,\ U_R = 0,\ U_D = 1 \tag{14}$$

RESULTS AND DISCUSSION

For filling the objective of the paper, we opted for Brown triple exponential smoothing model as a main research method. For extrapolation the trend till 2020, we have used second order polynomial.

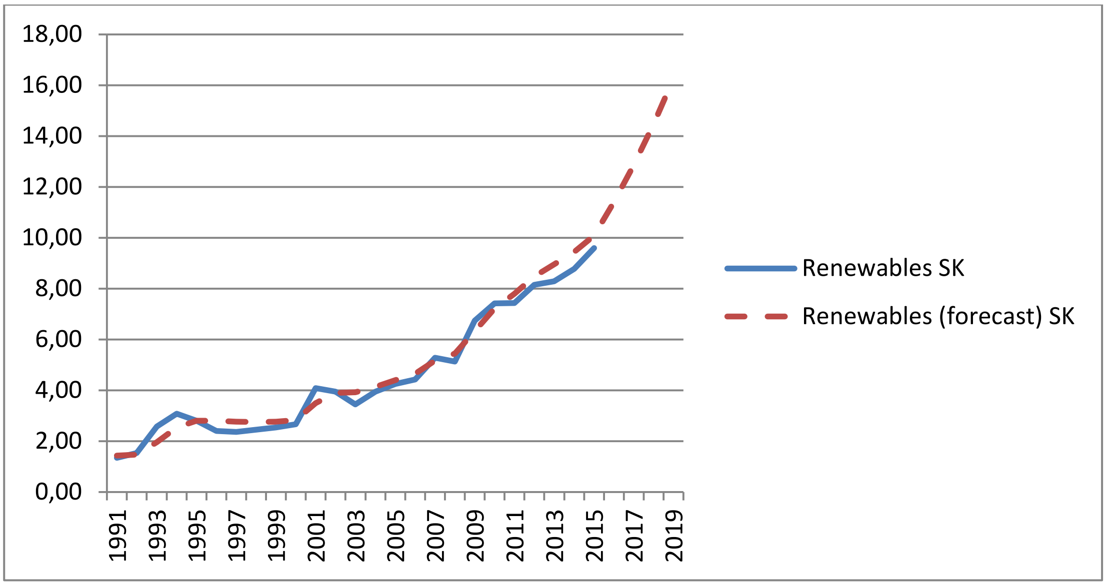

Figure 1 Extrapolation of the trend of the contribution of the RES on gross inland energy consumption in Slovakia in %
Source: Own processing in Excel, based on Eurostat data, 2017

In Figure 1 we can observe the potential development of the RES on gross inland energy consumption till 2020 in Slovakia. We can observe the steady rising trend since 2004 and there are good perspective of achieving national targets. However, the model is able to determine the value of the RES with highest confidence only one period ex ante and we can only try to determine it till 2020 with very low probability.

Table 1 Forecast ex ante for the share of the RES on gross inland energy consumption with 90% confidence interval

Period	Forecast	Lower 90.0% limit	Upper 90.0% Limit
2016	**10.08**	9.36	10.8
2017	**11.41**	10.34	12.47
2018	**12.86**	11.36	14.36
2019	**14.47**	12.49	16.45
2020	**16.25**	13.98	18.53

Source: Author

Table 1 shows forecast ex ante for the contribution of the RES on gross inland energy consumption in % in Slovakia, including 90% confidence interval. According the model the share of the RES should increase and in 2020 might stand around the 16.25% level.

Regarding the Theil decomposition of the mean square error (MSE) we have found out the following results:

$$1 = U_m + U_R + U_D = 0.041 + 0.212 + 0.746$$

Which means that according to the literature source the largest deviation of the model was caused by the non-systematic error, which is positive.The mean absolute percentage error is relatively high, becomes 37%.

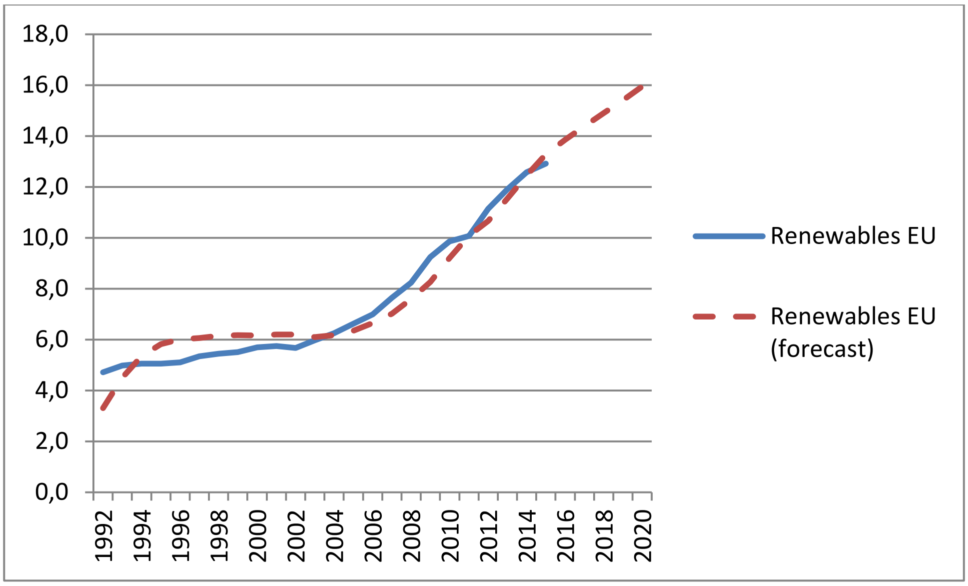

Figure 2 Extrapolation of the trend of the contribution of the RES in gross inland energy consumption in EU-28 in %
Source: Own processing in Excel, based on Eurostat data, 2017

In Figure 2 we can observe the potential development of the RES on gross inland energy consumption till 2020 in EU in average. We can observe the steady rising trend since 2002 and there are good perspective of achieving national targets of the member states, however in EU average it might fail short of expectations according the Strategy 2020. However the same is valid as in previous case, the probability of the "correct" estimation up to 2020 is diminishing.

Regarding the Theil decomposition of the mean square error (MSE) we have found out the following results:

$$1 = U_m + U_R + U_D = 0.006 + 0.016 + 0.977$$

Which means that the deviation of the model was overwhelmingly caused by the non-systematic error. The mean absolute percentage error is relatively low, becomes only 7.2%.

Table 2 Forecast ex ante for the share of the RES on gross inland energy consumption with 90% confidence interval

Period	Forecast	Lower 90.0% limit	Upper 90.0% limit
2016	**13.86**	12.84	14.87
2017	**14.39**	12.96	15.81
2018	**14.90**	13.04	16.76
2019	**15.41**	13.13	17.69
2020	**15.94**	13.08	18.8

Source: Author

Table 2 shows forecast ex ante for the contribution of the RES on gross inland energy consumption in % in EU average, including 90% confidence interval. According the model the share of the RES should increase and in 2020 might stand around the 15.94% level.

CONCLUSIONS

The aim of the paper was to point on use of forecasting model for extrapolating the trend of ex ante based on time series of development of the RES on gross inland energy consumption for the Slovakia and EU. For this purpose we opted to use of Brown triple exponential smoothing model which takes into account seasonal changes as well as trends. For underlying trend we have taken second-order polynomial model. The estimation was based on time series observation between the respective years of 1992 - 2015, with extrapolation of the trend up to 2020.

In despite of limited possibilities of the model to make an accurate prognosis up to 2020 we came to the conclusion, that the Slovakia might fill its national objective to reach 14% share of RES on gross energy consumption till 2020. In case of the EU, according the model in average the EU member states fail short of filling the strategic objective of the EU to reach 20% share of RES on gross energy consumption till 2020.

In the paper we have been dealing only with the prognostic application of the model and thus we have relaxed some assumptions like possible residual autocorrelation, which might be expected. Moreover, the model is capable with some level of the confidence to make prognosis only on such a short time periods ex ante (best $h = 1$). In case of our model for the purpose of making the estimation of the parameters, we have made before the estimation of the observing values $y_{t+n} = 1,2 \ldots, n$ based on the average coefficient of the growth of the variable, which might be in real condition unrealistic. However the prognostic applications are always connected with distortion and uncertainty, but have practical value in terms of the planning and adjusting the condition in order to achieve the desired results.

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Original Paper

The ability to apply numerical concepts in financial tasks

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ABSTRACT

Person's capability to manage financial issues has become important in today's world. Mathematical knowledge is a part of consumer and financial literacy. According the OECD study, there is a high correlation between financial literacy and mathematics. The aim of the paper is to examine the impact of gender, level of education and age on the ability to use mathematical tools to solve financial tasks in a sample of population. The examined sample consisted of 350 inhabitants of the Galanta region in the Slovak Republic. We distributed a questionnaire which consisted of 6 multiple choice questions (financial tasks focused on ability to calculate the value added tax and gross income, to use the exchange rate list, to compare the offers of insurance companies, to calculate simple interest). We found statistically significant differences in the answers of respondents according to their gender, level of education and also age. Respondents of both genders were most successful in calculation of the value added tax and in use of simple interest.

KEYWORDS: numerical skills, financial literacy, association, Slovakia

JEL CLASSIFICATION: K40, M30

INTRODUCTION

With an increasingly complex and diverse array of financial product choices available to individual small investors, today's people need to be financially literate. In personal and professional lives, active citizens need to make a number of complex financial decisions, many of which have a lasting impact on their welfare. In 2008, the OECD established the organization named the International Network on Financial Education, which is directly focused on supporting of financial education in the OECD countries. Providing students with the opportunity to work with a variety of real-world financial examples may increase the likelihood that the decisions they make will be financially sound and savvy [10]. Thus, financial education without the ability to assess financial options in the real-world and, over one's lifetime, make sound financial decisions, does not make one financially literate [10]. A growing number of countries OECD have developed and have implemented national strategies for financial education in order to improve the financial literacy of their

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populations. Financial literacy requires familiarity with fundamental concepts from economics and finance [5], practical experience, and the ability to apply the knowledge one has gained [6, 8], and the ability to perform a range of elementary mathematical computations such as calculating simple and compound interest or depreciation [1, 5, 7]. The ability to use mathematical tools to solve numerical tasks in financial decision making (numerical literacy) goes "hand to hand" with financial literacy [4]. Educational organizations on mathematics all over the world support including real-world financial problems in the curriculum and emphasize the connection between mathematics, numeracy and financial literacy [10]. On average, across the 13 OECD countries and economies, the correlation between financial literacy and mathematics was 0.83 [9]. This fact indicates that financial literacy is strongly correlated with mathematics. In 2008 the Ministry of Education, Science, Research and Sport of the Slovak Republic emphasized the importance of financial literacy development by formulating the National Standard of Financial Literacy, the initial document for incorporating financial literacy into The National Education Programs – Mathematics ISCED 2, ISCED 3A. The aim of the paper is to examine the impact of gender, level of education and age on the ability to use mathematical tools to solve financial tasks on a sample of population of the Galanta region in Slovakia.

MATERIAL AND METHODS

The examined sample consisted of 350 inhabitants of the Galanta region in Slovakia. We aimed to compare numerical ability of people of different education, age and gender. The sample structure is shown in Table 1.

Table 1 Numbers of respondents participating in research

Education	age 15-19		age 20-26		age 27-40		age 41-61		age over 62		Total
	women	*men*	*women*	*men*	*women*	*men*	*women*	*men*	*women*	*men*	
elementary	15	10	1	3	7	4	5	1	2	2	50
secondary	16	8	17	11	8	12	17	28	7	26	150
university	0	0	15	28	23	14	37	25	5	3	150
Total	31	18	33	42	38	30	59	54	14	31	350

We distributed a questionnaire which consisted of 6 multiple choice questions. Questions were formulated as financial tasks. The tasks were focused on the ability to calculate value added tax and gross income, to use the exchange rate list, to compare the offers of insurance companies and to calculate simple interest.
We created index of successfulness of respondents ($I - SR$) for each question according to selected determinants (age, education, gender). It is an average score of correct answers of respondents. The highest possible $I - SR$ value can be 1, the lowest 0. We calculated $I - SR$ by formulas:

$$I - SR = \frac{Number\ \ of\ \ correct\ \ answers\ \ of\ \ respondents}{Number\ \ of\ \ respondents\ according\ to\ determinant}$$

We used SAS software to process statistical analysis of obtained data and created contingency tables. Values in contingency tables present the frequency, expected frequency, table percentage, row percentage and column percentage according to selected determinants. The analysis of contingency tables includes chi-square tests and measures of association.

RESULTS AND DISCUSSION

The index of successfulness of respondents in each question according to selected determinants (gender, education, age) is shown in Table 2. The overall successfulness of our sample measured by $I - SR$ was 57 %. It means that, on average, each respondent answered correctly more than 3 answers out of 6.

Table 2 Index of successfulness of respondents according to selected determinants

Determinants		Questions						
		1	2	3	4	5	6	*average*
gender	women	0.47	0.69	0.64	0.54	0.35	0.78	0.58
	men	0.42	0.66	0.44	0.62	0.46	0.72	0.55
education	elementary	0.56	0.76	0.58	0.36	0.48	0.68	0.57
	secondary	0.49	0.66	0.54	0.58	0.41	0.77	0.57
	university	0.36	0.66	0.53	0.65	0.37	0.76	0.55
age	15-19	0.49	0.78	0.55	0.35	0.41	0.61	0.53
	20-26	0.40	0.60	0.55	0.65	0.51	0.81	0.59
	27-40	0.46	0.63	0.56	0.53	0.41	0.85	0.57
	41-61	0.44	0.71	0.58	0.61	0.30	0.71	0.56
	over 62	0.47	0.66	0.40	0.69	0.47	0.76	0.57

As we can see in Table 2, the differences in the index of successfulness calculated for selected determinants are minimal. The obtained values show a beneficial effect of female gender on the ability to solve financial tasks (see Table 2, Table 3). Mancebón et al. [7] according to their research claim that gender is a factor which affects the level of financial literacy. In their research they observed better scores of girls than boys. However Bhushan & Medury [2], Krechovská [4], Tóth et al. [11] described higher financial literacy of men compared to women in their research.

We wanted to find associations between gender and correct answers, therefore we created contingency tables with the SAS software. Values in Table 3 present the expected frequency, table percentage, row percentage and column percentage according to gender by questions. We can see differences in answers in Table 3. Respondents of both genders were the most successful in questions 2 and 6 (calculate value added tax, use simple interest). The first and the second questions were related to value added tax. Respondents did not know that VAT is an indirect tax (index of successfulness for the first question was only 45 %, Table 2) but they knew its value, because the index of successfulness for the second question was 67 %. Men in our research were not able to calculate correctly the gross income (question 3). On the other hand, men were more successful in solving tasks 4 (use of the exchange rate list) and 5 (comparing the offers of insurance companies). We tested an association between gender and answers (correct, wrong). Using the chi-square test we verified the differences between real and expected frequencies. The chi-square statistic is 23.73 with 11 degrees of freedom. The associated p-value is 0.014, which means that there is a significant association between gender and answers to questions. Measures of association (Phi Coefficient, Contingency Coefficient, and Cramer's V) have a value of 0.11, it is a weak association.

Table 3 Contingency table for gender and questions

Gender	Questions												Total
	1c	*2w*	*2c*	*2w*	*3c*	*3w*	*4c*	*4w*	*5c*	*5w*	*6c*	*6w*	
Men	74	101	115	60	77	98	108	67	80	95	126	49	1050
	78	97	118	57	94.5	80.5	101	74	70.5	104.5	131.5	43.5	
	3.56	4.81	5.48	2.86	3.67	4.67	5.14	3.19	3.81	4.52	6.00	2.33	50.00
	7.05	9.62	10.95	5.71	7.33	9.33	10.29	6.38	7.62	9.05	12.00	4.67	
	47.44	52.06	48.73	52.63	40.74	60.87	53.47	45.27	56.74	45.45	47.91	56.32	
Women	82	93	121	54	112	63	94	81	61	114	137	38	1050
	78	97	118	57	94.5	80.5	101	74	70.5	104.5	131.5	43.5	
	3.90	4.43	5.76	2.57	5.33	3.00	4.48	3.86	2.90	5.43	6.52	1.81	50.00
	7.81	8.86	11.52	5.14	10.67	6.00	8.95	7.71	5.81	10.86	13.05	3.62	
	52.56	47.94	51.27	47.37	59.26	39.13	46.53	54.73	43.26	54.55	52.09	43.68	
Total	156	194	236	114	189	161	202	148	141	209	263	87	2100
	7.43	9.24	11.24	5.43	9.00	7.67	9.62	7.05	6.71	9.95	12.52	4.14	100.00

c - correct answers, w - wrong answers

In the next part of the paper we present contingency tables with statistically significant associations between the observed determinants. The differences in answers of primary school, secondary school and university graduates can be seen in Table 2 and Table 4.

Table 4 Contingency table for education and questions for women

Education	Questions												Total
	1c	*2w*	*2c*	*2w*	*3c*	*3w*	*4c*	*4w*	*5c*	*5w*	*6c*	*6w*	
elementary	22	8	23	7	22	8	6	24	17	13	20	10	180
	14.06	15.94	20.74	9.26	19.2	10.8	16.11	13.89	10.46	19.54	23.49	6.51	
	2.10	0.76	2.19	0.67	2.10	0.76	0.57	2.29	1.62	1.24	1.90	0.95	17.14
	12.22	4.44	12.78	3.89	12.22	4.44	3.33	13.33	9.44	7.22	11.11	5.56	
	26.83	8.60	19.01	12.96	19.64	12.70	6.38	29.63	27.87	11.40	14.60	26.32	
secondary	32	33	40	25	41	24	29	36	19	46	50	15	390
	30.46	34.54	44.94	20.06	41.6	23.4	34.91	30.09	22.66	42.34	50.89	14.11	
	3.05	3.14	3.81	2.38	3.90	2.29	2.76	3.43	1.81	4.38	4.76	1.43	37.14
	8.21	8.46	10.26	6.41	10.51	6.15	7.44	9.23	4.87	11.79	12.82	3.85	
	39.02	35.48	33.06	46.30	36.61	38.10	30.85	44.44	31.15	40.35	36.50	39.47	
university	28	52	58	22	49	31	59	21	25	55	67	13	480
	37.49	42.51	55.31	24.69	51.2	28.8	42.97	37.03	27.89	52.11	62.63	17.37	
	2.67	4.95	5.52	2.10	4.67	2.95	5.62	2.00	2.38	5.24	6.38	1.24	45.71
	5.83	10.83	12.08	4.58	10.21	6.46	12.29	4.38	5.21	11.46	13.96	2.71	
	34.15	55.91	47.93	40.74	43.75	49.21	62.77	25.93	40.98	48.25	48.91	34.21	
Total	82	93	121	54	112	63	94	81	61	114	137	38	1050
	7.81	8.86	11.52	5.14	10.67	6.00	8.95	7.71	5.81	10.86	13.05	3.62	100.00

c - correct answers, w - wrong answers

We tested an association between the level of education and answers to questions. The chi-square statistic (a value of 26.90 with 22 DF) does not provide evidence for association between answers of respondents of both sexes and the level of education ($p = 0.2151$). When we focused only on answers of women, we found significant associations between answers and level of education. Table 4 presents the contingency table for the level of education by answers (correct, wrong) for women. The chi-square statistic (a value of 57.81 with 22 DF) provides evidence for association between answers of women and level of education ($p < 0.0001$). Measures of association (Phi Coefficient, Contingency Coefficient, and Cramer's V) have a value between 0.17 - 0.23, it is a weak association.

The differences in answers of respondents of different ages can be seen in Table 2 and Table 5. We tested an association between age and answers to questions. The *chi*-square statistic (a value of 47.13 with 44 DF) does not provide evidence for association between age and answers (p = 0.3459). But we found significant associations between gender and answers of respondents of age between 41 - 61 with university education. The chi-square statistic (a value of 40.55 with 11 DF) provides evidence for an association between gender and answers of above mentioned respondents (p < 0.0001). Measures of association (Phi Coefficient, Contingency Coefficient, and Cramer's V) have a value of 0.33, it is an intermediate association. When we focused only on answers of women, we found one significant associations between age and answers. Table 5 presents the contingency table for age between 41- 61 by answers (correct, wrong) for women. The chi-square statistic (a value of 32.31 with 11 DF) provides evidence for an association between answers of women and level of education (p = 0.0007). Measures of association (Phi Coefficient, Contingency Coefficient, and Cramer's V) have a value 0.22, it is a weak association.

Table 5 Contingency table for gender and questions for age 41-61

	Questions												
Gender	*1c*	*2w*	*2c*	*2w*	*3c*	*3w*	*4c*	*4w*	*5c*	*5w*	*6c*	*6w*	Total
Men	29	25	34	20	25	29	31	23	25	29	32	22	324
	23.89	30.11	38.23	15.77	31.06	22.94	32.97	21.03	16.25	37.75	38.23	15.77	
	4.28	3.69	5.01	2.95	3.69	4.28	4.57	3.39	3.69	4.28	4.72	3.24	47.79
	8.95	7.72	10.49	6.17	7.72	8.95	9.57	7.10	7.72	8.95	9.88	6.79	
	58.00	39.68	42.50	60.61	38.46	60.42	44.93	52.27	73.53	36.71	40.00	66.67	
Women	21	38	46	13	40	19	38	21	9	50	48	11	354
	26.11	32.89	41.77	17.23	33.94	25.06	36.03	22.98	17.75	41.25	41.77	17.23	
	3.10	5.60	6.78	1.92	5.90	2.80	5.60	3.10	1.33	7.37	7.08	1.62	52.21
	5.93	10.73	12.99	3.67	11.30	5.37	10.73	5.93	2.54	14.12	13.56	3.11	
	42.00	60.32	57.50	39.39	61.54	39.58	55.07	47.73	26.47	63.29	60.00	33.33	
Total	50	63	80	33	65	48	69	44	34	79	80	33	678
	7.37	9.29	11.80	4.87	9.59	7.08	10.18	6.49	5.01	11.65	11.80	4.87	100.00

c - correct answers, w - wrong answers

Financial literacy is being examined by a number of scientists who focus on various factors that affect it. For example the research by Bhushan & Medury [2] has shown that financial literacy level gets affected by gender, education, income, nature of employment and place of work whereas it does not get affected by age. Fraczek and Klimontowicz [3] concluded that gender has an effect on financial decisions. They found that women approach financial decisions in a more conservative manner compared to males and also conclude that males invest more frequently in the financial market than women. Krechovská [4], Tóth et al. [11] studied the effect of economic education focused on financial literacy of university students. They found that economic education focused on education has an impact on financial literacy. Lusardi, Mitchell and Curto [6] observed that financial literacy was strongly related to sociodemographic characteristics and financial sophistication of the family.

CONCLUSIONS

Nowadays financial literacy has become a universally necessary skill for life because financial decisions and personal money management is more challenging than ever before. A lot of scientists suppose that we must not only have a degree of familiarity with economic terms, but also a high level of reading comprehension and strong mathematical skills when we want to read financial documents. Our research has examined the impact of gender, level of education and age on the ability to use mathematical tools to solve financial tasks on a sample of population of the Galanta region in Slovakia. The obtained indices of successfulness show a beneficial effect of female gender, university education and age between 41 - 61 on the ability of investigated respondents to solve financial tasks.

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Original Paper

Financial literacy of students of the Slovak University of Agriculture in Nitra

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ABSTRACT

In the paper we analyze with questionnaire survey which was conducted in the study subject "Basics of Insurance Mathematics" of the Faculty of Economics and Management (FEM) of the Slovak University of Agriculture (SUA) in Nitra. Financial literacy of students is the important part of the educational process at universities in Slovak Republic. The objective of the paper is comparison and evaluation of the financial literacy of students of FEM SUA in Nitra. The basic methods of the mathematical statistics and hypotheses testing were utilized in the assessment of the survey results. The existence of the statistically significant relations among the acquired assessments was verified by the χ^2–test. In case of dependence confirmation the intensity of assessed dependence was determined. The survey results found out that there is the influence of specialized subjects study on the students' perception and showed some significant interactions between studied knowledge and practical application.

KEYWORDS: mathematics, financial and insurance mathematics, financial literacy, questionnaire survey, hypotheses testing, χ^2–test

JEL CLASSIFICATION: I 21, C12

INTRODUCTION

The quality of higher education and the increasing competitiveness of universities are subject to continuous updating of the academic content as a result of interaction with the requirements for university graduates in the labour market [8].

Financial education is an important type of education. People come into contact and deal with the world of finance on a daily basis. Financial education is becoming a key aspect in decision making on all the issues related to our day-to-day life. The ability to understand financial

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products offered in a variety of forms or financial behavior itself both reflects the financial literacy of each of us [9].

A key element in people's decision making in all areas of their lives is financial literacy. Ability to understand financial products which normally people come into a contact is a reflection of financial literacy of everyone. General problem of society is inadequate level of financial literacy, therefore it is appropriate to search this issue in depth [12].

The subject "Basics of insurance mathematics" includes the fundamentals of financial and insurance mathematics. This subject is provided for accredited study programs Quantitative Methods in Economics as a compulsory subject and for Accounting as a compulsory optional subject. Financial Mathematics provides useful applications of mathematics in the financial field. Insurance mathematics provides the basics of actuarial methods and the most common types of insurance products in the life and pension insurance. Knowledge and methods of financial and insurance mathematics can be applied not only in the context of business decisions, but also in the private decision-making process in life. The inclusion of the subject "Basics of insurance mathematics" into context of education at FEM SUA in Nitra helps to prepare more complex graduates to practice [6].

The principal objective of the university study is to teach the students the mathematical apparatus and the methods of solving mathematical and application assignments that can be useful in the specialist subjects.
The assignments with the application elements have impact on:

- the students´ motivation to study also the theoretical methods of mathematics,
- development of students´ creativity,
- knowledge durability,
- connection of the mathematical theory and its practical application [10].

MATERIAL AND METHODS

The following basic methods of descriptive statistics and hypotheses testing were utilized in the assessment of survey results. The existence of statistically significant relations between acquired assessments was verified by mean of χ^2–test. The chi-square statistic is most appropriate for use with categorical variables, such as marital status [1].

Statistically demonstrated differences in the assessment were based on the significance of testing (p-value), presenting the error probability which is reached when the H0 hypothesis is rejected even it is true. In case the p-value of testing characteristic is lower than 0.05, a null hypothesis about the equality of observed features is rejected and the difference in values of a statistical feature is considered as statistically significant [11].

In our case we dealt with the statistical samples of range n and analyzed two statistical features – the first observed feature X presents student exam results classified according to study program and the second observed feature Y present the results of total assessment of student knowledge conducted in a regular term of before mentioned subjects.

We tested the following null hypothesis H0: There in no dependence between the observed features X and Y. The alternate hypothesis H1 as opposite: There is dependence between the observed features X and Y.

Pearson was looking for a simple statistic, a value that could be easily computed and that would indicate whether the results of an experiment deviated from expected results [2].

The statistics χ^2 is used as a testing criterion and is presented by the following ratio

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{r}\frac{(a_i b_j - (a_i b_j)_0)^2}{(a_i b_j)_0}$$

The testing statistics χ^2 has the χ^2- division with the number of variance levels (m-1)*(r-1) under the validity of testing hypothesis H0. The testing hypothesis H0 is rejected on the significance level, if the value of testing criterion χ^2 exceeds the critical value $\chi^2(\alpha; (m-1)*(r-1))$. The critical value χ^2, respectively KH can be found in the table of critical values [7].

After dependence confirmation we have used Pearson correlation coefficient so called ϕ coefficient in order to determine the intensity of dependence. The intensity of assessed dependence is higher as the value of coefficient approximates to 1 [11].
Coefficient value is calculated as follows:

$$C = \sqrt{\frac{\Phi^2}{1+\Phi^2}} = \sqrt{\frac{\chi^2}{n+\chi^2}}$$

The program Microsoft Excel 2010 and SAS was used for the realization of calculations and determination of critical values, ϕ coefficients.

RESULTS AND DISCUSSION

Survey subject was to determine the financial literacy of students in FEM SPU in Nitra at the beginning of winter term before the graduation of subject “Basics of insurance mathematics”. The survey task was to find out what is the students financial literacy after the graduation of two years study in FEM SPU in Nitra in academic years 2016/17 or 2017/18.

The students of bachelor study which should graduate from the subject “Basics of insurance mathematics” were addressed by the questionnaire consisting of 13 closed questions and one open question. From the total number of 76 students 51 students were at daily study form (DS), thereof 35 students of study program Accounting (UCT), respectively 16 students of study program Quantitative methods in economy (KME) and 25 students of study program UCT in an external study form (ES).

In Figure 1 the educational respondents structure is presented where 65.79 % (50 students) of all students graduated the specialized secondary school like business or hotel academy (BA or HA) where as regards the school type we assume the graduation of specialized subjects with the emphasize on practical calculations. Other students from which 25 % (19 students) graduated gymnasium and 9.21 % (7 students) of other type of secondary school, probably the subjects of such a character were not the subject of graduation.

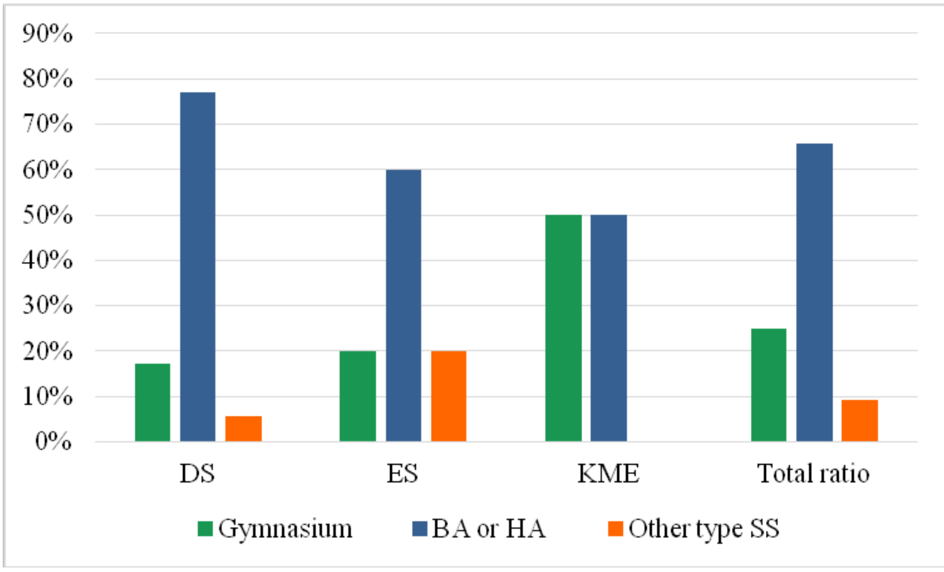

Figure 1 Respondents structure in accordance with the graduated secondary school

For the purpose of determination of respondents financial literacy level the survey was realized on the first lesson. As the subjects is include in the category of optional subjects, the fact was verified whether the students selected the subject based on the fact that they did not graduated the similar subject at the secondary school and the graduation of subject focused on the questions of financial literacy is considered to be relevant for them. The students' answers resulted in the following, 61 respondents (80.26 %) did not graduate the subject dealt with the issue of financial or insurance mathematics during their secondary school studies. Only three gymnasium students determined that this subject was graduated by them. During the secondary school studies almost 82.89 % (63 respondents), resp. 86.84 % (66 respondents) of students did not graduate the subject focused on financial respectively insurance mathematics (Table 1).

Table 1 Respondents structure in accordance with the graduation of financial and insurance mathematics

Subject graduation		Yes		No		Do not know	
		FM	IM	FM	IM	FM	IM
SS type	Gymnasium	4	3	14	15	1	1
	BA / HA	5	2	42	44	3	4
	Other SS	-	-	7	7	-	-
Total		9	5	63	66	4	5

Despite the before mentioned in average almost 64.47 % (49 respondents) respectively 57.89 % (44 respondents) of students answered the question whether they have already met with the tasks of financial respectively insurance mathematics in a practical life positively and 31.58 % (24 respondents) respectively 36.84 % (28 respondents) answered negatively. In the last part of questionnaire it was determined whether the students would be willing to accept the job offer in the area utilizing the knowledge of financial respectively insurance mathematics. After the study termination at FEM 55.26 % (in the number of 42) respondents would accept the job offer with the use of financial mathematics and equally 22.37 % (in the number of 17) students would take negative or indecisive attitude towards the job offer in such an area. The job using the insurance mathematics would be refused by almost 52.63 % (in the number of 40) students and only 22.37 % (in the number of 19) respondents would accept such a job.

Table 2 Results of mutual interactions determined in the survey

		Acquired assessment	Value of testing statistics	Pearson correlation coefficient
		p-value	χ^2	**ϕ coefficient**
Subject graduation at SS vs. application in a practical life	FM	**0.0058**	14.5298	0.4372
	IM	0.0625	8.9436	0.3430
Subject graduation at SS vs. job after the study termination with use	FM	0.9323	0.8448	0.1054
	IM	0.1284	7.1462	0.3066
Experience in a practical life vs. job after the study termination with use	FM	0.1429	6.8697	0.3026
	IM	0.0423	9.8895	0.3607
Knowledge application in a practical life FM vs. IM		**1.29 E-06**	43.7327	0.7586
Willingness to work after study termination in the area utilizing FM, resp. IM		**4.58 E-07**	34.3586	0.6724

By a more detailed analysis of survey results and investigation of mutual interactions between selected respondents answers it was determined that there exists the influence of specialized subject graduation on the students perception whether there have occurred some needs for application of knowledge from these subjects in a practical life. The survey approved the ability of students to correctly identify the application of knowledge from before mentioned subjects in a practical life (p-value = 0.0058). Simultaneously the willingness of future graduates to work in such an area where the knowledge of financial mathematics is used was observed. The vision to work in observed areas is only slightly influenced by the fact whether the students graduated the subject with the content focused on financial mathematics at the secondary school (ϕ coefficient equals 0.11) or they stated that in their practical life they met with the tasks of before mentioned issue (ϕ coefficient equals 0.3). While observing the similar relations in the insurance mathematics we can state that there exists medium strong dependence between the respondents' answers when ϕ coefficient obtains the value equaling 0.31, resp. 0.36 (Table 2).

Mathematics and its methods have multilateral application in specialized subjects of economics. The topics from the areas of finances and investments represent an interesting and important part of mathematical applications and can improve students' financial literacy [3].

Methods of financial mathematics can be applied in a lot of economic branches. Accounting, financial planning and decision making is the part of many professional courses and specialized subjects [4].

It is important to acquire key competences which they will use in future practice. The lifelong learning programs can be aimed at acquiring language skills, communication skills, diplomatic protocol and social literacy, computer literacy, legal issues, management and marketing, economics, finance and accounting, and so on [5].

CONCLUSIONS

Graduation of specialized subjects at all levels influence the increase in the financial literacy of population referring the correct identification of situation in a practical life which presents the basis for correct decision making.

The survey resulted in the fact that almost 63.27 % of students, who met with the financial mathematics in a practical life, were interested to work after their study termination in a financial area where the obtained knowledge is applied. Vice versa 61.36 % of students, who met with the insurance mathematics in a practical life, would refuse to work in an insurance area after their study termination.

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Original Paper

Statistical arbitrage regarding trade in goods of agricultural origin

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ABSTRACT

In our work we concentrate on the use of statistical arbitrage, mainly the trade in goods of agricultural origin model, which considers high frequency trading mechanisms of goods of agricultural origin. The strategy usefulness was measured by the information ratio. We considered both, options in the horizon and the time interval. The best results were achieved with respect to the historical time horizon, where the benchmark proportion of securities and one-minute time interval were appointed. Although we have to admit the investment issue constitutes a very complex problem influenced by a large number of factors. So there is not any universal mode of conduct guaranteeing profits that may be unequivocally indicated. We can only define a scenario, which will be effective with a substantial degree of probability.

KEYWORDS: statistical arbitrage, frequency trading, Kalman filter, simulation, investment strategy

JEL CLASSIFICATION: Q02, Q13, Q17

INTRODUCTION

Algorithmic trading stems from uncomplicated applications, which make it possible to divide large orders into a few smaller ones and realize them in an optimal manner. The development of this technique was possible only when the on-line stock exchange market allowing sending orders by e-mail became common and when personal computers became available. The current programmes use complex algorithms containing mathematical tools, especially in the field of statistics, optimisation or probability theory, presented Esfahanipour and Mousavi [5]. By observing quotations and other sources having a direct influence on the stock exchange markets, transaction programmes give adequate investment signals. As a result of the robust development of the IT services and an easy access to the Internet, the access to algorithmic trading is not reserved for large and significant investors anymore, but it is also available for individual investors. Enterprises, which provide such software, compete on many levels. First of all, they endeavour to generate highest possible profits for the customer (consequently, they

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increase their nominal commission by increasing the number of licenses and the amount of capital). Secondly, they try to indicate incorrect conduct of robots applied by the competition. Finally, and most importantly, they modify the operation of their algorithms with regard to moves and strategy adopted by alternative algorithms, was presented by Chavarnakul and Enke [3]. Thus, one may very rarely find in literature details concerning algorithms used by robots. And even if one succeeds, these are mainly publications relating to the application of artificial intelligence, the co-called 'black boxes'.

Statistical arbitrage trading has previously been examined by various authors, Dunis and other [4]. The goal of this type of trading is to develop highly automated trading strategies. Bertram [1] presented analytic formulae and solutions for calculating optimal statistical arbitrage strategies with transaction costs. The author assumed that the traded security had been described by an Ornstein–Uhlenbeck process. Broussard and Vaihekoski [2] investigated the practical issues of implementing the self-financing pairs portfolio trading strategy. These strategies engage in high frequency trading using algorithms based on stochastic or deterministic methods to identify price inefficiencies in the market [6]. A common approach when performing this type of trading is to construct a stationary, mean reverting synthetic asset.

MATERIAL AND METHODS

The Kalman filter has been used in order to forecast the cointegration coefficient in an optimal way. In the 1960's Kalman introduced the recursive estimation algorithm using the least squares method of parameters variable in time. Its fundamental idea is based on the predictor-corrector system and shown in the flow chart presented in Figure 1.

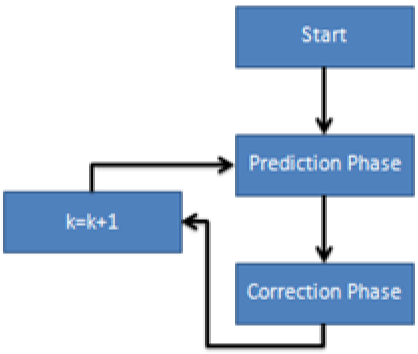

Figure 1 The idea of the Kalman filter operation. Source: authors

The a priori condition vector is predicated during the first phase.

$$\beta_{(t|t-1)} = \beta_t$$

Then it undergoes correction.

$$v_t = Y_t - X_t\beta_t$$

$$F_t = X_t P_t X'_t + H$$

$$\beta_{t+1} = \beta_t + P_t X'_t \frac{v_t}{F_t}$$

Finally, the covariance matrix is recalculated:

$$P_{t+1} = P_t + P_t X'_t X_t P_t \frac{1}{F_t} + Q$$

The H and Q parameters used in the procedure are responsible for smoothing time errors. The F matrix is the so-called state transition matrix.

Pair trading

The initial value is taken as

$$\beta_1 = \frac{Y_1}{X_1}.$$

The only condition of the adopted strategy was that the pair of commodities was supposed to originate from the same economic sector. The idea of pair trading itself is based on two pillars: the purchase of a commodity, whose prise is determined as too low and a short sale of a commodity, whose price is too high, according to the investment algorithm. Thus, the strategy is sometimes called the hedging strategy. The future direction of price changes of the respective goods is of no importance, i.e. it is not essential whether the price of a single commodity shall increase or decrease. What is important is the direction of the changes of the price ratio between both of them.
Let's assume that we have at our disposal two time series (P_{Y_t} and P_{X_t}) of the prices of selected goods. Moreover, let's suppose that they are cointegrated with each other. With the help of the Kalman filter we may calculate the forecast price of the first of them as

$$P_{Y_t}^{*} = \beta_t P_{X_t}$$

and the difference in prices

$$z_t = P_{Y_t} - P_{Y_t}^{*} = P_{Y_t} - \beta_t P_{X_t}.$$

The time series determined this way undergoes normalisation by cutting the average and dividing by the standard deviation. It should be stressed that the procedure of determining the average and the standard deviation is applied only with respect to the historical sample (in-sample), these data are used only to establish the constant proportion of goods prices. The out-of-sample specimen also undergoes normalisation, but on the basis of the historical average and the standard deviation. Here we may already make investment decisions.

Issues of optimizing the threshold of position opening and closing have not been considered in the study. Therefore, for the purposes of this work it has been assumed that exceeding the 'spread' i.e. the difference in price and its estimation, regarding the absolute value by the standard deviation value, constitutes the level of position opening (purchase of one commodity and short sale of the other, where ±1 signal mark decides which commodity is purchased and which is subject to short sale). Furthermore, it has been assumed that the actual position opening should occur only following two successive signals in order not to have the position opened too early. The position closing (sale of one commodity and purchase and return to the broker of the other commodity) is only possible when the difference in prices decreases below half of the standard deviation value.

Data collection method

Data concerning quotations of selected securities have been collected for the period of one year from April 1st 2013 to March 31st 2014 with one minute interval.

Statistical measures of investment strategy evaluation

The most common investment ratios have been applied for the purpose of evaluating quality of the investment strategy using the statistical arbitrage in high frequency trading.

Annualised return

This ratio is applied when the strategy testing period is different from the period of one year. It allows adjusting the rate of return from any period to one-year time horizon. Owing to this, it is possible to compare strategies of different time horizons

$$RA = 252 \cdot \frac{1}{N} \sum_{t=1}^{N} R_t ,$$

where R_t - daily return on investment, N - number of days under analysis.

Annualised volatility

The annualised volatility ratio is a measure of statistical return dispersion for a given security or market index. It may be measured with the help of the standard deviation or variation between revenues derived from the same security or market index. It shows the extent of risk and uncertainty, which may occur on the stock exchange

$$\sigma_A = \sqrt{\frac{252}{N-1} \cdot \sum_{t=1}^{N} (R_t - \bar{R})^2} ,$$

where:

R_t - daily return on investment,

$\bar{R}$ - average daily return on investment,

N - number of days under analysis.

Information ratio

The Information ratio (IR) is one of the most common indexes used for the purpose of comparing the risk level of various investment strategies. It is a measure of results achieved by the administrator with respect to the adopted risk level

$$IR = \frac{RA}{\sigma_A} .$$

It may be expressed in negative and positive values. The positive values are the most beneficial. The IR values within the 0,50 – 0,75 range are regarded a good investment and within the 0,75 – 1 range - a very good investment. The coefficient values exceeding 1 are evidence of an unusually good investment.

Maximum drawdown

The maximum drawdown ratio allows estimating the maximum loss that might flow from the investment. It describes the worst possible investment scenario, i.e. purchase at maximum prices and sale at minimum prices within the examined period:

$$MD = \min_{i=1,\ldots,t;\, t=1,\ldots,N} \sum_{j=1}^{t} R_j \, .$$

Implementation of statistical arbitrage in Matlab 2014 environment

The prototype of the statistical arbitrage consists of ten modules. Start constitutes the main module, which steers the realisation succession and the communication between the respective modules. The flow chart of component realisation succession of the prototype is shown in Figure 2.

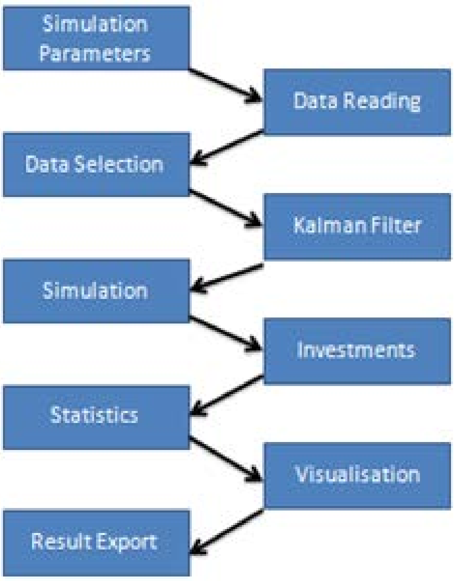

Figure 2 Scheme of communication between respective modules of the prototype. Source: authors

RESULTS AND DISCUSSION

Determination of simulation parameters is the module responsible for determining simulation parameters. The names of variables and their possible values and description are demonstrated in Table 1.

The objective of this module is to determine statistics concerning the investment profitability. The return on investment, the basic parameter, is calculated in the following way. The **Annualised return** (Annualised_Return) is the second parameter, which allows determining the profitability, but this time on a yearly basis. Determining **volatility** of the expected **Annualised return** (Annualised_Volatility), and consequently, the risk the investor should take into account, conducting such transactions, is the next factor. Furthermore, the **IR** (Information ratio) coefficient has been determined as the ratio of the annualised return to the annualised volatility. The **Maximum drawdown** (MaxDrawDown) has also been defined.
Figure 3 demonstrates the total pecuniary value of available funds and means invested in commodities within the successive time moments of quotations. This diagram derives from the prototype of the statistical arbitrage strategy in High Frequency Trading implemented in the Matlab environment.

Table 1 Simulation parameters

Variable	Possible Value	Description
startDate	'03/03/2013'	The initial date from which the quotation analysis begins. This solution allows selecting from long-term historical data only the period concerned with respect to the analysis, without the necessity to additionally process the data.
startTime	'9:30'	Listing initial time. Records including pre-session transactions may be found in the historical data.
endTime	'16:00'	Listing finishing time. Records including post-session transactions may be found in the historical data.
closeTime	'15:45'	Determination of time, after which transactions are not performed, despite the signal. The risk of unfavourable rate fluctuations between the next session days is limited then. \ Entering data into sale transactions is possible only between *closeTime* and *endTime*.
endTime	100	Number of days of simulation duration.
N	2	Number of days included in the history. The parameter of simulation memory.
Interval	5	Time interval, within which transactions may be entered into. In this case the change of the security number may be conducted every 5 minutes.
K	4	The procedure regarding temporary quotation suspension has been included in the simulation. In such cases data gaps are estimated with the help of the weighted average from the last quotations.
stopLoss	20	The parameter expressed in percentage, which prevents the phenomenon of dip on the stock exchange. In case of the sudden drawdown the possessed securities become sold out.
Capital	10000	The original value of funds intended for investments.
Times	100	The multiple of stock number (the number of stocks in the block).
brokerCost	0.0005	Broker's commission - amount per each block of stock.
bidaskCost	0.0005	Commission per transaction.
oSTreshold	-1.0	The threshold, below which the security purchase signal is generated (the market value is underestimated).
oLTreshold	1.0	The threshold, above which the security sale signal is generated (the market value is overestimated).
cTreshold	0.5	The threshold, at which the signal was generated earlier and remains binding.
H	0.01	The parameter used with the Kalman filter.
NoiseRatio	0.0001	The parameter used with the Kalman filter.

Source: authors

It may be noticed that the strategy after two days of operation has resulted in the profits at the level of 1.2 %. It is worth emphasizing that the strategy does not always guarantee profits. The investor should also take into account the possibility of suffering losses, which is reflected in the Figure 3 above.

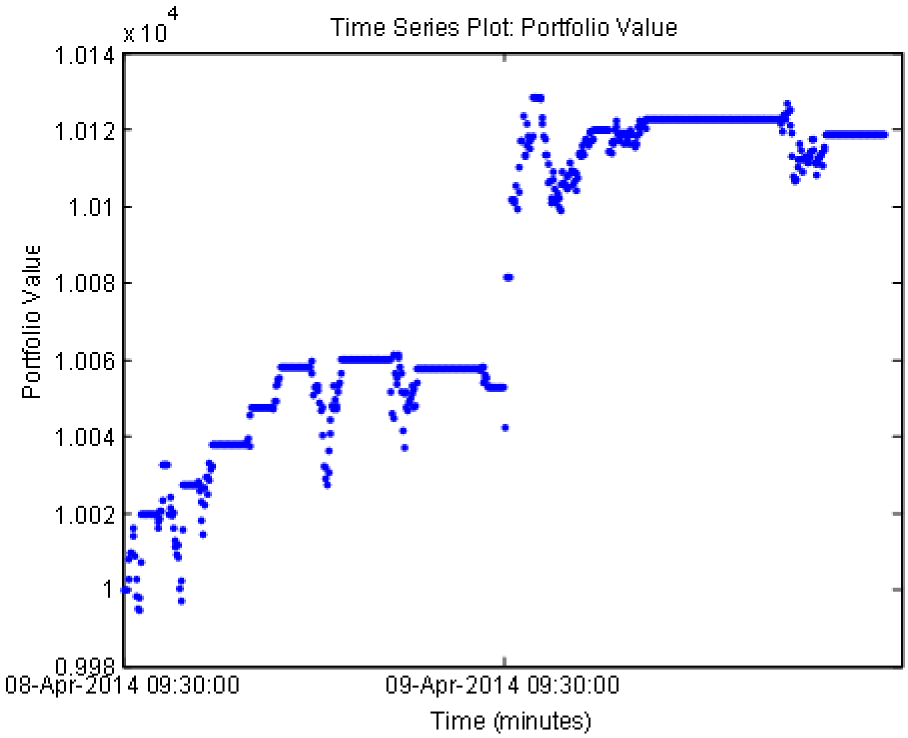

Figure 3 The portfolio value depending on listing time (the 1-minute time interval)

Table 2 demonstrates the basic statistics evaluating its quality.

Table 2 Statistics summarising the strategy simulation

Ratio	**Value**
Annualised Return	149.64%
Annualised Volatility	9.46%
Information Ratio	1.58
MaxDrawDown	$9.70

Source: authors

One should, first of all, concentrate on the high annualised return. Assuming the strategy operation analogical to the operation presented earlier and the capital amounting to $10,000.00, the investor could count on the profit amounting to $14,900.00. The IR ratio above 1, reflecting a very good investment, serves as the confirmation of quality. The maximum drawdown at the level below $10.00 should not deter the investor, taking into consideration the invested capital. In addition to this, the risk connected with the investment is significantly lower than expected profits.

The next step of the analysis is to check if the time interval length influences the strategy quality. Figure 4 show that the shorter time interval does not always mean a higher return on investment.

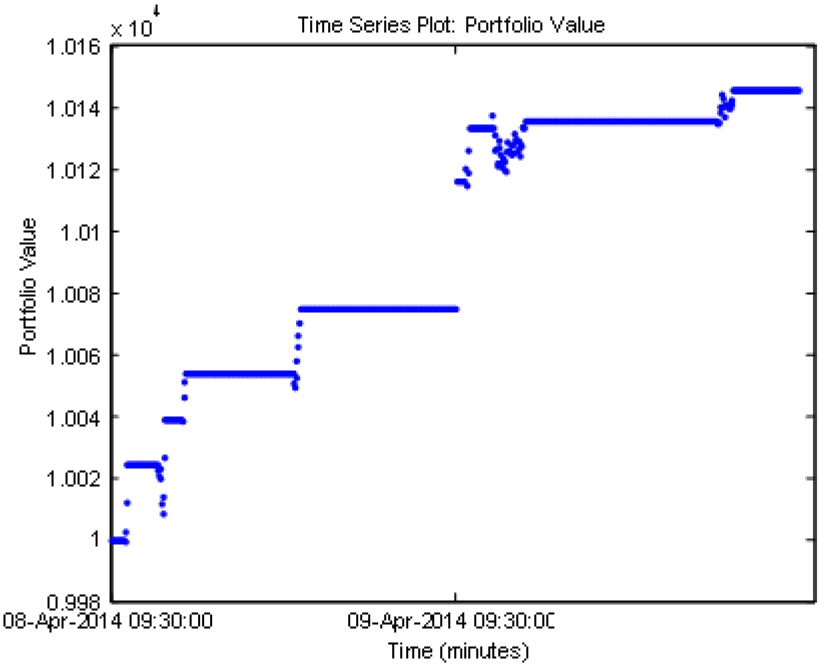

Figure 4 The portfolio value depending on listing time (the 5-minute time interval) Source: authors

Due to the fact that Figure 4 is for reference only (they are based on one arbitrarily chosen day), the researchers have tried to verify the influence of the time interval on the strategy quality. For this purpose the time horizon of simulation duration of 100 days has been adopted in the analysis, whereas 1, 5, 10 and 15 minutes respectively have been adopted as the interval variable. Other input parameters are shown in Table 3.

Table 3 Simulation Input Parameters

Parameter	**Value**
startDate	01-04-2013
start time	09:30
endTime	16:00
closeTime	15:45
N	1
sym_len	100
Capital	10000
Times	10
oSTreshold	-1
oLTreshold	1
cTreshold	0.5

Source: authors

The results are presented in Table 4. The highest rate of return have been achieved in case of the 15-minute time interval (over 20 % of return within a year), however, the result is burdened with a considerable risk (64 % annualised volatility). A slightly lower annualised return (nearly 18 %) has been recorded for the 1-minute interval, but the result has been more beneficial with respect to the investment risk (55 % annualised volatility). A significantly lower annualised return has been recorded for the interval (6 %). The relatively low risk suffered (25 %) argues in favour of this option. The 5-minute time interval, where a loss over 2 % has been recorded, has turned out to be the worst of the subject options. None of these investments has turned out to be good (IR < 0.5), although the 1-, 10- and 15-minute intervals have undoubtedly been favourable. Despite the fact that the 10-minute option has resulted in a small profit, it has been characterised by the maximum drawdown. Interestingly, this coefficient is comparable in terms of the most favourable and the worst annualised return.

Table 4 Summarising statistics depending on the time interval

Time interval [min]	Ratio			
	Annualised Return [%]	**Annualised Volatility** [%]	**Information Ratio** [ ]	**Maximum DrawDown** [$]
1	17.81	55.12	0.32	384.69
5	-2.32	34.65	-0.06	465.54
10	6.46	25.29	0.25	523.65
15	20.19	64.02	0.31	479.81

The other parameters have not been modified, just as in Table 5. It may be observed that owing to the increase in the number of days considered for determining the proportion model, the increase in the annualised return has been recorded, where the sample with the 5-day time horizon has been the highest (42 %). However, a high level of risk (44 %) has been recorded here. It is worth emphasizing that lenghtening the time horizon has not always translated into the increase in the annualised return. Each investment with the horizon longer than one day has been evaluated either as a very good investment (IR [0.75; 1)) or an unusually good investment (IR >1).

Table 5 Summarising statistics depending on the number of days in the history

Number of days in the history	Ratio			
	Annualised Return [%]	**Annualised Volatility** [%]	**Information Ratio** [ ]	**Maximum DrawDown** [$]
1	17.81	55.12	0.32	384.69
2	30.34	24.84	1.22	177.52
3	31.35	22.13	1.41	45.11
4	25.18	24.34	1.03	59.88
5	41.75	43.88	0.95	59.12

Source: authors

CONCLUSIONS

In view of the above arguments, one may not clearly state which time interval is the most favourable. Monitoring historical data of several time interval options may constitute the solution of this problem, whereas adopting the time interval which has recently turned out to be the best one may serve for the purpose of investing. Formulation of a periodical mechanism of the time option change might be proposed as a constructive solution. The model of the statistical arbitrage with high frequency trading in goods of agricultural origin may constitute an interesting alternative for traditional selling methods.

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Original Paper

Algorithm of determination of centre of gravity of agricultural machine with error estimation

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ABSTRACT

In this contribution we are dealing with determination the center of gravity of agricultural machine in experimental way. The mathematical algorithm was based on the basic principles of the static equilibrium equations of the mass. The algorithm was implemented to the environment of PTC® Mathcad Prime® 4. Processing of experimental data was realized with Microsoft™ Excel® format table importing to the Mathcad Prime® software. Dislocation of center of gravity we set up for universal systemic carrier Reform Metrac H6X with front end mounted adapter. Measurements was realized with respect to the Slovak technical standard STN 27 8154. To measuring the mass of machine we used the scales Evocar 2000R manufactured by Tecnoscale Oy. Measuring was cooperated with the authorized subject Sloveko Ltd. From each measuring was created the weight statement. Measured data was processed with published mathematical procedure implemented to software and solved the coordinates of center of gravity with error estimation. Obtained result was compared with the manufacturer specification.

KEYWORDS: center of gravity, machine mass, error estimation

JEL CLASSIFICATION: C63, C88, C93

INTRODUCTION

Dislocation of center of gravity belongs between the basic parameters of vehicle. For static and dynamic measuring this parameter is ultimate. In the Czech and Slovak agricultural research as well as the area of the design and testing of agricultural machines the position of center of gravity was often obtained from experimental measuring. The basic methodology was defined by [3]. The center of gravity is a point which locates the resultant weight of a system of particles or body as defined [7]. The sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at the center of gravity. The sum of moments due to the individual particles weights about center of gravity is equal to zero. Similarly, the center of mass is a point which locates the resultant mass of a system of particles or body. The center of gravity of a body is the point at which the

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total moment of the force of gravity is zero [5]. Theoretical analysis of vehicle dynamics, it is a common practice to define the motion equations in reference to the body of the vehicle, and so vehicle-fixed coordinate systems are often used to describe the fundamental dynamics of vehicles [10]. As depicted in Figure 5 a conventional vehicle coordinate system consists of body-fixed coordinates (hereafter the body coordinates) and steering wheel-fixed coordinates (hereafter the wheel coordinates). The construction of the vehicle refers to the usual configuration as published in [11]. The utilization of center of gravity in technical practice and educational process was published by [8, 9]. The origin of body coordinates is often defined as the center of gravity (CG) of the vehicle, with its XCG direction pointing in the direction of travel (or longitudinal motion), its YCG coordinate following the left-side direction (also denoted the lateral motion), and its ZCG direction indicating the vertical direction. Ride stability on vehicle critical maneuvers has a high dependency on position of center of gravity as determined by [6] and [13]. The different method of measuring of dislocation of center of gravity was published by [1]. The method based on the application of the Newton's moving equations where the vehicle acceleration is obtained from sensor of acceleration. The vehicle is moving on the slip plane often along the x axis. The sensor of acceleration is not located in the CG plane. This method is useful for the very weighty vehicles. Determination of dislocation of center of gravity is available only the moving direction with respect to the front end and rear axle. The error and uncertainty of experimental data are many possible sources defined in [4], as follows:

a) incomplete definition of the measurand,
b) imperfect realization of the definition of the measurand,
c) no representative sampling — the sample measured may not represent the defined measurand,
d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions,
e) personal bias in reading analogue instruments,
f) finite instrument resolution or discrimination threshold,
g) inexact values of measurement standards and reference materials,
h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm,
i) approximations and assumptions incorporated in the measurement method and procedure,
j) variations in repeated observations of the measurand under apparently identical conditions.

These sources are not necessarily independent, and some of sources from a) to i) may contribute to source j). Of course, an unrecognized systematic effect cannot be taken into account in the evaluation of the uncertainty of the result of a measurement but contributes to its error.

MATERIAL AND METHODS

Measurement system and object

Object for measurement was a systemic carrier Reform Metrac H6X. The basic parameters of machine are listed in the Table 1. The used measurement steps are defined in the technical standard STN 27 8154 [12]. This method is based on the weighing under all wheels and

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jacking the front end axle against the rear axle. Measurements of the agricultural machine were realized in the two variations. To measure the weight we used the scales Evocar 2000R (see Fig.4) manufactured by Tecnoscale Oy. Measuring was provided with the authorized subject Sloveko Ltd.

Table1 Parameters of off-road machine

Parameter	Value	Unit
Manufacturer	Reform-Werke	
Type	Metrac H6 X	
Engine	VM-D 754 SE 3	
Tyres	33x15.50-15	
Weight	2370	kg
Weight with mulch device	2750	kg
Wheel base	1.995	m
Wheel track	1.630	m

Table 2 Parameters of mounted adapter

Parameter	Value	Unit
Manufacturer	DSP Production Sas	
Mark	Carroy	
Type	GF 2072 RE7F C6 2S	
Description	mulch-laying adapter	
Weight	410	kg
Length	1.0	m
Width	2.270	m
Height	1.10	m

The first one was realized without the front end mounted adapter and the second one without the adapter. Mounted adapter was a mulch device Carroy GF 2072 which technical parameters are the Table 2. Weighting was realized in eight positions including the lateral position with respect to the longitudinal axis of machine. Other positions were varied from 3 to 4 deg. Measured weights without and with mounted adapter are depicted in the Figures 1 and 2. Detail of the weighing measurement is displayed in the Figure 4.

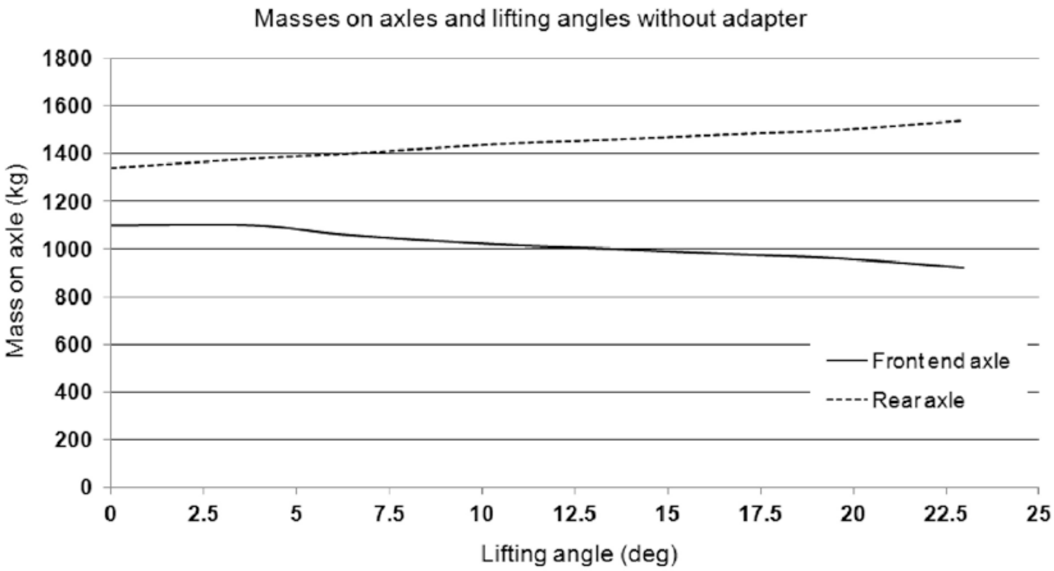

Figure 1 Masses on axles and lifting angles without adapter

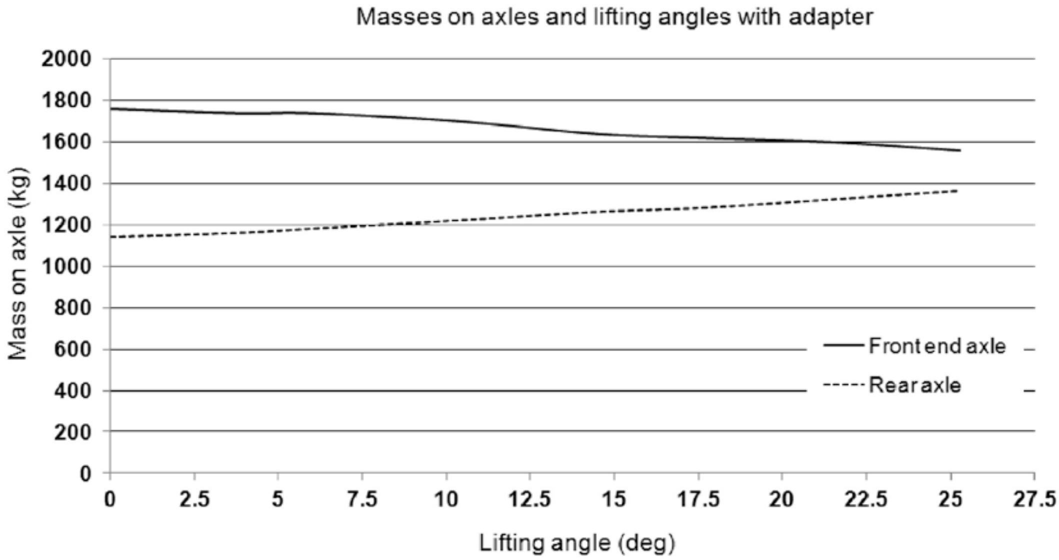

Figure 2 Masses on axles and lifting angles with adapter


Figure 3 Off-road vehicle measurement


Figure 4 Detail of vehicle wheel on scale

Center of gravity solving – mathematical procedure

Static disposition of the machine in the lifted position is depicted in the Figure 3. Mathematical identification was derived from the Figure 5 as follows. Reactions solving was realized with equation 1.

$$_{l,r}R_{f,r,(i)} = {}_{l,r}W_{f,r,(i)}.g\,, \quad (1)$$

Equation condition of vehicle with respect to the pole $P_{f(0)}$, if index $i = 0$ (longitudinal axis of vehicle is parallel with ground) is:

$$G_{(0)}.x_{f(0)} - R_{r(0)}.L = 0\,, \quad (2)$$

Gravity force will be:

$$G_{(0)} = R_{r(0)}.\frac{L}{x_{f(0)}} \quad (3)$$

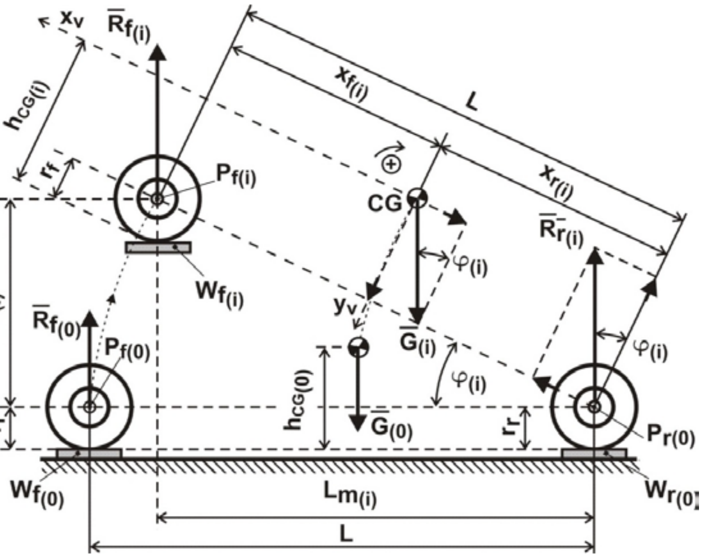

Figure 5 Machine in lifting position

Where :

CG - center of gravity

$\overline{G}_{(i)}$ - mass vector

x_v, y_v -vehicle line coordinates , m

$x_{f(i)}$ - CG front dislocation, m

$x_{r(i)}$ - CG rear dislocation, m

$W_{f(i)}$ - front weigh, kg

$W_{r(i)}$ - rear weigh, kg

$\overline{R}_{f(i)}$ - front reaction, N

$\overline{R}_{r(i)}$ - rear reaction, N

$h_{(i)}$ - height (right, left), m

$h_{CG(i)}$ - CG height, m

r_f - front wheel radius, m

r_r - rear wheel radius, m

$\varphi_{(i)}$ - angle, deg

L - wheel base, m

$L_{m(i)}$ - measured projection of wheel base, m

$P_{f(i)}$ - pole of rotation, front

$P_{r(i)}$ - pole of rotation, rear

With respect to the Figure 5 we were arranging the equation of condition to the pole $P_{f(i)}$ in the next form:

$$-R_{r(i)}.\cos\varphi_{(i)}.L + G_{(i)}.\cos\varphi_{(i)}.x_{f(i)} + G_{(i)}.\sin\varphi_{(i)}.\left[h_{CG(i)} - r_f\right] = 0\ . \tag{4}$$

With respect of indexing in equation (4) for using the measured data we modify the second member of equation (4) as follows:

$$G_{(0)}.\cos\varphi_{(i)}.x_{f(i)}\ . \tag{5}$$

Substituting the member $G_{(0)}$ from equation (3) to the equation (5) and putting back to the equation (4) we get:

$$h_{CG(i)} = -\left[R_{r(0)} - R_{r(i)}\right].\frac{L}{G_{(i)}.\tan\varphi_{(i)}} + r_f\ . \tag{6}$$

From Figure 5 we get the implicit goniometric relationship:

$$\tan\varphi_{(i)} = \frac{h_{(i)}}{L_{m(i)}},\ L_{m(i)} = \sqrt{L^2 - h_{(i)}^2}\ . \tag{7}$$

We set to the equation (6) and with arrangement we get:

$$h_{CG(i)} = \left[R_{r(i)} - R_{r(0)}\right].\frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\ , \tag{8}$$

whereby the interval for index is $i(1,n\rangle$, where n is the count of measurements and $G_{(i)} = W_{(i)}.g$, $R_{r(i)} = W_{r(i)}.g$, where $g = 9.81 m.s^{-2}$ is the gravitation acceleration.

Dislocation of center of gravity with respect to the front end (rear) axle in the all measured positions we get if deriving x_f from equation (3) and simultaneously eliminating the variable g, we get:

$$x_{f(i)} = \frac{W_{r(i)}}{W}.L - \frac{H.(h - r_r)}{\sqrt{L^2 - H^2}}\ . \tag{9}$$

To determine the transversal position of center of gravity with respect to the median longitudinal plane of vehicle we deriving formula in accordance to the Figure 5. In vector form we get:

$${}_l\overline{R} = {}_l\overline{R}_f + {}_l\overline{R}_r\ ,\ {}_r\overline{R} = {}_r\overline{R}_f + {}_r\overline{R}_r\ . \tag{10}$$

Equation of equilibrium to the left side of the machine will be:

$$G.y_l - {}_rR.B = 0\ . \tag{11}$$

Distance from the longitudinal axis of vehicle to the left sided wheels longitudinal axis will be:

$$y_l = \frac{{}_rR}{G}.B \tag{12}$$

The referenced values of dislocation of center of gravity are listed in the Table 3.

Table 3 Reference values of center of gravity dislocations

Parameter		Value	Unit
Machine	x	0.9	m
	y	0.021	m
	z	0.7	m
Machine with adapter	x	1.180	m
	y	+0.007	m
	z	0.74	m

Error estimation

To determine an error we used the $A-type$ error estimation method. Standard error will be solved from dataset of n count (x_1, x_2,..., x_i,... x_n), *as* defined in [2]. Measured data are considered as statistically independent random data and assume there were measured with the same conditions. Error of measurement is in relationship with the sample mean in this form:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \,. \tag{13}$$

Standard error of type A is equal to the experimental standard deviation of the sample mean.

$$u_A(x) = \sqrt{\frac{1}{n.(n-1)}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \,. \tag{14}$$

Let us assume the function of height of center of gravity with many variables:

$$f_{h_{CG(i)}}\left(R_{(i)}, h_{(i)}, G_{(i)}\right) = \Delta R_{r(i)} . \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r \text{, where} \tag{15}$$

$$\Delta R_{r(i)} = R_{r(i)} - R_{r(0)} \,.$$

As defined in [4] the uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects. Squared uncertainty of this function will be defined in [4] as follows:

$$\Delta f_{h_{CG(i)}}\left(\Delta R_{(i)}, h_{(i)}, G_{(i)}\right) = \sqrt{pd_{\Delta R}{}^2 + pd_h{}^2 + pd_G{}^2} \,. \tag{16}$$

The partial derivations of members will be next:

$$pd_{\Delta R} = \frac{\partial\left(\Delta R_{r(i)} . \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial \Delta R_{r(i)}} = \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} , \tag{17}$$

$$pd_h = \frac{\partial\left(\Delta R_{r(i)} . \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial h_{(i)}} = -\frac{L^3.\Delta R_{r(i)}}{G_{(i)}.h_{(i)}^2.\sqrt{L^2 - h_{(i)}^2}} , \tag{18}$$

$$pd_G = \frac{\partial\left(\Delta R_{r(i)} . \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial G_{(i)}} = -\frac{\Delta R_{r(i)}.L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}^2.h_{(i)}} , \tag{19}$$

For uncertainty finally we get:

$$\Delta f_{h_{CG(i)}}\left(\Delta R_{(i)}, h_{(i)}, G_{(i)}\right) = \sqrt{pd_{\Delta R}{}^2 + pd_h{}^2 + pd_G{}^2} \tag{20}$$

RESULTS AND DISCUSSION

Experimental measuring and the measured data processing is very unstable process. In this paper we are dealing with measuring the masses of agricultural machine. The measuring process is based on the standard STN 27 8154. The aim of the measuring process is the determination of the location of center of gravity of systemic carrier Reform Metrac H6X. We set up the measuring devices and realized the experimental measuring with cooperation with certified subject. From measured data we get the locations of the center of gravity for two configurations of the machine. For all measured data we determine the standard error and the percentage error. To processing the experimental data with presented algorithm we used the PTC® Mathcad Prime® 4 environment. For comparison we had referenced values from manufacturer. The difference from solved results and the referenced values is given by the different conditions in the measuring process. The solved standard errors are in the Table 3.

Table 3 Standard error of type A

Coordinate	Machine	$u_A\left(x_f, y_l, h_{CG}\right)$	Machine with adapter	$u_A\left(x_f, y_l, h_{CG}\right)$
x_f	0.785 m	3.388E-03 m	1.083 m	2.302E-03 m
y_l	0.821 m	1.771E-03 m	0.792 m	1.492E-03 m
h_{CG}	0.628 m	1.705E-02 m	0.57 m	1.598E-02 m

The squared uncertainty was solved in value $39.86.10^{-3} m$ for machine without adapter and with adapter was solved in value $4.429.10^{-2} m$. For experimental measuring of this kind of application the result is acceptable.

CONCLUSIONS

Experimental measuring and the measured data processing is very unstable process. In this paper we are dealing with measuring the masses of agricultural machine. The measuring process is performed according to the standard STN 27 8154. The aim of the measuring process is the determination of the location of center of gravity of systemic carrier Reform Metrac H6X. We set up the measuring devices and realized the experimental measuring with cooperation with certified subject. From measured data we got the locations of the center of gravity for two configurations of the machine. To solve mathematical equations we used the PTC® Mathcad Prime® 4 environment. For all measured data we determine the standard error and the squared uncertainty. For comparison we had referenced values from manufacturer. The difference from solved results and the referenced values are the different conditions in the measuring process. For experimental measuring of this kind of application the result is acceptable because the standard error of type A is in interval $\left\langle 1.492.10^{-3} m - 1.705.10^{-2} m \right\rangle$. The squared uncertainty was solved for the machine in value $39.86.10^{-3} m$ and machine with mounted adapter in value $4.429.10^{-2} m$.

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Original Paper

Analysis of SMEs development and their innovation activities in EU countries

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ABSTRACT

Small and medium enterprises form the basis of each highly-developed economics. They create the biggest part of the number of all business enterprises. They have significant influence on offering job opportunities and creating of surplus value. Except these aspects, SMEs is also attributed to high innovation and developing potential. The ability of SMEs to innovate introduces one of the key factors. The aim of this article was to compare the number of SMEs in EU countries with respect to old and new member countries. In order to achieve the goal and analyze the data, we used the statistical methods: basic statistical characteristics, Student's t-test and one factor analysis of variance. We found that there are still significant disparities in the development of SMEs between old and new member countries. We believe that there is a similar gap in SME innovation activities as well.

KEYWORDS: SMEs, analysis of variance, t-test, development, innovations

JEL CLASSIFICATION: C00, O10, O31

INTRODUCTION

Business is the driving force of a market economy. Small and medium-sized enterprises (SMEs) are an important part of the national economy, job creation, added value or foreign trade. Newly created SMEs generate new jobs, new SMEs that are emerging as a means of commercializing new technologies or innovative ideas are often the carriers of positive structural changes in the economy, increasing its productivity and contributing to economic growth. SME development increases the intensity of competition on the market, acts against monopolistic tendencies and has the potential to reduce social and regional polarization [8]. Small and medium-sized enterprises are characterized by transparent organizational structure, enabling direct management and information flow without significant impact of negative aspects. Small and medium-sized enterprises (SMEs) have significant potential when they perform several important functions (e.g. social, economic, export-import, etc.). The

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importance of these companies is not only at national level, but at the level of transnational [7]. With the advancing process of globalization, small and medium-sized enterprises are still more confronted with international competition. On the other hand, globalization also provides SMEs with other opportunities and possibilities for doing business on foreign markets. There is even a direct link between the degree of involvement of SMEs in international relations and their performance. Businesses engaged in international relations are more competitive and more efficient than those that do not engage in international activities. The innovation activities of these enterprises represent one of the main prerequisites for their competitiveness on international markets [3]. Also important is the importance of building a brand in SMEs, which is confirmed by Graa and Andelhak [6]. The importance of SMEs can also be perceived at EU level when, for example, in a document called the European Charter for Small and Medium-Sized Enterprises, SMEs are characterized as the EU's basic economic support and as a key source of employment.

The expansion of the business and economic base and the development of the entrepreneurial business with the support of the EU and national sources of finance initiated the growth of the national economy at the level of Slovakia [5]. However, there is still a certain negative in the development of the business environment. Entrepreneurs negatively perceive, in particular, the high tax burden, the weak law enforcement, administrative burdens of business, many unnecessary environmental standards, the level of regulated prices, complicated and often changing legislation, corruption and the increase in regulatory burdens in relation to the approximation of Slovak legislation to EU law [10].

In addition to these aspects, SMEs are also attributed to the high innovation and development potential. The ability of enterprises to innovate is one of the key factors of positive structural change. The widespread use of new (progressive) knowledge in all business activities has a positive impact on the growth of labor productivity, the growth of added value of production and the increase in competitiveness (based on production quality), not only at the enterprise level but also at the national level. However, innovative business SMEs face a number of obstacles compared to large companies. For example, a shortage of educated workers, the cost of own research and development, a lack of research infrastructure, difficulties in establishing new contacts with research organizations, lack of investment capital, limited administrative capacity, and so on [8]. The government should minimize administrative barriers, which could hamper businesses in their activities [9]. The importance of innovation is also supported by Fenyvesi [4].

As SMEs are the driving force of the economy and hence significant potential, it is desirable to support them, whether financial or advisory. Veber et al. [11] states that activities aimed at supporting SMEs can be broadly divided into two basic groups: financial support and information support. Financial support and development of SMEs in Slovakia is mainly implemented through loans, micro-loans, risk capital use, the use of guarantees (for example bank loans), non-repayable financial assistance, funding of selected activities from national funds, funding of selected activities from transnational funds so on. Support, whether financial or informational, invested in the development of SMEs will return in the short term in the form of higher competitiveness and the creation of further prerequisites for economic growth.

MATERIAL AND METHODS

The aim of the thesis was to compare the number of SMEs in EU countries for the years 2010 to 2015 with respect to old and new member countries. Source data were obtained from Eurostat database (2017). Methodology division and definition of SMEs can be various in different countries. In this article we deal with definition SMEs by EU. The reason is mainly consecutive comparison. For this comparison we need uniform methodology for all countries. By mentioned definition into the category SMEs we include all enterprises, which ones have 0 to 250 employees and turnover or balance sheet total less than 50 million EUR. We can sectionalize SMEs even more in detail on micro enterprise (have less than 10 employees and turnover or balance sheet total less than 2 million EUR), small enterprise (have 10 to 49 employees and turnover or balance sheet total less than 10 million EUR) and medium-sized enterprise (have 50 to 249 employees and turnover less than 50 million EUR or balance sheet total less than 43 million EUR).

In order to achieve the goal and analyze the data, we used the statistical methods and the obtained observations were evaluated as follows:

- We have determined the basic variation-statistical characteristics by states and years.
- Comparison of observation numbers with respect to the size of MS states and years was evaluated using Student's t-test and by one and two factor analyses of variance with fixed effects.

Mathematical and statistical evaluation was carried out using the statistical package Statistix version 9.0 [12].

RESULTS AND DISCUSSION

In the European Union, SMEs represent for more than 99% of the total number of enterprises. Similarly, in the Slovak Republic. In the Eurostat databases for the Slovak Republic we can find an extremely large difference between 2008 - 2009 and 2010 - 2017 in the number of micro-enterprises. However, it has an artificial cause, since 2010 there have been some methodological and legislative changes, which subsequently affected the overall statistics. For the purposes of the analysis in this article, therefore, for all SMEs, we chose the 2010 - 2015 timeframe to make the data comparable and not to have distorted results. For the primary countries, for the purposes of our article, we considered those countries that joined the EU in the 20th century (until 1995). For new countries, we consider those who joined the EU in the 21st century (after 2004). This means that for the purposes of this article, we will consider the old member states Austria, Belgium, Germany, Denmark, Greece, Spain, Finland, France, Ireland, Italy, Luxembourg, Netherlands, Portugal, Sweden and United Kingdom. The latter country leaves the European Union in a short time, but it is still a part of the EU and so we included it, of course, in our analyzes. For the new EU member states we consider Bulgaria, Czech Republic, Cyprus, Estonia, Croatia, Hungary, Lithuania, Latvia, Malta, Poland, Romania, Slovakia and Slovenia. As the new member states are made up of many states of the former Eastern Bloc, we assumed that there will be significant differences in the development and abundance of SMEs.

Table 1 Means and standard errors of means for numbers of SMEs in 2010 - 2015 for each country and also for old and new EU countries

No	State	Abb.	$\bar{y}$	$s_{\bar{y}}$	No	State	Abb.	$\bar{y}$	$s_{\bar{y}}$
1.	Austria	AT	103532	29048	16.	Bulgaria	BG	103464	30917
2.	Belgium	BE	186190	58471	17.	Czech R.	CZ	328605	106141
3.	Germany	DE	720767	184375	18.	Cyprus	CY	15333	4702.9
4.	Denmark	DK	70436	20412	19.	Estonia	EE	19684	5790.8
5.	Greece	EL	240493	78653	20.	Croatia	HR	49894	15095
6.	Spain	ES	799788	251966	21.	Hungary	HU	173947	54857
7.	Finland	FI	75435	23061	22.	Lithuania	LT	46599	13992
8.	France	FR	938067	298674	23.	Latvia	LV	30527	9087.7
9.	Ireland	IE	14062	634.22	24.	Malta	MT	8895.1	2776.3
10.	Italy	IT	1.257E+06	398797	25.	Poland	PL	499236	159264
11.	Luxemb.	LU	9901.4	2778.6	26.	Romania	RO	143153	40537
12.	Nether.	NL	307725	98293	27.	Slovakia	SK	133472	43381
13.	Portugal	PT	266078	84872	28.	Slovenia	SI	40767	12865
14.	Sweden	SE	218238	69001					
15.	U. Kingd.	UK	582142	168791					
Tot	Old	n			Tot	New	n	122583	18275
		90	386002	47160			78		

Source: Own processing based on Eurostat (2017)

Tab. 1 shows means and standard errors of means for the analyzed countries for the years 2010 to 2015. We have made calculations for both the old and the new EU countries. The average number of SMEs per old state was 386002 with a standard error of ± 47160. For the new states, the average number of SMEs was 122583 with a standard error of ± 18275. The lowest number of SMEs in the old states was 9901.4 (Luxembourg) with a standard error of ± 2778.6. The largest number was found in the old states in Italy 1.257E + 06 with a standard error of ± 398797. For the new states, we found the smallest value of 8895.1 (Malta) with a standard error of ± 2776.3 and a maximum of 499236 (Poland) with a standard error of ± 159264. We believe that the number of businesses seems to be related to the geographical size of the analyzed states. The average number of SMEs in the old member countries has more than three times the number in the new member countries. As the old Member States also have a higher average population, we believe that this will be related. We have not, however, evaluated this impact. However, such a calculation also brings with it some shortcomings. Take, for example, a number of countries with a very similar population: Belgium, Greece, Portugal, Czech Republic and Hungary. Despite the similar population, it seems that the number of SMEs is more different than we would expect. For example, in Hungary, it is only 173947 and in the Czech Republic up to 328605. Another example could be Denmark, Ireland, Finland and Slovakia, which have a similar population. However, when looking at the average number of SMEs, the differences are again greater than we would expect. While in Ireland it is only 14062, in Slovakia it is up to 133472. However, we have not discussed this topic in detail.

Table 2 shows the averages and their standard errors according to the size of SMEs in the old and new EU states, indicating the significance of the differences between SME1 to SME3, as well as between old and new states. We observed the largest number of enterprises over the monitored period with SME1. The lowest number we found for SME1 in 2010. It was 714788 with a standard error of ± 166505. The highest abundance was 758058 with a standard error of ± 174420 in 2015. For SME2, we found the smallest number of businesses in 2013. It was 48073 with a standard error of ± 13405. The largest number was 49209 with a standard error of ± 13736 in 2015. For SME3, we found the smallest number of businesses in 2013. It was 8356.6 with a standard error of ± 2134.2. The largest number was 8675.6 with a standard error of ± 2224.2 in 2015.

Table 2 Means and standard errors of means for number of SMEs by size, for old and new EU countries and by years

Year	HistC		SME1		SME2		SME3			SME			
	State	n	$\bar{y}$	$s_{\bar{y}}$	$\bar{y}$	$s_{\bar{y}}$	$\bar{y}$	$s_{\bar{y}}$	N	$\bar{y}$	$s_{\bar{y}}$	F	SC
2010	1O	15	1.038E+06	272531	75236	22099	12504	3525.6	45	375281	113745		
	2N	13	341732	112790	17629	4286.7	3757.5	1164.9	39	121039	44530		
	Total	28	714788	166505	48490	13039	8443.1	2106.5	84	257240	65494	16.93	1:.(2,3)**
	t		2.36*	Satt	2.56		2.36*	Satt	F	3.88	FWelch	4.33*	
2011	1	15	1.042E+06	272459	76071	23060	12659	3640.0	45	377183	113935		
	2	13	344218	115987	17883	4538.3	3772.8	1163.4	39	121958	45489		
	Total	28	718468	167010	49055	13534	8533.5	2165.0	84	258686	65739	16.98	1:.(2,3)**
	t		2.36*	Satt	2.48*	Satt	2.33*	Satt	F	3.88	FWelch	4.33*	
2012	1	15	1.062E+06	278556	75150	22765	12482	3656.5	45	383152	116340		
	2	13	344940	115174	17815	4657.5	3729.0	1149.4	39	122162	45306		
	Total	28	728985	170368	48530	13369	8418.0	2166.2	84	261978	66965	16.84	1:.(2,3)**
	t		2.38*	Satt	2.47*	Satt	2.28*	Satt	F	3.91	FWelch	4.37*	
2013	1	15	1.081E+06	280091	74264	22904	12426	3596.0	45	389114	117586		
	2	13	341987	112481	17853	4750.9	3661.5	1112.7	39	121167	44461		
	Total	28	737701	171549	48073	13405	8356.6	2134.2	84	264710	67542	17.04	1:.(2,3)**
	t		2.45*	Satt	2.41*	Satt	2.33*	Satt	F	4.06*	FWelch	4.54*	
2014	1	15	1.089E+06	280876	75267	23296	12621	3669.5	45	392450	118160		
	2	13	348996	115410	17999	4839.4	3662.8	1124.1	39	123552	45544		
	Total	28	745674	172376	48678	13631	8461.8	2177.2	84	267605	67990	17.24	1:.(2,3)**
	t		2.44*	Satt	2.41*	Satt	2.33*	Satt	F	4.03*	FWelch	4.51*	
2015	1	15	1.107E+06	283931	76069	23455	13023	3736.5	45	398829	119710		
	2	13	354976	116768	18218	4938.3	3658.9	1124.5	39	125618	46163		
	Total	28	758058	174420	49209	13736	8675.6	2224.2	84	271981	68894	17.40	1:.(2,3)**
	t		2.45*	Satt	2.41*	Satt	2.40*	Satt	F	4.06*	FWelch	4.53*	

Source: Own processing based on Eurostat (2017)

From the comparison of the old and the new Member States for the sizes SME1, SME2 and SME3 we found that practically up to SME2 in 2010, the differences in numbers were statistically significant. When comparing SME1, SME2 and SME3 over the years, we have found that the number of SME1 enterprises is statistically high compared to the number of SMEs2 and SME3 in all the years under review.

The number of enterprises in the old as well as the new EU member states has risen practically with years, with the lowest value of the number of enterprises we have found in 2010. It was 257240 with a standard error of ± 65494. The maximum number was 271981 with a standard error of ± 68894 in 2015. Using the analysis of variance, we found, that the number of enterprises in the old EU countries was statistically significantly higher than in the new countries.

From the point of view of the overall assessment and development of SMEs in Slovakia, we can say that their overall growth is only small compared to the pre-crisis period and Slovakia does not record any significant growth dynamics. There are several reasons for this, for example, deterioration of the business environment, increase of the tax burden and so on. Further development of SMEs requires a change of access in a number of areas, including a more efficient allocation of subsidy resources to support SMEs' competitiveness, growth and innovation. Help can be, for example, The National Business Center in charge of the Slovak Business Agency, whose project with an impact on individual regions is beginning to develop in Slovakia. It can also help the Center of better regulation to reduce the impact of legislation on the business environment. These solutions can bring further positive development of the number and scope of SME activity in Slovakia.

CONCLUSIONS

The position of small and medium enterprises of knowledge intensive services and their dynamics of growth in the EU economy and within the Slovak Republic can be monitored through performance indicators such as employment, number of enterprises and added value [1]. In our article, we focused mainly on the analysis of the number of enterprises in European Union in 2010 - 2015 and the comparison between old and new EU member states. The increasing number of SMEs is important for a number of reasons. One example is the fact that new SMEs generate new jobs. It is clear from the above findings that there are still significant differences between the primary and the new member countries in the area of SME development, new member countries are still lagging behind. For future investigations, it might be interesting to take into account the number of inhabitants in each country. Even more interesting would be the comparison of countries in terms of innovation activities of SMEs, since innovation is one of the most important factors of competitiveness. However, in the article, we have not explored innovative business activities, but we believe that the differences between the new and old EU states are also in this area. The new member states are composed mainly of Eastern Bloc countries that have experienced the transition from command to market economy. These effects are still perceptible and therefore the results are expected. An exception is not Slovakia, whose innovation activity and innovation performance of SMEs is on the imaginary "tail" within the EU.

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