Acta Mechatronica - International Scientific Journal about Mechatronics
Volume: 3 2018 Issue: 1 Pages: 1-6 ISSN 2453-7306

doi:10.22306/am.v3i1.27
Received: 12 Jan. 2018
Accepted: 23 Jan. 2018

CONDITION OF MEASUREMENT CHAIN WITH POSITION SENSOR

Tatiana Kelemenová
Technical University of Kosice, Faculty of Mechanical Engineering, Letna 9, Kosice, Slovak Republic,
tatiana.kelemenova@tuke.sk (corresponding author)
Eduard Jakubkovič
Technical University of Kosice, Faculty of Mechanical Engineering, Letna 9, Kosice, Slovak Republic,
eduard.jakubkovic@tuke.sk

Keywords: measurement, calibration, uncertainty, gauge
Abstract: The paper deals with experimental identification of condition of measurement chain with position sensor. The position sensor is industrial version used mainly for measurement of position of any moving parts. It uses technology of conductive plastic. Gauge length blocks are used for identification of its condition.

1 Introduction

Measurement of position is very frequently measured quantity in practice. The potentiometer sensor has a long tradition of using. Last years the using of this principle has increased, because of its disadvantage. A lot of books have mentioned about its disadvantages like noise, oxidation of wiper and resistive road, short life etc. In this days situation is changes, because the new technologies and materials have been developed. Modern potentiometer sensors have an excellent properties, low noise, long life, low uncertainty etc [1-13].

2 Position sensor measurement chain

The sensor has body with vibration damped element and no wiper bounce in high vibration environments. It has smooth operation under large misalignment. Wiper is made from precious metal with high corrosion resistance, low noise and high performance. The simplicity of this sensor enables to use it with simple controllers. It is possible to execute absolute continuous measurement. Selected properties of the sensor are listed in the table 1 below [1].

Table 1. Selected parameters of the tested sensor [1]

Total Mechanical dimension	70x 62 x 900 mm
Total Mechanical Travel	780 mm
Independent Linearity	0,1%
Total Resistance	10kΩ
Operating Temperature	-60°C až 100°C
Resolution	Infinite

Calibration of the sensor has been executed in accordance with standards (EA-4-02rev01) [2]. Position of the wiper has been adjusted with length gauges. Length gauges has been composed into the block of the length gauges. Full range of the sensor has been compared with block of the gauges at every millimetre. Electrical resistance between wiper and one end has been assigned to every block of length gauges (every millimetre ten times).

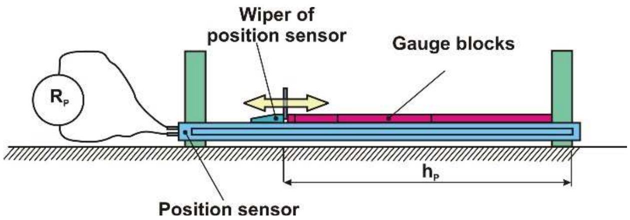


Figure 1 Position sensor measurement chain


Figure 2 Position sensor

It is recommended to do calibration for every millimetre ten times in industrial practise. Ten times measured every value is minimum, which enables to evaluate standard uncertainty of type A (see standards EA-4-02rev01) [2].

3 Calibration of the measurement chain

Two packages of the length gauges have to be used for the calibration process. The Gloves has been necessary for the manipulation with these gauges. Process needs very high attention and a lot of time. Temperature if the room has to be regulated via air condition at the 20°C. Sensor and package of the length gauges has to be placed in laboratory with stabilized air temperature all day before measurements. Every piece of the length gauges is conserved with vaseline to avoid the corrosion of the length gauges. So, every piece is necessary to unconserve with denatured alcohol before using.

Consequently, observance of every these mentioned rules causes that calibration process is very complicated and difficult for time.

Measured data have been stored into the evaluation table. It is possible to evaluate static characteristic shown on figure 3.

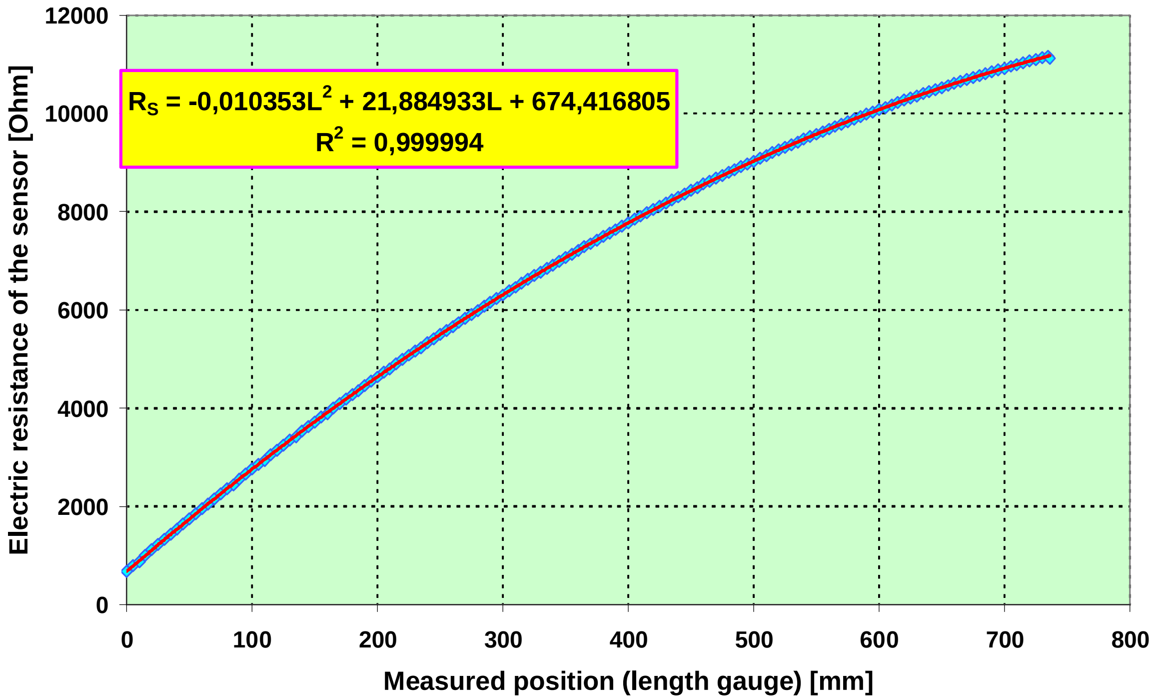


Figure 3 Static characteristic of the measurement chain

Measured data has nonlinear dependence. That is difference from information mentioned via producer noted in table 1. Dependence can be fitted with polynom of 2nd degree. Also maximum range of the output electrical resistance exceeds the mentioned total resistance 10kΩ.

This characteristic enables to recalculate the measured electrical resistance to linear position of the wiper from the end of the sensor. The approximation regression equation (shown on figure 3) can be inserted into the evaluation subsystem for calculation of the measured position. But, how we can believe it? How is the measured data and equation exactly? It is necessary to give answers for these questions.

4 Uncertainty balance

The uncertainty of measurement is a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand. Term uncertainty is also used for uncertainty of measurement if there is no risk of misunderstanding.

Sensor producer doesn’t note uncertainty of measurement. Consequently, it is necessary to obtain this information from calibration process.

For a random variable the variance of its distribution or the positive square root of the variance, called standard deviation, is used as a measure of the dispersion of values. The standard uncertainty of measurement associated with the output estimate or measurement result y, denoted by *u(y)*, is the standard deviation of the measurand *Y* [2].

The uncertainty of measurement associated with the input estimates is evaluated according to either a 'Type A' or a 'Type B' method of evaluation. The Type A evaluation of standard uncertainty is the method of evaluating the uncertainty by the statistical analysis of a series of observations. In this case the standard uncertainty is the experimental standard deviation of the mean that follows from an averaging procedure or an appropriate regression analysis. The Type B evaluation of standard uncertainty is the method of evaluating the uncertainty by means other than the statistical analysis of a series of observations. In this case the evaluation of the standard uncertainty is based on some other scientific knowledge [2].

The Type A evaluation of standard uncertainty can be applied when several independent observations have been made for one of the input quantities under the same conditions of measurement (minimum of 10 samples of measurement). If there is sufficient resolution in the measurement process there will be an observable scatter or spread in the values obtained [2].

The proper use of the available information for a Type B evaluation of standard uncertainty of measurement calls for insight based on experience and general knowledge. It is a skill that can be learned with practice. Type B evaluation of standard uncertainty can be obtained from various sources as [2]:

- previous measurement data,
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments,
- manufacturer's specifications,
- data provided in calibration and other certificates,
- uncertainties assigned to reference data taken from handbooks.

Electrical resistivity has been measured via multimeter and manufacturer provide specification for type B evaluation of the standard uncertainty of measurement. It is possible to specify equation (1):

$$u_B = \pm(0{,}0025\% measured\_value + 0{,}0005\% scale\_range) \quad (1)$$

Figure 4 shows the standard uncertainty of measurement for values of electrical resistance measured via multimeter. Type B evaluation is much smaller then type A evaluation. So, it is possible the evaluation B neglected in the next evaluation process. It means that multimeter used in calibration process has been well selected.

Recalculation of the standard uncertainty of electrical resistance measurement to standard uncertainty of position measurement is possible via using regression math model obtained from analysis shown on figure 3. Figure 5 shows the standard uncertainty for position measurement.

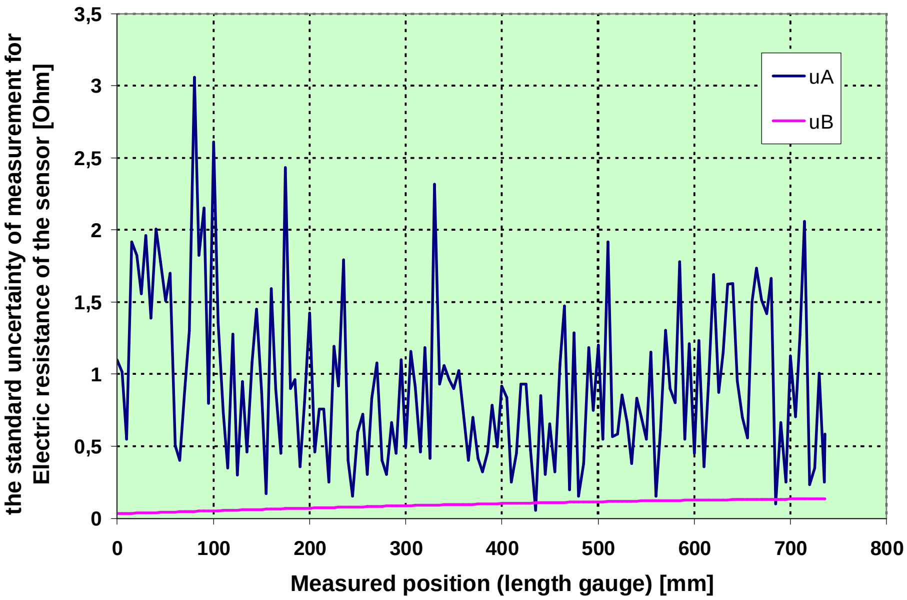


Figure 4 The standard uncertainty of measurement for electric resistance of the sensor measured in calibration process

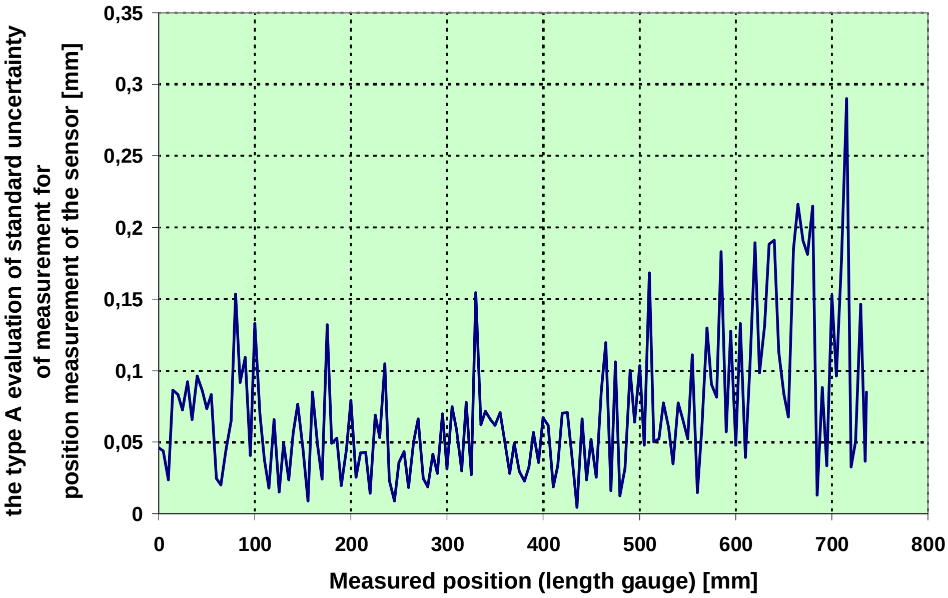


Figure 5 The standard uncertainty of position measurement

Temperature in laboratory has been maintained on value 20°C±1°C.

Expanded uncertainty of measurement U (2), obtained by multiplying the standard uncertainty $u(y)$ of the output estimate y by a coverage factor k [2],

$$U = k \cdot u \tag{2}$$

Coverage factor should be defined via sensor manufacturer, but datasheet has no information about it. Best way how to find value of coverage factor is experiment. It is known that coverage factor depends on measurement data distribution.

Identification of the measurement data distribution has done for four random selected values from sensor range. Every value has been measured 100 times at the same conditions. These values have been evaluated into histograms. One of them is shown on figure 6. All explored values are distributed according to Normal law of distribution of measured values. It means that for significance level P=0.95 is coverage factor equals to value 2.

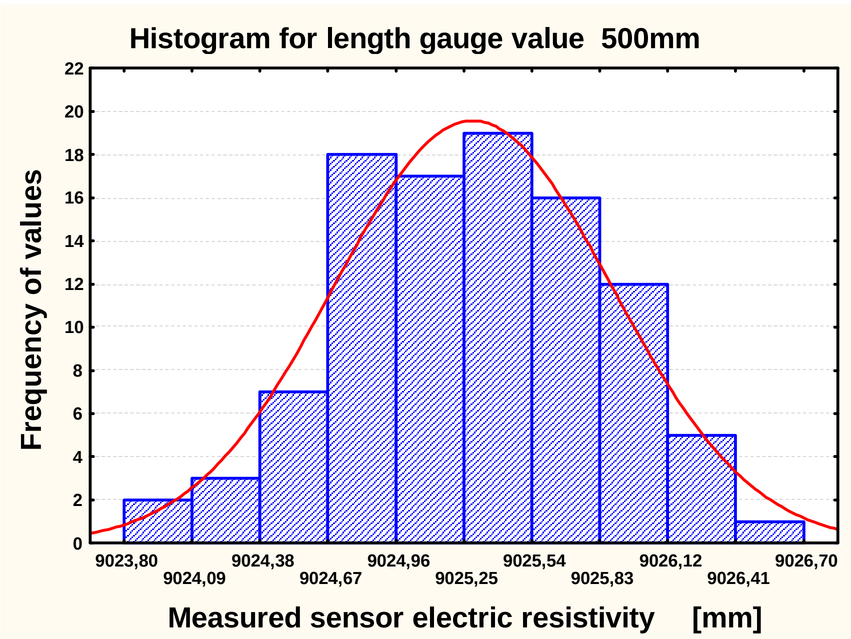


Figure 6 Measurement sensor chain - measurement data distribution

5 Conclusion

Figure 7 shows the expanded uncertainty for position measurement. Expanded uncertainty means the interval about mean value (obtained as average of measured data) where is located true value of measurement with probability 95%.

The expanded uncertainty means how can we believe to examined sensor in measurement process. The expanded uncertainty is as inseparable part of measurement result [14-32].

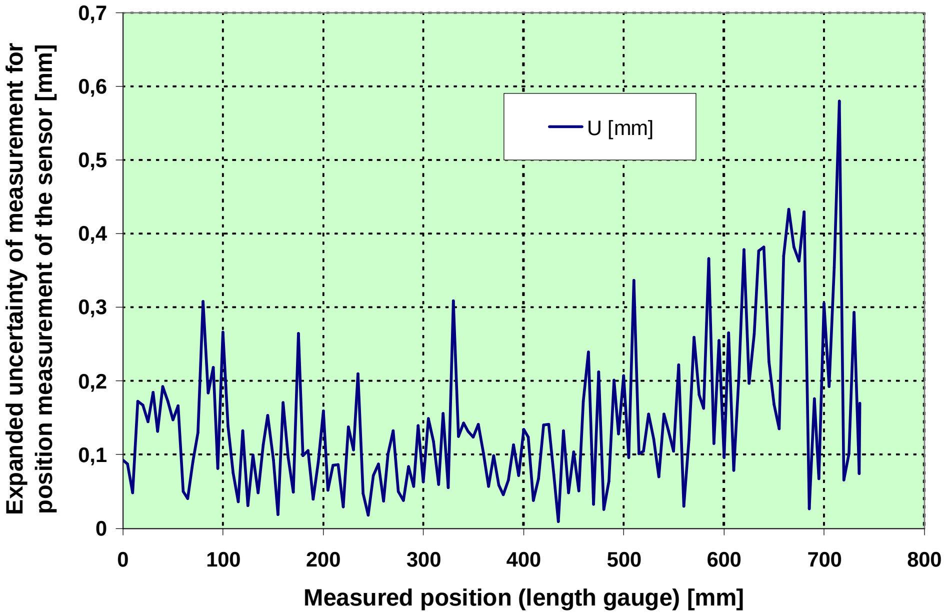


Figure 7 Measurement sensor chain – expanded uncertainty

Acknowledgement

The work has been accomplished under the research project APVV-15-0149, VEGA 1/0224/18, KEGA 006STU-4/2018 financed by the Slovak Ministry of Education.

References

[1] PAWLAK, A. M.: *Sensors and Actuators in Mechatronics*, CRC Press, Taylor & Francis Group, 2007.

[2] EA-4/02 *Expression of the Uncertainty of Measurement in Calibration*, European co-operation Accreditation Publication Reference, December 1999.

[3] JURIŠICA, L., VITKO, A., DUCHOŇ, F., KAŠTAN, D.: Statistical Approach to GPS Positioning of Mobile Robot, *Control Engineering and Applied Informatics*, Vol. 12, No. 2, p. 44-51, 2010.

[4] VITKO, A., JURIŠICA, L., KĽÚČIK, M., MURÁR, R., DUCHOŇ, F.: Sensor Integration and Context Detection in Mechatronic Systems, In: *Mechatronika* 2008, Proceedings of 11th International Conference on Mechatronics. Slovakia, Trenčianske Teplice, June 4-6, 2008. - Trenčín : Trenčianska univerzita Alexandra Dubčeka v Trenčíne, pp. 49-53, 2008.

[5] KONIAR, D., HARGAŠ, L. and HRIANKA, M.: Application of standard DICOM in LabVIEW, Proc. of 7th conf. *Trends in Biomedical Engineering*, Kladno 11. – 13. 9. 2007, 2007.

[6] VITKO, A., JURIŠICA, L., KĽÚČIK, M., MURÁR, R., Duchoň, F.: Embedding Intelligence Into a Mobile Robot, *AT&P Journal Plus*, No. 1: Mobile robotic systems (2008), pp. 42-44, 2008.

[7] BOŽEK, P.: Robot path optimization for spot welding applications in automotive industry, *Tehnicki vjesnik / Technical Gazette*, Sep/Oct2013, Vol. 20, Issue 5, pp. 913-917, 2013.

[8] DUCHOŇ, F., BABINEC, A., KAJAN, M., BEŇO, P., FLOREK, M., FICO, T., JURIŠICA, L.: Path planning with modified a star algorithm for a mobile robot, *Procedia Engineering*, Vol. 96, pp. 59-69, 2014.

[9] PÁSZTÓ, P., HUBINSKÝ, P.: Mobile robot navigation based on circle recognition, *Journal of Electrical Engineering, Vol.* 64, No. 2, pp. 84-91, 2013.

[10] ABRAMOV, I. V., NIKITIN, Y. R., ABRAMOV, A. I., SOSNOVICH, E. V., BOŽEK, P.: Control and Diagnostic Model of Brushless DC Motor, *Journal of Electrical Engineering*, Vol. 65, Issue 5, pp. 277–282, 2014.

[11] KONIAR, D., HARGAŠ, L., ŠTOFAN, S.: Segmentation of Motion Regions for Biomechanical Systems, *Procedia Engineering*, Vol. 48, pp. 304-311, 2012.

[12] FATIKOW, S. & REMBOLD. U.: *Microsystem Technology and Microrobotics*. Berlin Heidelberg, Springer-Verlag, 1997.
[13] CHUDÝ, V., PALENČÁR, R., KUREKOVÁ, E., HALAJ, M.: *Measurement of technical quantities*, Edition of STU, 1st ed., 1999. (Original in Slovak)
[14] JCGM 100 – *Evaluation of measurement data – Guide to the expression of uncertainty in measurement* (ISO/IEC Guide 98-3). First edition September 2008. Online, Available: http://www.iso.org/sites/JCGM/GUM-JCGM100.htm; http://www.bipm.org/en/publications/guides/gum_print.html
[15] JCGM 104 – *Evaluation of measurement data – An introduction to the "Guide to the expression of uncertainty in measurement"* (ISO/IEC Guide 98-1). First edition July 2009. Online, Available: http://www.bipm.org/en/publications/guides/gum_print.html
[16] JCGM 200 - *International vocabulary of metrology – Basic and general concepts and associated terms (VIM) 3rd edition* (2008 version with minor corrections). © JCGM 2012, Online, Available: http://www.iso.org/sites/JCGM/VIM-JCGM200.htm
[17] KREITH, F.: *The Mechanical Engineering Handbook* Series, CRC PRESS, New York, 1999.
[18] MELOUN, M., MILITKÝ, J.: *Statistical analysis of experimental data*, Praha: Academia, 2004. (Original in Czech)
[19] EA-4/02 *Expression of the Uncertainty of Measurement in Calibration*, European co-operation Accreditation Publication Reference, December 1999.
[20] MSA 104/97 *Expression of the Uncertainty of Measurement in Calibration*, (EAL-R2) - Expression of the Uncertainty of Measurement in Calibration, Slovenská národná akreditačná služba, SNAS BRATISLAVA, December 1997.
[21] MSA 104/D1-98 *Appendix 1 for MSA 104-97 Expressing of measurement uncertainties in Calibration* (in Slovak) (EAL-R2-S1), (EA-4/02-S1) Supplement 1 to EAL-R2 Expression of the uncertainty of measurement in calibration. Slovak national accreditation service, SNAS BRATISLAVA, October 1998.
[22] MSA-L/11 *Guidelines on the expression of uncertainty in quantitative testing (In Slovak)* (EA - 4/16: 2003). Guidelines on the expression of uncertainty in quantitative testing. Slovak national accreditation service, SNAS BRATISLAVA, August 2009.
[23] MSA–L/12 *Expression of the uncertainty of measurement in calibration (In Slovak)* (EA-4/02) - Expression of the uncertainty of measurement in calibration, Slovak national accreditation service, SNAS BRATISLAVA, November 2010.
[24] TAYLOR, B. N. and KUYATT, C. E.: *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994.
[25] TPM 0050-92 *Etalons. Expressing of errors and uncertainties*, Metrological Technical Directive, Slovak Metrological Institute, Bratislava, 1992. (Original in Slovak)
[26] TPM 0051-93 *Expressing of uncertainties in measurement*, Metrological Technical Directive, Slovak Metrological Institute, Bratislava, 1993. (Original in Slovak)
[27] WIMMER, G., PALENČÁR, R., WITKOVSKÝ, V.: *Stochastic models of measurement*, Graphic Studio Ing. Peter Juriga, Ľ. Fullu 13, 841 05 Bratislava. 1st ed., 2001. (Original in Slovak)
[28] PALENCAR, R., SOPKULIAK, P., PALENCAR, J. et al.: *Application of Monte Carlo Method for Evaluation of Uncertainties of ITS-90 by Standard Platinum Resistance Thermometer*, Measurement Science Review, Vol. 17, Issue 3, pp. 108-116, Jun 2017.
[29] SOPKULIAK, P., PALENCAR, R., PALENCAR, J., et al.: Evaluation of Uncertainties of ITS-90 by Monte Carlo Method, Conference: *6th Computer Science On-Line Conference* (CSOC) Location: Zlin, CZECH REPUBLIC Date: Apr., 2017, CSOC2017, Vol. 2, Book Series: Advances in Intelligent Systems and Computing , Vol. 574, pp. 46-56, 2017.
[30] SPANIKOVA, G., SPANIK, P., FRIVALDSKY, M. et al.: Electric model of liver tissue for investigation of electrosurgical impacts, *Electrical Engineering*, Vol. 99, Issue 4. pp 1185–1194, 2017. https://doi.org/10.1007/s00202-017-0625-0.
[31] KURIC, I., BULEJ, V., SAGA, M., POKORNÝ, P.: Development of simulation software for mobile robot path planning within multilayer map system based on metric and topological maps, *International Journal of Advanced Robotic Systems* Vol. 14, Issue 6, pp. 1-14, 2017. https://doi.org/10.1177/1729881417743029.
[32] HARGAS, L., KONIAR, D., SIMONOVA, A., HRIANKA, M., LONCOVA, Z.: Novel Machine Vision Tools Applied in Biomechatronic Tasks, *Procedia Engineering*, Vol. 96, pp. 148-156, 2014.

Review process
Single blind peer review process.

Acta Mechatronica – International Scientific Journal about Mechatronics
Volume: 3 2018 Issue: 1 Pages: 7-12 ISSN 2453-7306

doi:10.22306/am.v3i1.28

Received: 05 Feb. 2018
Accepted: 19 Feb. 2018

NEW METHOD OF THREE-POINT VIBRATION MEASUREMENTS OF TENSILE MODULUS OF THIN SAMPLES – AND ITS APPLICATION TO THE VARIETY OF SPECIMENS

Karol Kvetan
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, karol.kvetan@stuba.sk (corresponding author)
Janette Kotianová
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Applied Informatics, Automation and Mechatronics, Bottova 25, 917 24 Trnava, Slovak Republic, janette.kotianova@stuba.sk
Ondrej Bošák
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, ondrej.bošák@stuba.sk
Marián Kubliha
Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Materials Science, Bottova 25, 917 24 Trnava, Slovak Republic, marian.kubliha@stuba.sk
Martin Kotian
Slovak University of Technology, Faculty of Materials Science and Technology
Bottova 25, 917 24 Trnava, Slovak Republic, xkotian@is.stuba.sk

Keywords: Tensile modulus, three-point bending, thin samples, dynamic and static methods, uncertainty of measurement

Abstract: We describe an improved method of measuring the modulus of elasticity by means of three-point bending, based on dynamic approach. This method is particularly suitable for relatively short thin specimens, and – in addition - with a wide range of shaped cross-sectional variety. In conclusion, we present a comparison of this method with the classical static one for a standard circular sample.

1 Introduction

Tensile modulus (also called elastic modulus or Young's modulus) E is a constant that describes the material´s mechanical property of stiffness and is expressed as the ratio of stress to strain for a material experiencing tensile or compressive stress.

There exist several ways for measuring this quantity. Mechanical bending phenomena belong to the most beneficial principles. The three-point bend method, for example, is particularly suited for measuring thin samples such as bars, wires, strings, fibers, stalks, and the like.

A sample of length l is firmly attached to its end points. When extinguished by the force F, the sample bends into the arc., as it is issustrated in Figure 1.

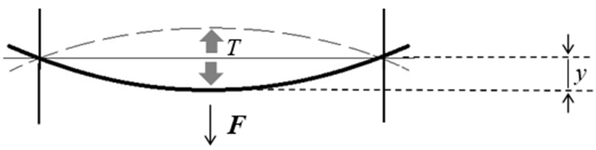


Figure 1 Arrangement principle at three-point bending experiments. A wire sample is bent by acting of force F, and vibrates with the period T when it is released

We can determine the modulus of elasticity in two ways:

1) Statically – by measuring the deflection y. After mechanical calculations, we get a relationship for the modulus of elasticity

$$E = \frac{Fl^3}{48yJ_A} \tag{1}$$

2) Dynamically - we release a tension force and we let the sample with the mass m to vibrate freely. We determine the modulus using the period T of oscillations being obtained from the relationship

$$E = \frac{4\pi^2 l^3 m}{3T^2 J_A} \tag{2}$$

The quantity of J_A means the areal moment of inertia with respect to the bending axis and is different for each shape of the sample. An overview of relations for its calculation is given in the table 1 (see section *2.3*).

The first method has a relatively wide use in practice. The second method is used less in the classical configuration due to several disadvantages: the magnitudes

Acta Mechatronica – International Scientific Journal about Mechatronics
Volume: 3 2018 Issue: 1 Pages: 7-12 ISSN 2453-7306

are fast, they have a small amplitude and they drop out quite quickly. The stabilization solution by stretching the sample into sides (like it is in the cases of guitar or violin strings, for example) is not appropriate; the magnitudes will be even faster and with less amplitude like before.

A very good and simple solution is the slowing of vibrations with damping flywheels. The magnitudes will be significantly slower, and the freewheeling of the flywheels and their permissible rotation in terms of sample bends will also create a large amplitude of vibrations and their longer duration, too. Such the counting of oscillations is even manageable "by the naked eyes", without the necessity of electronic instruments.

2 Technical and theoretical analysis of measurements

2.1 Description of the apparatus

Such a device (also known as Searle's pendulum) is illustrated in Figure 2.

It consists of three main parts: between two hinge yarns 1 are fixed horizontally the flywheels (cylindrical or prismatic) 2, they are connected by the measured sample 3; this one basically represents the element of "coupling". Usually it is in the form of wire, but it can also be a thin rod, thread or thin prismatic tape. Symmetrical deflection of the flywheels in the horizontal direction by the angle φ performs the bending oscillating movement of the sample that is reversely transmitted to the oscillating rotary motion of flywheels - and vice versa. Both parts of a pendulum – i.e. flywheels and sample - oscillate synchronously, with the same frequency and phase.

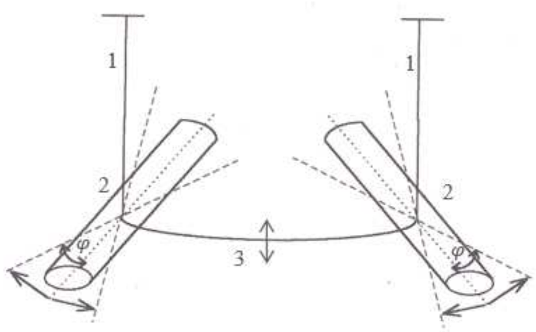


Figure 2 Three-point bending pendulum with the slowing flywheels
1 – hanging threads, 2 – cylinder flywheels, 3– measured sample (arrows indicate the direction of the oscillations)

2.2 Dynamic analysis of vibrations

The physical principle of measurement is based on the transformation of the sample bending vibrations into the oscillations of the flywheels. We proceed from the assumption that the bending moment of the wire is equal to the mechanical momentum of the force causing the rotation of the flywheels.

As we know from the mechanics, the total bending moment of the wire when deflected by the angle φ is

$$M = K \cdot \varphi = \frac{2EJ_A}{l}\varphi \,, \tag{3}$$

where K is so-called directional moment, i.e. moment of power, required for bending of wire by unit angel. The mechanical moment of the rotation force depends on the momentum of the inertia J of the flywheels; they are deflecting the same angle. We can express this process of energy transformation by the motion equation of the rotating body as

$$M = -J \cdot \varepsilon \,, \tag{4}$$

where $\varepsilon = d^2\varphi/dt^2$ is the angular acceleration of vibrations. After incorporating these statements into one relationship, we get them

$$\frac{d^2\varphi}{dt^2} + \frac{2EJ_A}{lJ}\varphi = 0 \,. \tag{5}$$

It is a known differential equation for oscillating motion; its solution is

$$\varphi = \varphi_0 \sin(\omega t + \alpha) \,, \tag{6}$$

where φ_0 is the amplitude of the oscillating motion, and α is a phase shift between zero time and moment of minimum deflection. In this relation, the new variable ω has appeared. It is the circular frequency of the oscillations that it is the result of

$$\omega = \sqrt{\frac{K}{J}} = \sqrt{\frac{2EJ_A}{lJ}} \tag{7}$$

and which is related to the oscillation period T by a known relationship

$$\omega = \frac{2\pi}{T} \tag{8}$$

We shall get the resulting relationship for E from both of these expressions

$$E = \frac{4\pi^2}{T^2} \cdot \frac{lJ}{2J_A} \tag{9}$$

The desired modulus of elasticity thus can be determined by measuring the oscillations of the sample in particular.

Acta Mechatronica - International Scientific Journal about Mechatronics
Volume: 3 2018 Issue: 1 Pages: 7-12 ISSN 2453-7306

In addition to the areal moment of inertia $J_{A,}$ there is acting also the moment oj inertia J of the damping flywheels. Nnow we must differentiate the type of flywheels, too. In the case of cylindrical flywheels, as well as in our picture, the moment of inertia is given by the known relationship

$$J = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right). \qquad (10)$$

The parameters R and L are the diameter and length of flywheels, and m means their (single) mass. In the case of square flywheels it would be

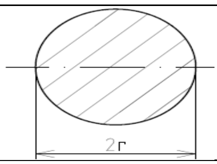
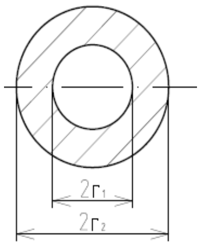
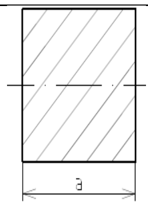
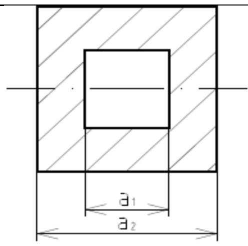
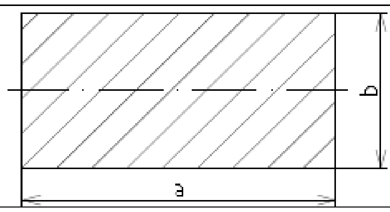
$$J = \frac{1}{12} m(A^2 + B^2), \qquad (11)$$

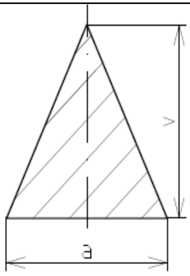
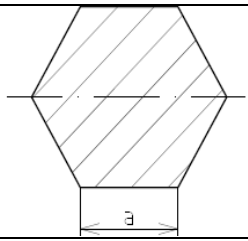
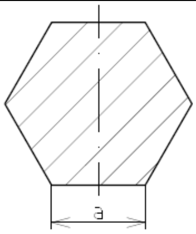
wherein A and B are the length and the width of the prism.

2.3 *Variety of samples*

In practice, the wire samples of round shape are mostly used. However, there exist also samples with different cross-sectional shapes. An overview of the most common ones is given in tab. 1.

Table 1 Overview of possible cross-sectional shapes of wire samples

Cross-section		Areal moment of inertia
Circle - full		$J_A = \frac{1}{4}\pi r^4$
Circle - hollow		$J_A = \frac{1}{4}\pi(r_1^4 - r_2^4)$
Square - full		$J_A = \frac{1}{12}a^4$
Square - hollow		$J_A = \frac{1}{12}(a_1^4 - a_2^4)$
Rectangle		$J_A = \frac{1}{12}ab^3$

Triangle		$J_A = \frac{1}{48} v a^3$
Hexagon (top axis)		$J_A = \frac{5\sqrt{3}}{16} a^4$
Hexagon (side axis)		$J_A = \frac{\sqrt{3}}{3} a^4$

In the last column there are the relations for the calculation of the quantity J_A, which is different for each sample and which stands out in relevant relations for E.

3 Experimental procedure

3.1 Measuring assembly

Our measuring device consist of two homogeneous steel rollers in the role of flywheels, each having a mass m = 0,72 kg, a length L = 137 mm and a radius r = 14,6 mm.

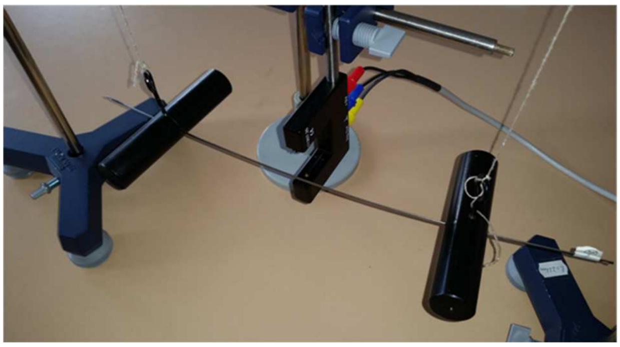


Figure 3 Experimental assembly. Vibrating wire sample crosses the infrared beam of an optical sensor (prismatic body with the shape of figure U in the centre of operation)

Size of moment of inertia of each of them, determined from the relation (10), had a value of $J = 1{,}15.10^{-3}$ kg.m^2. These were connected to one another via a wire sample being measured.

The oscillation times were scanned electronically, or by using a high-speed camera, respectively.

The photo of our device is shown in the Figure 3.

3.2 Results of measurements

We performed measurements of several samples of wires with different shapes of cross sections. All the samples had the same "active" length (i.e. the distance between the points of attachment to flywheels) l = 0,3 m. Relationships for determining the modulus of elasticity of individual samples were obtained using the presented dynamic analysis - namely the relation (9). In this relation we put the expressions for J_A for the sample in Table 1. (see the last column).

An overview of the measured samples, including the relevant geometric parameters and the measured values, is given in Table 2.

Table 2 Parameters of samples and the results of measurements

Sample	Period of oscillation T (s)	Modulus of elasticity (measured) E_{meas} (GPa)	Table valued parameter E_{tab} (GPa)
Steel I – circle full r = 1,00 mm	0,203	210	180 – 210
Steel II – circle full r = 1,25 mm	0,135	196	180 – 210
Copper – circle full r = 1 mm	0,281	110	103 – 126
Alluminium – circle full r = 2 mm	0,126	68	65 – 70
Brass – circle full r = 1,5 mm	0,131	101	86 – 105
Steel – circle hollow r_1 = 1,1 mm; r_2 = 0,75 mm	0,195	198	180 – 210
PVC – circle full r = 1,1 mm	1,429	2,9	2,5 – 3,0
Polyamide – circle full r = 1,0 mm	2,031	2,1	1 – 2,6
Polystyrene – circle full r = 1,5 mm	0,680	3,7	3,2 – 3,5
Polypropylene – circle hollow r_1 = 1,5 mm; r_2 = 1,0 mm	1,033	2,0	1,3 – 2
Polypropylene – square full a = 2 mm	1,559	2,1	1,3 – 2
Polypropylene – square hollow a_1 = 2,0 mm; a_2 = 1,0 mm	1,740	1,8	1,3 – 2
Steel – rectangular a = 1,2 mm; b = 2,7 mm	0,130	204	180 – 210
Polypropylene – rectangular a = 1,0 mm; b = 2,7 mm	2,526	0,65	0,25 – 0,70
Polypropylene – triangle a = 2,5 mm; v = 2,2 mm	2,261	1,8	1,3 – 2
Polyamide – hexagonal a = 1,5 mm (top axis)	1,287	1,5	1 – 2,6
Polyamide – hexagonal a = 1,5 mm (side axis)	1,338	1,3	1 – 2,6
Oak wood – circle full r = 2 mm (along the fibres)	0,262	8,8	10 – 13
Corn stalk – circle hollow r_1 = 2,8 mm; r_2 = 1,0 mm	0,167	5,1	4,2 – 7
Grass stalk – circle hollow r_1 = 1,5 mm; r_2 = 1,2 mm	0,910	3,5	inaccessible value

3.3 Statistical analysis - determination of uncertainty

A "central" sample to be subjected to a more detailed analysis will be a sample of full circular cross-section.

Dynamic analysis of the process [2], [3] here gives for the measured modulus E a final relationship

$$E = \frac{8\pi l J}{r^4 T^2} \quad . \tag{12}$$

Evaluation of the corresponding uncertainty is as follows:

We must consider that the determination of the modulus of elasticity - as can be seen from equation (2) - is a function of four variables x_i; namely $E = f(l, J, r, T)$. In this case - in accordance with theory of measurements - the uncertainty is given by a root, containing partial derivatives with respect to all of the relevant variables and uncertainties of the following variables:

$$u_E = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial E}{\partial x_i}u_{x_i}\right)^2} =$$
$$\sqrt{\left(\frac{\partial E}{\partial l}u_l\right)^2 + \left(\frac{\partial E}{\partial J}u_J\right)^2 + \left(\frac{\partial E}{\partial r}u_r\right)^2 + \left(\frac{\partial E}{\partial T}u_T\right)^2} \qquad (13)$$

The relevant partial derivatives are

$$\frac{\partial E}{\partial l} = \frac{\partial \frac{8\pi l J}{r^4 T^2}}{\partial l} = \frac{8\pi J}{r^4 T^2}$$
$$\frac{\partial E}{\partial J} = \frac{\partial \frac{8\pi l J}{r^4 T^2}}{\partial J} = \frac{8\pi l}{r^4 T^2}$$
$$\frac{\partial E}{\partial r} = \frac{\partial \frac{8\pi l J}{r^4 T^2}}{\partial r} = -\frac{32\pi l J}{r^5 T^2}$$
$$\frac{\partial E}{\partial T} = \frac{\partial \frac{8\pi l J}{r^4 T^2}}{\partial T} = -\frac{16\pi l J}{r^4 T^3}$$

We felt the precision of measuring instruments for applying the uncertainties of them as the size of the smallest pieces on their scales. So:

u_L = 0,1 mm (sliding ruler)
u_r = 0,01 mm (micrometer)
u_l = 1 mm (ruler)
u_T = 0,005 s (stopwatch)
u_m = 1 g = 0,001 kg (laboratory scales).

(Quantities u_L and u_m had been used for deriving of the uncertainty u_J. Substituting into (5) gives a value of $u_J = 1{,}5.10^{-5}$ kg.m^2).

The value of a numerical expression of uncertainty u_E gives a value of u_E = 14,48 GPa, which represents about 7,8 % against the size of the module being measured.

So, the final result can be written as E = (185,64 ± 14,48) GPa, resp. E = 185,64 GPa ± 7,8 %.

The value of total uncertainty is given by the sum of partial uncertainties of types A and B. As we know from the theory of measurement, the causes of uncertainty A are unknown. However, the causes of uncertainty B it is not difficult to determine, they related with an accuracy of instruments, uncertainty in the readings and air resistance against the vibrating motion. Other factors, such as a non-uniformity of wire thickness, directional moment of the hanging threads, heating the samples as a result of oscillations etc. are negligible.

3.4 Comparison with classical bending methods

For aim of comparison, we have tried to determine the modulus of elasticity using the conventional bending methods, both static and dynamic (see the introduction to this article). Calculations were performed according to relations (1) and (2). Using the same statistical procedure as in the previous case, we obtained the following results:

Static method: E = 191,56 GPa ± 4,3 % .

Dynamic method: E = 188,26 GPa ± 6,4 % .

4 Conclusion

Our equipment is less accurate than standard bending methods [5], as we can see by comparison of measurement results, mainly by means of uncertainties. But, on the other hand, it has two significant benefits:

1. The speeds of vibrations are diminished by means of flywheels, which is particularly valuable for samples with fast free oscillations. The time periods are therefore easier to measure.
2. Our system is phase-stable, it does not “tune-out” even after several tens or hundreds of oscollations. The benefit is also the possibility of measuring non-standard samples, without the risk of permanent damage.

In addition, the results are sufficiently precise, as evidenced by the fact that the measured values are well correlated with the table values.

This method can be used successfully in the wires, plastic and textile industries (investigation of elasticity of thin materials), in botany (elasticity of stalks) and the like. As so as a demonstration chapter in university textbook (section of “Vibrating Movements” or “Solid State Physics”), or a task for laboratory exercises.

Anknowledgment
The article was created within framework of the project VEGA 1/0238/17 (Slovak Republic).

References

[1] FRIŠ, S.E., TIMOREVA, A.V.: *Kurz fysiky I*, NČSAV, 1957. (Original in Slovak)
[2] BRAWN, R.: *General Properties of Matter*, London, Butherwords, 1969.
[3] TIMOSHENKO S. , YOUNG D. H., WEAWER W.: *Vibration Problems in Engineering*, John Wiley and Sons, New York, 1974.
[4] MILJOJKOVIĆ, J., BIJELIĆ, I., VRANIČ, N., RADOVANOVIČ, N., ŽIVKOVIČ, M.: Determining Elastic Modulus of the Material by Measuring the Deflection of the Beam Loaded in Bending, *Tehnički vjesnik*, Vol. 24, No. 4, p.1227-1234, 2017.
[5] ŠTUBŇA, I., KOZÍK, T.: Factors Affecting the Accuracy of the Measurement Elastic Modulus and Mechanical Strength, *Sklář a keramik*, Vol. 1979, No. 8, p. 228-230, 1979. (Original in Czech)

Review process
Single-blind peer review process.