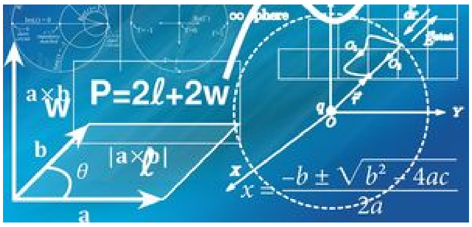
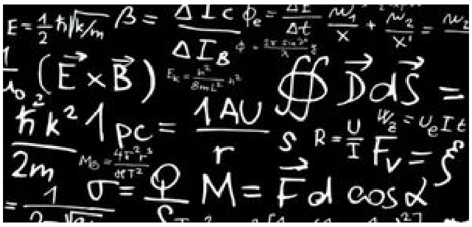
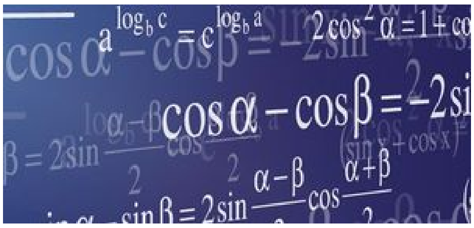




ISSN 2453-6881

2018, vol. 4, no. 1

{MERAA} Mathematics in Education, Research and Applications

Research in mathematics and related fields

Education in mathematics at all types of schools

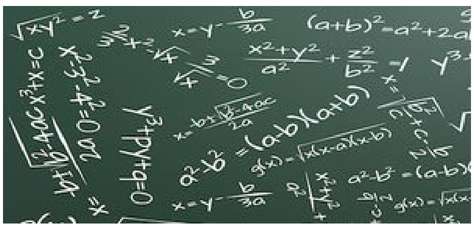
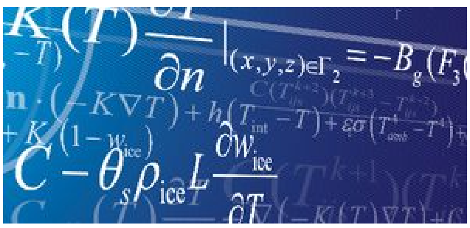
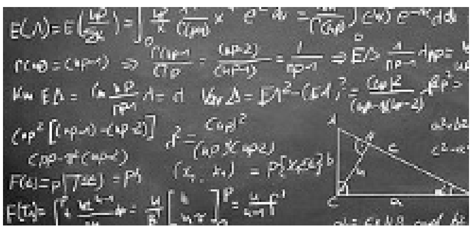
Applications of mathematics in teaching, research and practice

Methodology of mathematics

Information technology tools in mathematics

Cross-curricular relations with mathematics

Slovak University of Agriculture in Nitra :: Department of Mathematics, Faculty of Economics and Management

Editorial Team

Mathematical scholarly, peer-reviewed, open access, online journal.

Publisher: Slovak University of Agriculture in Nitra :: Department of Mathematics

ISSN: 2453-6881

Journal Website: http://meraa.uniag.sk

Journal acronym: MERAA

Journal title abbreviation: Math Educ Res Appl

Periodicity: 2 times per year.

Content

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Received: 2018-10-03
Accepted: 2018-10-29
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.1-8

Original Paper

The sum of the series of reciprocals of the cubic polynomials with three different positive integer roots

Radovan Potůček*

University of Defence in Brno, Faculty of Military Technology, Department of Mathematics and Physics, Czech Republic

ABSTRACT

In this contribution we deal with the sum of the series of reciprocals of the cubic polynomials with different positive integer roots. We derive the formula for the sum of these series and verify it by some examples using the basic programming language of the computer algebra system Maple 16. This paper can be an inspiration for teachers who are teaching the topic Infinite series or as a subject matter for work with talented students.

KEYWORDS: sum of the series, telescoping series, harmonic number, digamma function, Euler's constant, computer algebra system Maple

JEL CLASSIFICATION: I30

INTRODUCTION

This contribution is a follow-up to author's papers [2] and [3]. We deal with the sum of the series of reciprocals of the cubic polynomials with different positive integer roots.

Let us recall some basic terms. The series

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots$$

converges to a limit s if and only if the sequence of partial sums $\{s_n\} = \{a_1 + a_2 + \cdots + a_n\}$ converges to s, i.e. $\lim_{n\to\infty} s_n = s$. We say that the series has a *sum* s and write $\sum_{k=1}^{\infty} a_k = s$. The *sum of the reciprocals* of some positive integers is generally the sum of unit fractions. For example the sum of the reciprocals of the square numbers (the *Basel problem*) is

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6} \doteq 1.644934.$$

* Corresponding author: Radovan Potůček, Kounicova 65, 662 10 Brno, Czech Republic, e-mail: Radovan.Potucek@unob.cz

The n −th *harmonic number* is the sum of the reciprocals of the first n natural numbers:

$$H_n=\sum_{k=1}^{n}\frac{1}{k}=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n},$$

H_0 being defined as 0. Basic information about harmonic numbers can be found e.g. in the web-site [4] or in [1].

The *telescoping series* is any series where nearly every term cancels with a preceding or following term, so its partial sums eventually only have a fixed number of terms after cancellation. For example, the series

$$\sum_{k=1}^{\infty}\frac{1}{(k-1)(k-2)(k-3)},$$

where obviously the summation index $k\neq 1,2,3$, i.e. $k=4,5,\ldots$, has the general k −th term, after partial fraction decomposition, in a form

$$a_k=\frac{1}{(k-1)(k-2)(k-3)}=\frac{1}{2}\left(\frac{1}{k-1}-\frac{2}{k-2}+\frac{1}{k-3}\right).$$

After that we arrange the terms of the n −th partial sum $s_n=a_4+a_5+\cdots+a_n$ in a form where can be seen what is cancelling. Then we find the limit $\lim_{n\to\infty}s_n$ of the sequence of the partial sums s_n in order to find the sum s of the infinite telescoping series. In our case we get

$$s_n=\frac{1}{2}\left[\left(\frac{1}{3}-\frac{2}{2}+\frac{1}{1}\right)+\left(\frac{1}{4}-\frac{2}{3}+\frac{1}{2}\right)+\left(\frac{1}{5}-\frac{2}{4}+\frac{1}{3}\right)+\cdots+\left(\frac{1}{n-3}-\frac{2}{n-4}+\frac{1}{n-5}\right)+\right.$$
$$\left.+\left(\frac{1}{n-2}-\frac{2}{n-3}+\frac{1}{n-4}\right)+\left(\frac{1}{n-1}-\frac{2}{n-2}+\frac{1}{n-3}\right)\right]=\frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{n-2}+\frac{1}{n-1}\right).$$

So we have

$$s=\lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{n-2}+\frac{1}{n-1}\right)=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}.$$

THE SUM OF THE SERIES OF RECIPROCALS OF THE CUBIC POLYNOMIALS WITH THREE DIFFERENT POSITIVE INTEGER ROOTS

Let us consider the series of reciprocals of the normalized cubic polynomials with three different positive integer roots $a<b<c$, i.e. the series

$$\sum_{\substack{k=1\\k\neq a,b,c}}^{\infty}\frac{1}{(k-a)(k-b)(k-c)},\tag{1}$$

and let us determine its sum $s(a,b,c)$. This series can be split into four parts:

$$s(a,b,c)=\sum_{k=1}^{a-1}\frac{1}{(k-a)(k-b)(k-c)}+\sum_{k=a+1}^{b-1}\frac{1}{(k-a)(k-b)(k-c)}+$$
$$+\sum_{k=b+1}^{c-1}\frac{1}{(k-a)(k-b)(k-c)}+\sum_{k=c+1}^{\infty}\frac{1}{(k-a)(k-b)(k-c)}.\tag{2}$$

The symbol of vertical bar hereinafter used in some of three positions before three letters means that corresponding finite sum in the relation (2) is omitted. For example a notation $s(|a|b,c)$ denotes the case, where $a=1$, $b=2$ and $c-1\geq b+1=3$, i.e. the sum $s(|1|2,c)$.

In total, we differentiate $2^3=8$ following possible cases of the sums: $s(|1|2|3)$, $s(|1|2,c)$, $s(|1,b|b+1)$, $s(a|a+1|a+2)$, $s(a,b|b+1)$, $s(a|a+1,c)$, $s(|1,b,c)$, $s(a,b,c)$.
We focus on the last and most general case of the relation (2), where $0<a<b<c$, $a\geq 2$, $b-1\geq a+1$, $c-1\geq b+1$, and determine the sum $s(a,b,c)$ using the equality

$$\frac{1}{(k-a)(k-b)(k-c)}=\frac{1}{A(k-a)}-\frac{1}{B(k-b)}+\frac{1}{C(k-c)},$$

where $A=(b-a)(c-a)$, $B=(b-a)(c-b)$, $C=(c-a)(c-b)$.
Because

$$\sum_{k=1}^{a-1}\frac{1}{k-a}=\frac{1}{1-a}+\frac{1}{2-a}+\cdots-\frac{1}{2}-\frac{1}{1}=-\left(\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{a-2}+\frac{1}{a-1}\right)=-H_{a-1},$$

$$\sum_{k=1}^{a-1}\frac{1}{k-b}=\frac{1}{1-b}+\frac{1}{2-b}+\cdots+\frac{1}{a-b-2}+\frac{1}{a-b-1}=H_{b-a}-H_{b-1},$$

$$\sum_{k=1}^{a-1}\frac{1}{k-c}=\frac{1}{1-c}+\frac{1}{2-c}+\cdots+\frac{1}{a-c-2}+\frac{1}{a-c-1}=H_{c-a}-H_{c-1},$$

then the sum $s'(a,b,c)$ of the first finite part of the series (2) is

$$s'(a,b,c)=\frac{-H_{a-1}}{A}-\frac{H_{b-a}-H_{b-1}}{B}+\frac{H_{c-a}-H_{c-1}}{C}.$$

Now, let us determine the sum $s''(a,b,c)$ of the second finite part of the series (2). Because

$$\sum_{k=a+1}^{b-1}\frac{1}{k-a}=\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{b-a-2}+\frac{1}{b-a-1}=H_{b-a-1},$$

$$\sum_{k=a+1}^{b-1}\frac{1}{k-b}=\frac{1}{a-b+1}+\frac{1}{a-b+2}+\cdots-\frac{1}{2}-\frac{1}{1}=-H_{b-a-1},$$

$$\sum_{k=a+1}^{b-1}\frac{1}{k-c}=\frac{1}{a-c+1}+\frac{1}{a-c+2}+\cdots+\frac{1}{b-c-2}+\frac{1}{b-c-1}=H_{c-b}-H_{c-a-1},$$

then the sum $s''(a,b,c)$ of the second finite part of the series (2) is

$$s''(a,b,c)=\frac{H_{b-a-1}}{A}+\frac{H_{b-a-1}}{B}+\frac{H_{c-b}-H_{c-a-1}}{C}=\left(\frac{1}{A}+\frac{1}{B}\right)H_{b-a-1}+\frac{H_{c-b}-H_{c-a-1}}{C}.$$

Now, we determine the sum $s'''(a,b,c)$ of the third finite part of the series (2). Because

$$\sum_{k=b+1}^{c-1}\frac{1}{k-a}=\frac{1}{b-a+1}+\frac{1}{b-a+2}+\cdots+\frac{1}{c-a-2}+\frac{1}{c-a-1}=H_{c-a-1}-H_{b-a},$$

$$\sum_{k=b+1}^{c-1} \frac{1}{k-b} = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{c-b-2} + \frac{1}{c-b-1} = H_{c-b-1},$$

$$\sum_{k=b+1}^{c-1} \frac{1}{k-c} = \frac{1}{b-c+1} + \frac{1}{b-c+2} + \cdots - \frac{1}{2} - \frac{1}{1} = -H_{c-b-1},$$

then the sum $s'''(a,b,c)$ of the third finite part of the series (2) is

$$s'''(a,b,c) = \frac{H_{c-a-1} - H_{b-a}}{A} - \frac{H_{c-b-1}}{B} - \frac{H_{c-b-1}}{C} = \frac{H_{c-a-1} - H_{b-a}}{A} - \left(\frac{1}{B} + \frac{1}{C}\right) H_{c-b-1}.$$

Finally, let us express the partial sum $s_n(a,b,c)$ of the infinite part of the series (2). We have

$$s_n(a,b,c) = \sum_{k=c+1}^{n} \left[\frac{1}{(a^2 - ab - ac + bc)(k-a)} + \frac{1}{(b^2 - ab + ac - bc)(k-b)} + \frac{1}{(c^2 + ab - ac - bc)(k-c)}\right].$$

By means of the computer algebra system Maple 16 we get the following worksheet:

```
> sum((1/(((b-a)^2+(b-a)*(c-b))*(k-a))-1/((b-a)*(c-b)*(k-b))
+1/(((b-a)*(c-b)+(c-b)^2)*(k-c))),k=c+1..n);
```

$$-\frac{\Psi(n+1-b)}{(-c+b)(-b+a)} + \frac{\Psi(n+1-a)}{(a-c)(-b+a)} + \frac{\Psi(n+1-c)}{(-c+b)(a-c)} + \frac{\Psi(c+1-b)}{(-c+b)(-b+a)} - \frac{\Psi(c+1-a)}{(a-c)(-b+a)} + \frac{\gamma}{(-c+b)(a-c)}$$

```
> limit(Psi(n+1-a)/((a-b)*(a-c))-Psi(n+1-b)/((a-b)*(b-c))+Psi(n+1-
c)/((a-c)*(b-c)),n=infinity);
```

$$0$$

```
> simplify(Psi(c+1-b)/((a-b)*(b-c))-Psi(c+1-a)/((a-b)*(a-c))
+gamma/((a-c)*(b-c)));
```

$$\frac{\Psi(c+1-b)a - \Psi(c+1-b)c + \Psi(c+1-a)c - \Psi(c+1-a)b - \gamma b + a\gamma}{(-b+a)(a-c)(-c+b)}$$

Let us note that $\Psi(x)$ is the *digamma function*, $\Psi(x) = \frac{\mathrm{d}}{\mathrm{d}x}\ln(\Gamma(x)) = \frac{\frac{\mathrm{d}}{\mathrm{d}x}\Gamma(x)}{\Gamma(x)}$, where $\Gamma(x)$ is the *Gamma function* and that *Euler's constant* or *Euler-Mascheroni constant*

$$\gamma = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{i} - \ln(n)\right) \doteq 0.5772156649 \dots.$$

Altogether, for $0 < a < b < c$, $a \geq 2$, $b - 1 \geq a + 1$ and $c - 1 \geq b + 1$, we get the sum $s(a,b,c)$ of the series (2) in the form

$$s(a,b,c)=s'(a,b,c)+s''(a,b,c)+s'''(a,b,c)+\lim_{n\to\infty}s_n(a,b,c)=\frac{-H_{a-1}}{A}$$
$$-\frac{H_{b-a}-H_{b-1}}{B}+\frac{H_{c-a}-H_{c-1}}{C}+\left(\frac{1}{A}+\frac{1}{B}\right)H_{b-a-1}+\frac{H_{c-b}-H_{c-a-1}}{C}+\frac{H_{c-a-1}-H_{b-a}}{A}$$
$$+\frac{(a-c)\Psi(c+1-b)+(c-b)\Psi(c+1-a)+(a-b)\gamma}{(-b+a)(a-c)(-c+b)},$$

i.e.

$$s(a,b,c)=\left(\frac{1}{A}+\frac{1}{B}\right)H_{b-a-1}+\left(\frac{1}{A}-\frac{1}{C}\right)H_{c-a-1}-\left(\frac{1}{B}+\frac{1}{C}\right)H_{c-b-1}-\left(\frac{1}{A}+\frac{1}{B}\right)H_{b-a}$$
$$-\frac{H_{a-1}}{A}+\frac{H_{b-1}}{B}-\frac{H_{c-1}}{C}+\frac{H_{c-a}+H_{c-b}}{C}$$
$$+\frac{(a-c)\Psi(c+1-b)+(c-b)\Psi(c+1-a)+(a-b)\gamma}{(a-b)(a-c)(b-c)}.$$

Because

$$\frac{1}{A}+\frac{1}{B}=\frac{a+b-2c}{(a-c)(ab+bc-ac-b^2)},\qquad \frac{1}{B}+\frac{1}{C}=\frac{2a-b-c}{(a-b)(ab-ac-bc+c^2)},$$
$$\frac{1}{A}-\frac{1}{C}=\frac{-a+2b-c}{(a-b)(ab-ac-bc+c^2)},$$

we derived this statement:

Theorem 1. The series

$$\sum_{\substack{k=1\\k\neq a,b,c}}^{\infty}\frac{1}{(k-a)(k-b)(k-c)},$$

where $a\geq 2$, $b-1\geq a+1$ and $c-1\geq b+1$ are positive integers, has the sum

$$s(a,b,c)=\frac{(a+b-2c)(H_{b-a-1}-H_{b-a})}{(a-c)(ab+bc-ac-b^2)}+\frac{(-a+2b-c)H_{c-a-1}-(2a-b-c)H_{c-b-1}}{(a-b)(ab-ac-bc+c^2)}$$
$$+\frac{H_{c-a}+H_{c-b}-H_{c-1}}{ab-ac-bc+c^2}-\frac{H_{a-1}}{a^2-ab-ac+bc}+\frac{H_{b-1}}{ab+bc-ac-b^2}\qquad(3)$$
$$+\frac{(a-c)\Psi(c+1-b)+(c-b)\Psi(c+1-a)+(a-b)\gamma}{(a-b)(a-c)(b-c)},$$

where H_n is the $n-$th harmonic number, $\Psi(n)$ is digamma function, and γ is Euler's constant.

Remark 1. It can be shown that the first seven cases of the sums $s(|1|2|3)$, $s(|1|2,c)$, $s(|1,b|b+1)$, $s(a|a+1|a+2)$, $s(a,b|b+1)$, $s(a|a+1,c)$, $s(|1,b,c)$ are all special cases of the sum $s(a,b,c)$ expressed by the general formula (3).

Corollary 1. From Theorem 1 it follows that for $a \geq 1$ and integer $d \geq 1$ it holds

$$s(a, a+d, a+2d) = \frac{H_{2d} - H_{a-1} - 2H_d + 2H_{a+d-1} - H_{a+2d-1} + 2\Psi(d+1) - \Psi(2d+1) + \gamma}{2d^2}.$$

NUMERICAL VERIFICATION

We solve the problem to determine the values of the sum $s(a,b,c)$ for $a = 1,2,3,4$ and for $b = a+1, a+2, \dots, a+5$, $c = b+1, b+2, \dots, b+5$. We use on the one hand approximated evaluation of the sum

$$s(a,b,c,t) = \sum_{\substack{k=1 \\ k \neq a,b,c}}^{t} \frac{1}{(k-a)(k-b)(k-c)},$$

where $t = 10^7$, using the basic programming language of the computer algebra system Maple 16, and on the other hand the formula (3) for evaluation the sum $s(a,b,c)$. We compare 60 pairs of these ways obtained sums $s(a,b,c,t)$ and $s(a,b,c)$ to verify the formula (3). We use following simple procedure `rp3abcpos` and the following double repetition statement:

```
> rp3abcpos:=proc(a,b,c,t)
     local A,B,C,k,sabc,sabct; A:= a^2-a*b-a*c+b*c; B:=a*b+b*c-a*c-b^2;
     C:=a*b-a*c-b*c+c^2; sabct:=0;
     sabc:= (a+b-2*c)*(harmonic(b-a-1)-harmonic(b-a))/((a-c)*B)+((-a+2*b-c)
     *harmonic(c-a-1)-(2*a-b-c)*harmonic(c-b-1))/((a-b)*C)+(harmonic(c-a)
     +harmonic(c-b)-harmonic(c-1))/C-harmonic(a-1)/A+harmonic(b-1)/B+((a-c)
     *Psi(c+1-b)+(c-b)*Psi(c+1-a)+(a-b)*gamma)/((a-b)*(a-c)*(b-c));
     print("t=",t,"s(",a,b,c,")",evalf[12](sabc));
     for k from 1 to t do
         if k<>a then if k<>b then if k<>c then
            sabct:=sabct+1/((k-a)*(k-b)*(k-c))
         else sabct:=sabct+0; end if; end if; end if;
     end do;
     print("sum(",a,b,c,")=",evalf[12](sabct));
     print("diff=",evalf[12](abs(sabct-sabc)));
  end proc:

> for i from 1 to 4 do
     for j from i+1 to i+5 do
        for k from j+1 to i+6 do
           rp3abcpos(i,j,k,10000000);
        end do;
     end do;
  end do;
```

The approximated values of the sums $s(a,b,c)$ rounded to 10 decimals obtained by this procedure are written into the following table:

Table 1: The approximate values of the sums $s(a,b,c)$ for $a=1,2,3,4$, $b=a+1,a+2,\dots,a+5$ and $c=b+1,b+2,\dots,b+5$, obtained by means of the formula (3)

$\boldsymbol{a=1}$	$c=3$	$c=4$	$c=5$	$c=6$	$c=7$
$b=2$	0.2500000000	−0.3611111111	−0.3263888889	−0.2691666667	−0.2238888889
$b=3$	×	0.6944444444	0.1145833333	0.0355555556	0.0159722222
$b=4$	×	×	0.5763888889	0.1394444444	0.0675925926
$b=5$	×	×	×	0.4691666667	0.1256944444
$b=6$	×	×	×	×	0.3905555556
$\boldsymbol{a=2}$	$c=4$	$c=5$	$c=6$	$c=7$	$c=8$
$b=3$	0.0833333333	−0.4861111111	−0.4263888889	−0.3525000000	−0.2953174603
$b=4$	×	0.6111111111	0.0479166667	−0.0200000000	−0.0316468254
$b=5$	×	×	0.5263888889	0.0977777778	0.0318783069
$b=6$	×	×	×	0.4358333333	0.0971230159
$b=7$	×	×	×	×	0.3667460317
$\boldsymbol{a=3}$	$c=5$	$c=6$	$c=7$	$c=8$	$c=9$
$b=4$	0.0416666667	−0.5194444444	−0.4541666667	−0.3763095238	−0.3161507936
$b=5$	×	0.5861111111	0.0270833333	−0.0378571429	−0.0472718254
$b=6$	×	×	0.5097222222	0.0834920635	0.0193783069
$b=7$	×	×	×	0.4239285714	0.0867063492
$b=8$	×	×	×	×	0.3578174603
$\boldsymbol{a=4}$	$c=6$	$c=7$	$c=8$	$c=9$	$c=10$
$b=5$	0.0250000000	−0.5333333333	−0.4660714286	−0.3867261905	−0.3254100529
$b=6$	×	0.5750000000	0.0175595238	−0.0461904762	−0.0546792328
$b=7$	×	×	0.5017857143	0.0765476191	0.0132054674
$b=8$	×	×	×	0.4179761905	0.0814153439
$b=9$	×	×	×	×	0.3531878307

Source: own modelling in Maple 16

Computation of 60 pairs of the sums $s(a,b,c)$ and $s(a,b,c,10^7)$ took almost 2 hours. The absolute errors, i.e. the differences $|s(a,b,c)-s(a,b,c,10^7)|$, are all only about $5\cdot 10^{-15}$.

CONCLUSIONS

We dealt with the sum of the series of reciprocals of the cubic polynomials with different positive integer roots $a<b<c$, i.e. with the series

$$\sum_{\substack{k=1\\k\neq a,b,c}}^{\infty}\frac{1}{(k-a)(k-b)(k-c)}.$$

We derived that the sum $s(a,b,c)$ of this series is given by formula

$$s(a,b,c)=\frac{(a+b-2c)(H_{b-a-1}-H_{b-a})}{(a-c)(ab+bc-ac-b^2)}+\frac{(-a+2b-c)H_{c-a-1}-(2a-b-c)H_{c-b-1}}{(a-b)(ab-ac-bc+c^2)}$$
$$+\frac{H_{c-a}+H_{c-b}-H_{c-1}}{ab-ac-bc+c^2}-\frac{H_{a-1}}{a^2-ab-ac+bc}+\frac{H_{b-1}}{ab+bc-ac-b^2}$$
$$+\frac{(a-c)\Psi(c+1-b)+(c-b)\Psi(c+1-a)+(a-b)\gamma}{(a-b)(a-c)(b-c)}.$$

We verified this main result by computing 60 sums by using the computer algebra system Maple 16. We also stated a special case of the sum $s(a,b,c)$ for $a \geq 1$ and integer $d \geq 1$:

$$s(a, a+d, a+2d) = \frac{H_{2d} - H_{a-1} - 2H_d + 2H_{a+d-1} - H_{a+2d-1} + 2\Psi(d+1) - \Psi(2d+1) + \gamma}{2d^2}.$$

The series above so belong to special types of the series, such as geometric and telescoping ones, which sums are given analytically by means of a formula.

ACKNOWLEDGEMENTS
The work presented in this paper has been supported by the project "Rozvoj oblastí základního a aplikovaného výzkumu dlouhodobě rozvíjených na katedrách teoretického a aplikovaného základu FVT (K215, K217)".

REFERENCES

[1] Benjamin, A. T., Preston, G. O. & Quinn, J. J. (2002). A Stirling Encounter with Harmonic Numbers. *Mathematics Magazine*, 75 (2), 95 –103. Retrieved 2018-07-31 from https://www.math.hmc.edu/~benjamin/papers/harmonic.pdf

[2] Potůček, R. (2016). A formula for the sum of the series of reciprocals of the polynomial of degree two with different positive integer roots. *Mathematics, Information Technologies and Applied Sciences 2016* – post-conference proceedings of extended versions of selected papers. University of Defence, Brno, 71-83.

[3] Potůček, R. (2017). Sum of the series of reciprocals of the cubic polynomials with triple positive integer root. *Mathematics, Information Technologies and Applied Sciences 2017* – conference proceedings on CD, University of Defence in Brno, 12 p.

[4] Wikipedia contributors: *Harmonic number*. Wikipedia, the Free Encyclopaedia. Retrieved 2018-07-31 from https://en.wikipedia.org/wiki/Harmonic_number

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Received: 2018-11-13
Accepted: 2018-11-30
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.9-17

Original Paper

Processing of stress dataset with rain-flow counting method

Jozef Rédl*, Dušan Páleš

Slovak University of Agriculture in Nitra, Faculty of Engineering, Department of Machine Design, Slovak Republic

ABSTRACT

In this contribution we are dealing with application of rain-flow method to processing the experimental stress data. Vibration of agricultural plough on duty was measured by sensor of acceleration ADXL345 Inertial Sensor mounted on the Pottinger Plough frame. Measured acceleration data was transformed to the stress on the frame. The mathematical algorithm of rain-flow method was based on the recommendation of ASTM code. The simple mathematical model was developed and the algorithm was implemented to the environment of PTC® Mathcad Prime® 4. Processing of experimental data was realized with Microsoft™ Excel® format table importing to the Mathcad Prime® software. The results from raw data and filtered data were compared.

KEYWORDS: signal processing, frame vibration, counting algorithm

JEL CLASSIFICATION: C65

INTRODUCTION

In many application of constructions life analysis were used the strain gauges to measure the stress in the materials under the random loading. The measurement is complicated because the gauges must be glued to the whole construction in the certain points and directions. Data recording and processing must be provided with the very wide skills [10]. The time-domain approach was defined by [1, 2] in which the response time history is calculated by static stress analysis by superimposing all stress influences from the applied loads at each time step, lacks the dynamics of the structure especially for vibration-based problems when a loading excites the natural frequencies of the structure. The use of strain gauges is possible for limited locations only and moreover requires early knowledge of critical fatigue locations. On the other hand, using the sensors of acceleration is very useful in measurement of vibrations of beams, where the beams are the components of the framed structures. Mathematical definition of Rain Flow Cycle (RFC) was first time defined by [8]. The method presented by [8] attaches to each maximum of the strain function the amplitude of a corresponding cycle or two half cycles, which are evaluated independently from each other.

* Corresponding author: Assoc. Prof. Jozef Rédl, PhD., Department of Machine Design, Faculty of Engineering, Slovak University of Agriculture in Nitra, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic, e-mail: jozef.redl@uniag.sk

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881

Algorithm of RFC was analyzed by [3, 9]. Practical application of RFC in fatigue life prediction was published by many authors. We can include to major works [4, 5, 6, 7].

MATERIAL AND METHODS

Measurement system and object

Object for measurement was a plough Pottinger depicted on Figure 1. The basic parameters of plough are listed in the Table 1 [11]. Measurement on plough was realized on deep plow. The plowed ground was planar without rough parts. The plough was mounted on tractor Fendt Vario 930.

Table1 Parameters of plough

Parameter	Value	Unit
Manufacturer	Pottinger	
Type	Servo 65 Plus	
Mass (m_P)	2370	kg
Distance between bodies	1	m
Bodies	7	m
Beam ($a \times a \times t$)	0,16x0,16x0,010	m

Figure 1 Plough Pottinger

Figure 2 Detail of sensor location

Figure 3 Sensor Eval - ADXL 345Z-DB

For measurement of plough vibration was used the Eval - ADXL 345Z-DB data acquisition board. The board measured accelerations in the XYZ axes. Measured data was recorded to the MiniSD card. The relevant direction for us was the accelerations in Z axis direction.

Mathematical model

As mentioned below, the measured vibration data were transformed to the stress data set. For the transformation we designed the simple mathematical model based on the theory of cantilever beam design theory. We substituted the real plough with model as depicted on Figure 4. For utilizing the designed model we set up the basic assumption as follows:

- three-point linkage stiffness is similar as a fixed connection of the cantilever beam,
- for bending moment is used the effective length of plough,
- plough support wheel damping is contained in the acceleration data,
- used loading is the weight of the plough,
- neutral axis of the beam lay to the beam axis of symmetry,
- neglecting the shear deformation.

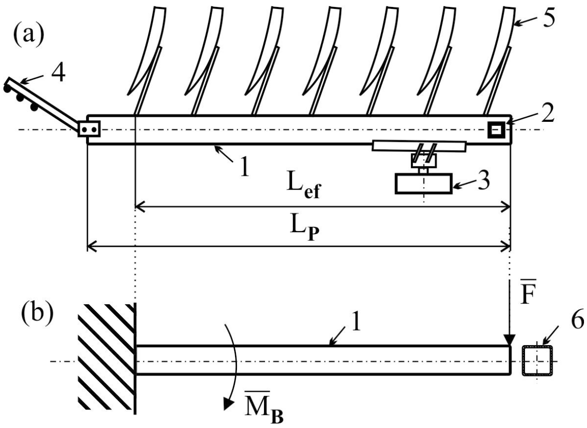


Figure 4 Plough and its model
(a) 1-plough beam, 2-acceleration gauge, 3-wheel, 4-three point linkage mechanism, 5-body
(b) 1-plough beam, 6-beam profile, $\overline{F}$ -force, $\overline{M}_B$ - bending moment

The algorithm was designed for computer processing and all variables are indexed. Bending moments dataset solving was realized with Equation 1. The beam cross section modulus was derived from equation of moment of inertia for square tube. Final Equation is 2. Stress formula was derived from stiffness condition equation for simple bending. Combination of Equations 1, 2 we get the Equation 3.

$M_{B(i)} = m_P . ac_{(i)} . L_{ef}$, where:

$M_{B(i)}$ - bending moment of the plough beam,
m_P - plough mass,
$ac_{(i)}$ - measured acceleration array,
L_{ef} - effective length of plough $= 6m$. (1)

Bending section modulus was solved by Equation 2.

$$W_B = \frac{a_1^4 - a_2^4}{6a_1} = \frac{1}{6}\left(a_1^3 - \frac{a_2^4}{a_1}\right), \tag{2}$$

where: $a_1 = a$, $a_2 = a - t$.

For purpose of processing the experimental data we transformed acceleration dataset to the stress data set. Finally we get a formula for stress of beam with Equation 3.

$$\sigma_{B(i)} = \frac{6.a_1 m_P.ac_{(i)}.L_{ef}}{a_1^4 - a_2^4} \tag{3}$$

We processed the raw accelerations data set and filtered data set. Stress dataset from raw data is depicted on Figure 5. The raw accelerations data was filtered with Butterworth maximally flat magnitude filter (see Equations 4). The transformed data from filtered dataset is depicted on Figure 6.

$$B_n(s) = \prod_{k=1}^{\frac{n}{2}} \left[s^2 - 2.s.\cos\left(\frac{2k+n-1}{2n}.\pi\right) + 1 \right], n = even$$

$$B_n(s) = (s+1) \prod_{k=1}^{\frac{n-1}{2}} \left[s^2 - 2.s.\cos\left(\frac{2k+n-1}{2n}.\pi\right) + 1 \right], n = odd\ , \tag{4}$$

where n - order of filter

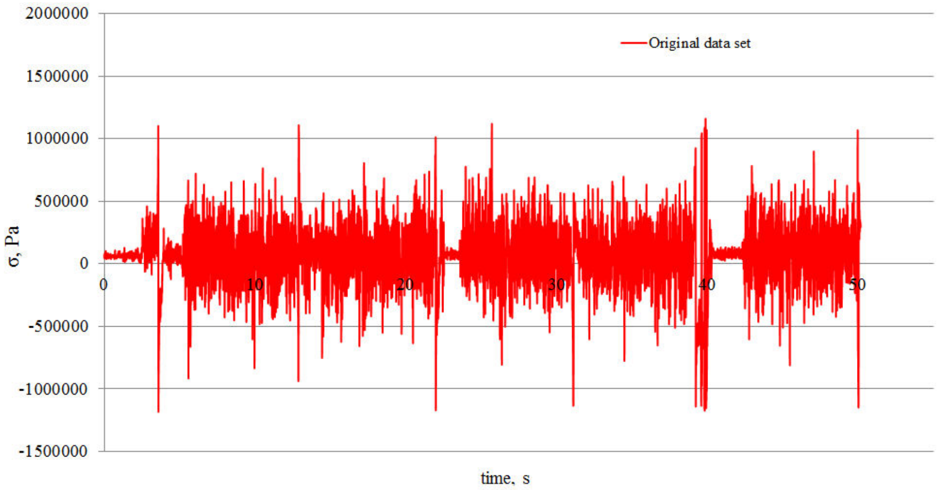


Figure 5 Stress from raw data

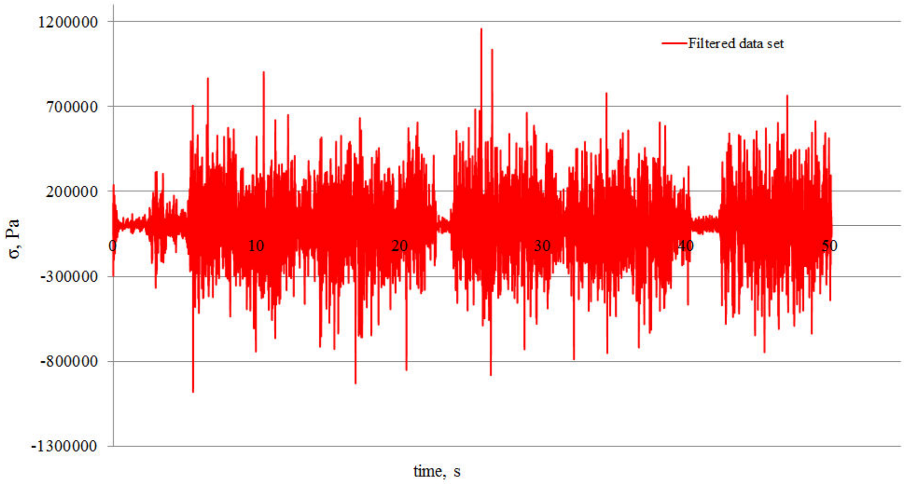


Figure 6 Stress from filtered data

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Rain flow counting cycle (RFC)

For processing the transformed data we used the Mathcad Prime® 4 software. Rain flow counting method is used to decomposition of the load signal. The load signal is represented by maxims and minima. Increasing and decreasing parts are counted as half cycles. Initial and final extreme do not have to follow behind, other half cycles can run between them. The name of the method is based on the idea, that after recording the load drops of water flow and drain as from the roof. Our algorithm is based on the [3, 5] descriptions. For method we set up basic rules for half cycle identification with respect the Figure 7, as follows:

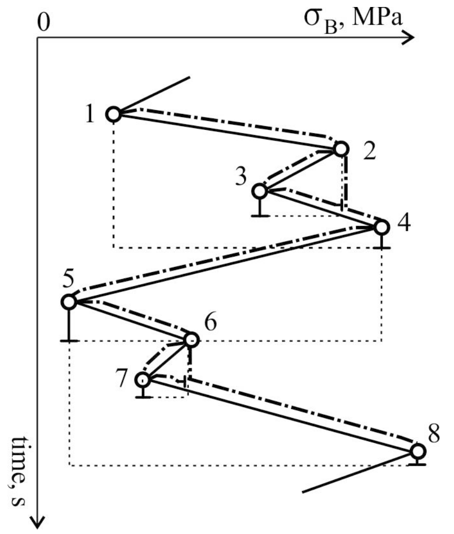


Figure 7 Rain flow method, where σ_B is stress of beam in Pa.

1) In every local extreme begins a half cycle,
2) half cycle ends if the edge of the outermost roof is reached(1-4, 2-3, 4-5, 5-8, 6-7),
3) half cycle ends if the drop strikes the dripping drop from the higher roof (3-4, 7-8).

The result of the method is the maximum and minimum stress, respectively, after recalculation, mean stresses in the rain-flow cycle (σ_B^M) and amplitude stress in the rain-flow cycle (σ_B^A) and these was solved by Equations 5, 6, where σ_B^{Max} is local maxima and σ_B^{Min} is the local minima. For the setup the cycle algorithm we design a flowchart with all conditions, variables and counts. The flowchart is depicted on Figure 8.

$$\sigma_B^A = \frac{\left|\sigma_B^{Max} - \sigma_B^{Min}\right|}{2} \tag{5}$$

$$\sigma_B^M = \frac{\sigma_B^{Max} + \sigma_B^{Min}}{2} \tag{6}$$

RESULTS AND DISCUSSION

Applying the RFC to the raw and filtered data we get the significant results about the both datasets. The values are solved by Equations 5, 6. For the raw data we set the results to the

Table 2. Tables 2 and 3 directly correspond with Figures 9, 10. Bin in the Tables 2, 3 means interval width of histogram.

Table 2 Results from rain-flow matrix histogram from raw data

	Min	Max	Bin
σ_B^M	-1.097451 x 10^6	7.43956 x 10^5	6.138022 x 10^4
σ_B^A	2.310422 x 10^3	1.162142 x 10^6	5.79916 x 10^4

Table 3 Results from rain-flow matrix histogram from filtered data

	Min	Max	Bin
σ_B^M	-3.805818 x 10^5	3.786286 x 10^5	2.530701 x 10^4
σ_B^A	4.9067252 x10^1	1.24555 x 10^6	6.227507 x 10^4

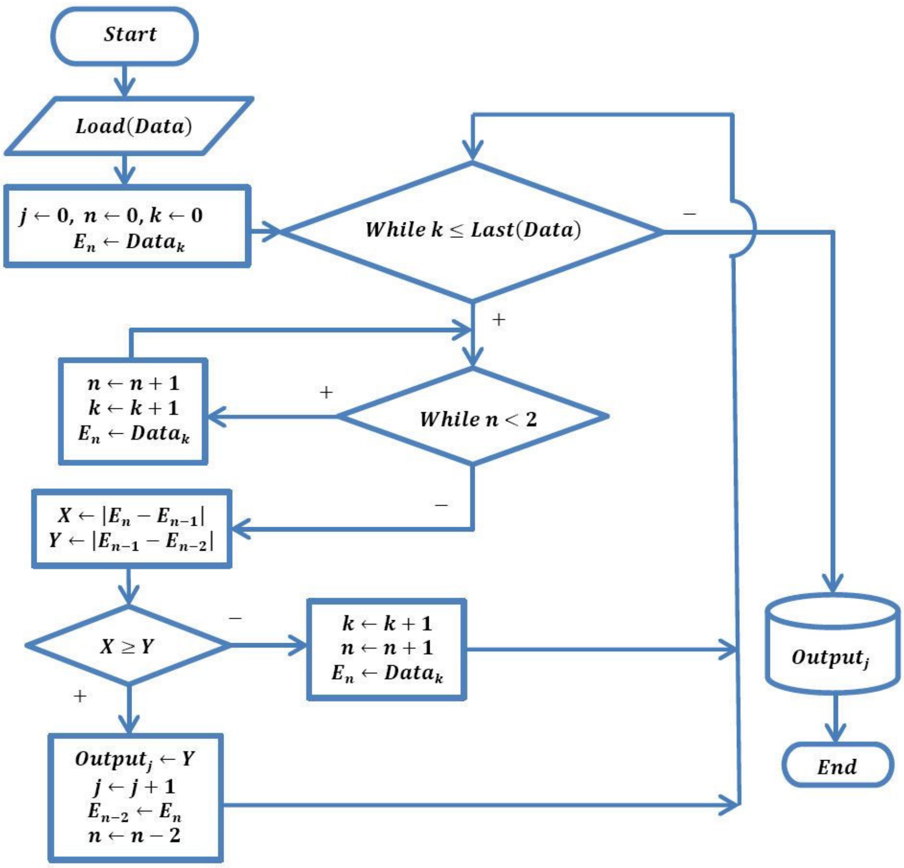


Figure 8 Flowchart of RFC

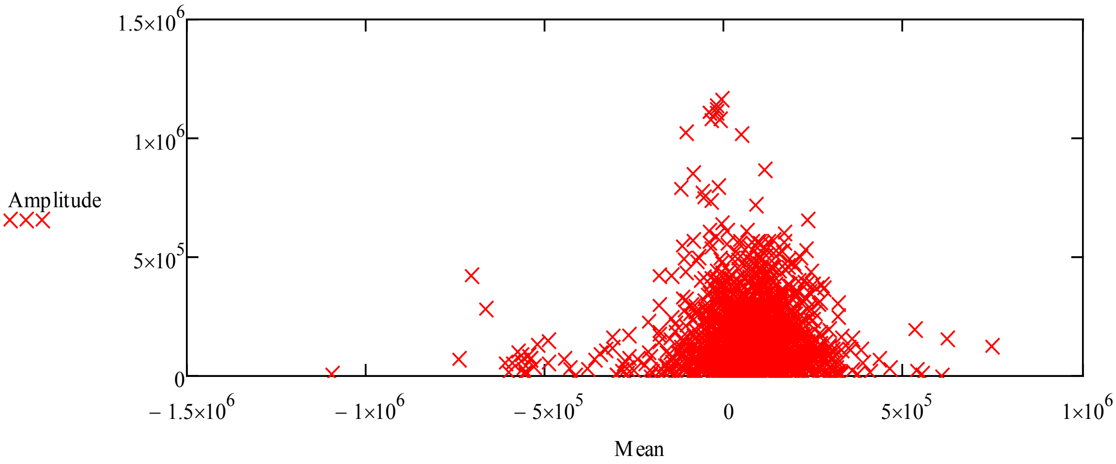


Figure 9 Stress raw data amplitudes and means layout

Description for the RFC flowchart algorithm (Figure 8):
X – range under consideration, Y – previous range under consideration,
range means difference between two following peaks of stress.
1) Read the next peak (if out of data stop)
2) Form ranges X and Y (if the vector contains less than 3 points go to step1)
3) Compare ranges X and Y, if $X < Y$ go to step 1, if $X \geq Y$ go to step 4
4) Count range Y, discard the peak of Y, go to step 2.

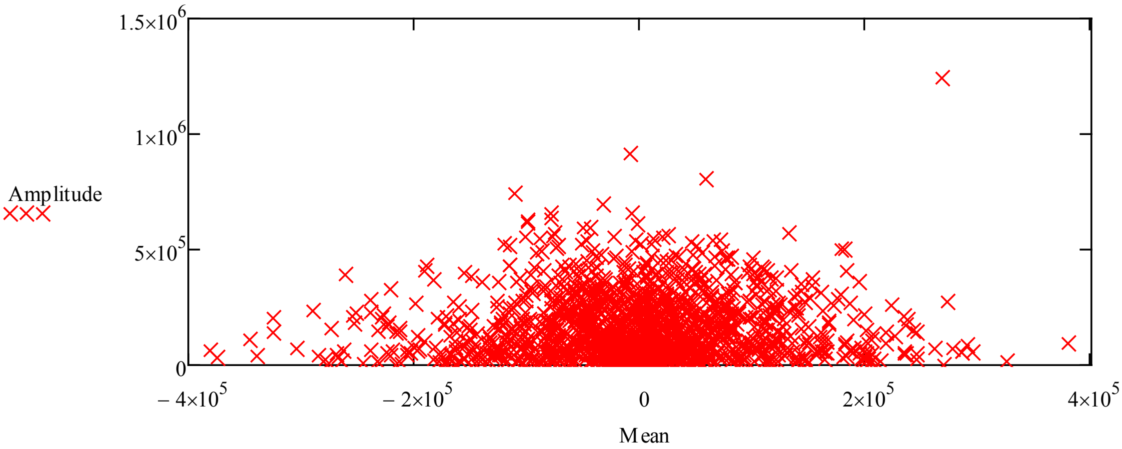


Figure 10 Stress filtered data amplitudes and means layout

RFC method gives the analysts the information about the concentrating the stress values in materials. From the Figure 9 and 10 we can see that the filtering data is valid because the range of raw stress data is in interval $\left\langle -1.5\cdot 10^6, 1\cdot 10^6 \right\rangle$ and the filtered stress data range is reduced to the interval $\left\langle -4\cdot 10^5, 4\cdot 10^5 \right\rangle$ in Pa. The negative values means that the loading is pressure and the positive values means that the loading is tension. With RFC algorithm we removed the no relevant values from stress data set. Another significant parameter about distribution of data is histogram. In Figure 11 we can see counts histogram of means and amplitudes for raw stress data. In Figure 12 we can see counts histogram of means and amplitudes for filtered stress data. The distribution data in rain-flow matrix is more uniformly as on Figure 11.

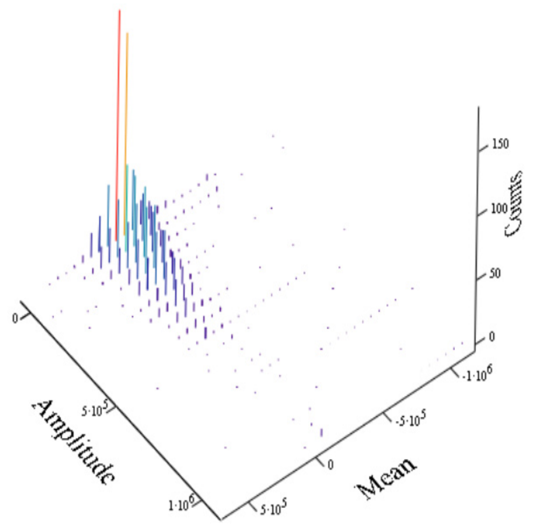


Figure 11 Rain-flow matrix histogram from raw data

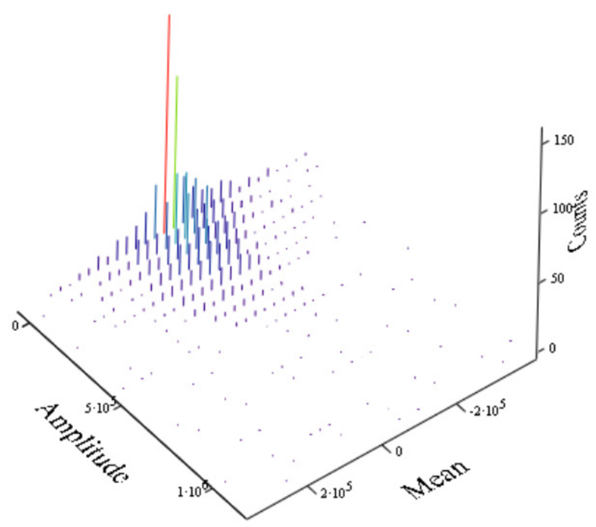


Figure 12 Rain-flow matrix histogram from filtered data

CONCLUSIONS

In this paper we are dealing with stress data processing by rain-flow counting algorithm. Data was obtained from experimental measurement of vibration of plough beam. The accelerations were converted to the stress time series. For signal processing we used the Butterworth maximally flat magnitude filter. In software Mathcad Prime 4 we designed the algorithm for establishing the relevant stress dataset by rain-flow counting algorithm. We processed the raw and filtered data. Result of RFC is rain-flow histogram matrix. From analysis we get the result that the raw data gives more irrelevant range of values than filtered data. The distribution of filtered data is more evenly distributed.

REFERENCES

[1] Anzai, H. (1992). Algorithm of the Rainflow Method. In *The Rainflow Method in Fatigue*, Butterworth-Heinemann, pp. 11-20. doi: https://doi.org/10.1016/B978-0-7506-0504-5.50010-2

[2] Aykan, M. & Çelik, M. (2009). Vibration fatigue analysis and multi-axial effect in testing of aerospace structures. *Mechanical Systems and Signal Processing*, Vol. 23, Issue 3, pp.897-907. doi: https://doi.org/10.1016/j.ymssp.2008.08.006

[3] Baek, S. H., Cho, S.S. & Joo, W.S. (2008). Fatigue Life Prediction Based on the Rainflow Cycle Counting Method for the End Beam of a Freight Car Bogie. *International Journal of Automotive Technology*, vol. 9, no. 1, pp. 95-101. doi: https://doi.org/10.1007/s12239-008-0012-y

[4] Downing, S. D. & Socie, D. F. (1982). Simple rainflow counting algorithms. *International Journal of Fatigue*, Vol. 4, Issue 1, pp. 31-40. doi: https://doi.org/10.1016/0142-1123(82)90018-4

[5] Lauschmann, H. (2007). *Mezní stavy I: únava materiálu*. 2nd ed. ČVUT: Praha (in Czech).

[6] Papagiannakis, T., Ancea, N. A., Chan, J. & Bergan, A. T. (1991). Application of ASTM E1049-85 Calculating Load Equivalence Factors from In Situ Strains. *Transportation Research Record*, 1307, pp. 82-89. Retrieved 2018-10-17 from http://onlinepubs.trb.org/Onlinepubs/trr/1991/1307/1307-009.pdf

[7] Prasad, S. R. & Sekhar, A, S. (2018). Life estimation of shafts using vibration based fatigue analysis. *Journal of Mechanical Science and Technology*, 32 (9), 4071-4078. doi: https://doi.org/10.1007/s12206-018-0806-4

[8] Rychlik, I. (1987). A new definition of the rainflow cycle counting method. *International Journal of Fatigue*, 9, No. 2, pp. 119-121.

[9] Schluter, L. L. (1991). *Programmer's Guide for LIFE2's Rainflow Counting Algorithm*. Sandia Report, SAND90–2260 UC–261, Unlimited Release, January 1991. Retrieved 2018-10-17 from https://prod.sandia.gov/techlib-noauth/access-control.cgi/1990/902260.pdf

[10] Šesták, J. (2002). *Life prediction of framed structures*. 1st ed. SPU: Nitra (in Slovak).

[11] POTTINGER. (n. d.) *Polonesené pluhy SERVO 65*. Retrieved 2018-10-17 from http://www.poettinger.at/landtechnik/download/servo65_cz.pdf

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Received: 2018-11-13
Accepted: 2018-11-30
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.18-22

Original Paper

Influence of microclimatic environment on production of pollutants from recycled manure solids in laboratory conditions

Ingrid Karandušovská*, Jana Lendelová, Štefan Boďo, Miroslav Žitňák

Slovak University of Agriculture in Nitra, Faculty of Engineering, Department of Building Equipment and Technology Safety, Slovak Republic

ABSTRACT

The aim of this experiment was to analyse influence of climatic factors on production of harmful gases from organic litter. The sample of separated sludge slurry was collected from lying area of dairy housing. Measurement of produced harmful gases was carried out in laboratory conditions in the environmental chamber, where the amount of released gas concentrations from sample of separated sludge was monitored under different microclimatic conditions by setting the temperature. The photoacoustic multi-channel analyser INNOVA 1309 with accessories was used for the measurement of gas concentrations. It has been shown, that increasing air temperature in the climatic chamber increase release of all monitored gases. Most of this was reflected CH_4 and NH_3 in the measurement of dry samples of separated manure, where the concentration growing rapidly at air temperature of 20 °C. Minimum ammonia concentration of 0.53 mg.m^{-3} was at air temperature of -10°C and maximum of 5.11 mg.m^{-3} at air temperature of 40°C. The concentrations of monitored gas increased continuously with the temperature of the air when measuring of moist samples of separated manure. Minimum ammonia concentration of 0.93 mg.m^{-3} was at air temperature of -10 °C and maximum of 5.8 mg.m^{-3} at 40 °C air temperature. Statistical analysis of the results showed there is a significant dependence between air temperature and ammonia concentrations and other monitored gases ($P < 0.05$). The gas production from the wet recycled manure solids sample was significantly higher ($P < 0.05$).

KEYWORDS*:* dairy cattle, recycled manure solids, bedding

JEL CLASSIFICATION: Q15

INTRODUCTION

In livestock production, the use of manure is currently being used not only as a source of nitrogenous fertilizers but to be recovered in other agricultural activities [4]. There were bedding livestock housing systems prevalent in our country, while the loose housing is housing with cubicle beddings or pens with bedding layer [1]. Bedding material is very a

* Corresponding author: doc. Ing. Ingrid Karandušovská, PhD., Department of Building Equipment and Technology Safety, Faculty of Engineering, Slovak University of Agriculture in Nitra, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic, e-mail: ingrid.karandusovska@uniag.sk

costly part of dairy farming and has a significant impact not only on animal health but also to the environment.

It is increasingly difficult problem to ensure a reliable source of bedding for farmers. Dairy farms are therefore constantly under increasing pressure to improve their management of manure. The separated slurry of manure - also called "green litter" is used for bedding in areas where the availability of classic bedding material is decreasing. It is a separated raw slurry or separated digested slurry from the biogas plant [5]. Farmers using this material report greater cow comfort than with other bedding materials they have used, and the potential financial savings are substantial [3]. They indicate a significant improvement of animals housing welfare, in that animals create natural bed in the organic plastic material and separated slurry handling is very simple [4]. With the arrival of recycled manure solids (RMS) use, the question forms associated with the risk of harmful gas production, because dairy cattle barns are a major source of NH_3 emissions and greenhouse gases (GHG) to the atmosphere. Previous studies have also shown that the bedding material used in the barn can influence the magnitude of NH_3 emissions, but little is known about which bedding characteristics are important in this respect [6].

MATERIAL AND METHODS

For our experiment, the samples of the separated slurry of manure were taken in the center of the lying cubicles in dairy barn. Repeatedly had been sampled 2 litters of 10 selected boxes for two weeks. The experiment was conducted under laboratory conditions in a climatic chamber FEUTRON 3522/51, where the amount of gas released from sample of recycled manure bedding was monitored under different microclimatic conditions by setting the air temperature from -10°C up to +40°C and humidity (RH = 50%). The photoacoustic multi-channel analyzer INNOVA 1309 with accessories was used for the measurement of gas concentrations of N_2O, NH_3, H_2S, CH_4. Five measurement points were chosen directly in the chamber and one place outside the chamber (Fig. 1). The measured data were evaluated by Statistica 10, the gas concentrations and air temperature dependence were determined by correlation analysis. Using the F test the differences in the production of pollutants from the bedding sample with a dry matter content of 58% and 24% were evaluated.

Figure 1 Sample of litter placed in a climatic chamber and instrument for measuring gas concentrations

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

RESULTS AND DISCUSSION

The air temperature is a very important variable in the development of ammonia and GHG production, because of temperature change can influence the mass flow of ammonia significantly. As follows from measurements, it has been shown, that increasing air temperature in the climatic chamber increase release of all monitored gases. Most of this was reflected CH_4 and NH_3 in the measurement of dry bedding samples (dry matter content 58 %), were the concentration growing rapidly from air temperature of 20 ° C (Fig. 2). Minimum ammonia concentration of 0.53 mg.m^{-3} was at air temperature of -10°C and maximum concentration of 5.11 mg.m^{-3} was at air temperature of 40 °C.

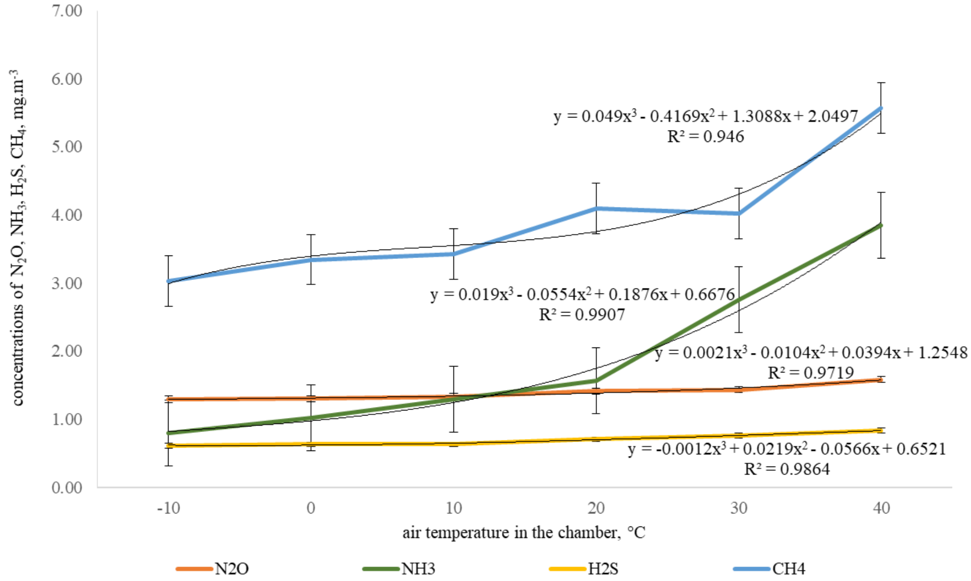


Figure 2 The course of gas concentrations in climatic chamber in raising of the air the temperature - measurement of the dry RMS

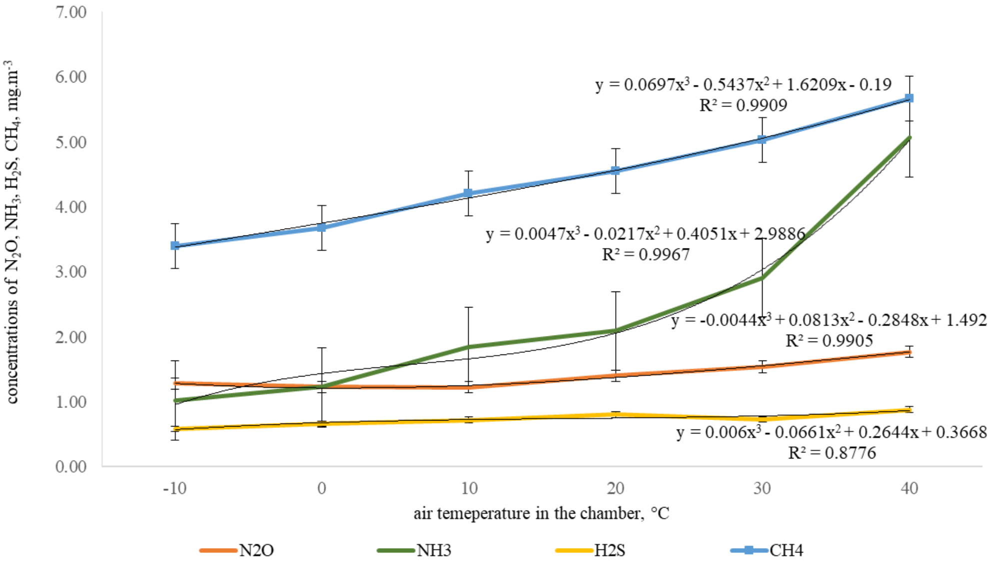


Figure 3 The course of gas concentrations in climatic chamber in raising of the air the temperature - measurement of the wet RMS

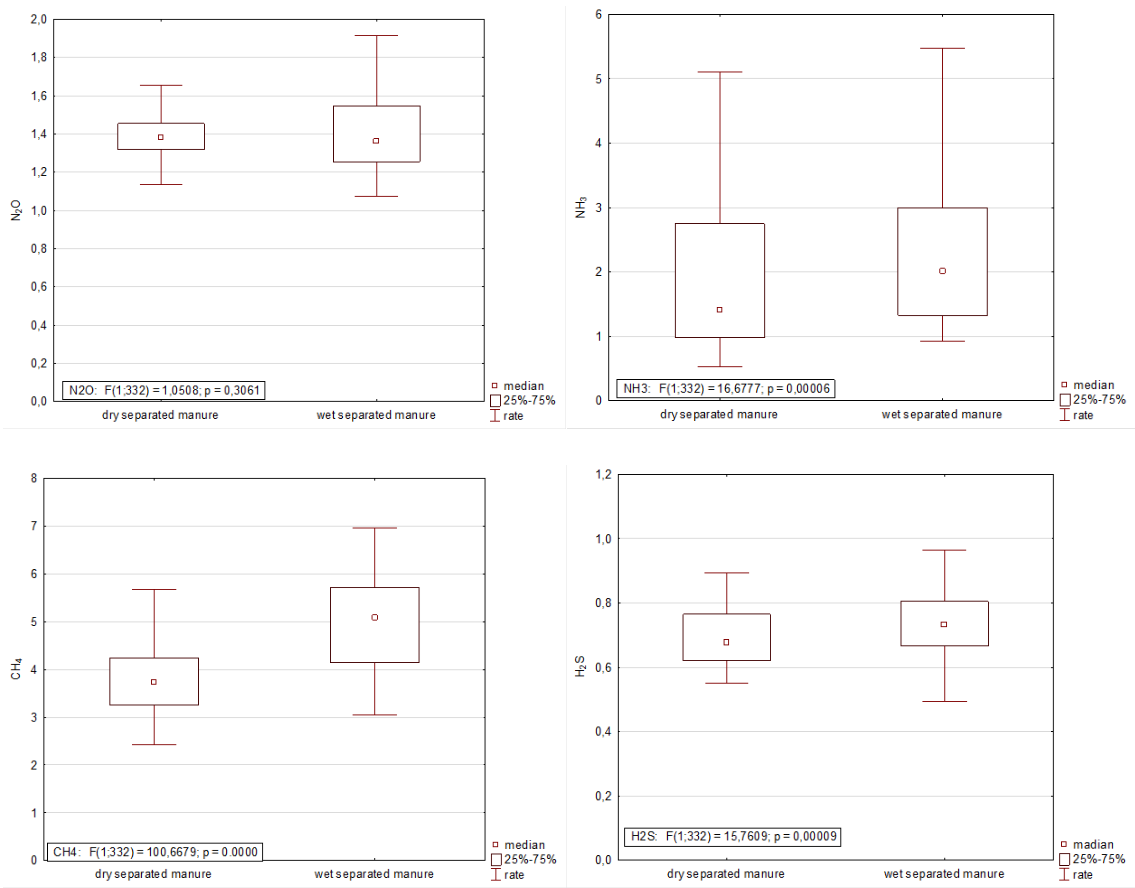


Figure 4 Comparison of gas production from a dry and wet sample in a climatic chamber (Statistica 10, F test, Anova, P <0.05)

The gas concentrations increased continuously with the temperature of the air when measuring of moist bedding samples (dry matter content 25 %). Minimum ammonia concentration of 0.93 $mg.m^{-3}$ was at air temperature of -10°C and maximum of 5.8 $mg.m^{-3}$ was at 40 ° C air temperature (Fig. 3). Statistical analysis of the results showed that there is a significant dependence between air temperature and ammonia concentrations and other monitored gases ($P < 0.05$). The gas production from the wet recycled manure solids sample was therefore somewhat higher. In Figure 4 there are shown significant differences in the production of NH_3, CH_4 and N_2O from dry and wet sample ($P < 0.05$). As reported Redwine et al. [8] litter temperature and humidity are factors that influencing amounts of the released ammonia. The optimum conditions for this process are temperature above 20 °C, humidity of bedding 40 – 60 %. Temperature and humidity are stressors that occur at increased levels in confined spaces and poorly ventilated housing systems.

The results of our experiment are in accordance with the conclusions of Zhao et al. [10] and Harper et al. [2], who claimed that ammonia production is increasing with increasing air temperatures. Wu et al. [9] indicated that the concentration of ammonia in naturally ventilated barns is mostly influenced by temperature and air movement. The growth of temperature increases urease activity thus raising ammonia emission from manure [7]. Research acknowledged that the level of ammonia is influenced by the air temperature and the humidity of the bedding material in the case of the use of the separated manure solids as litter, as it is in the case with using of straw litter. However, the levels of the produced gas concentrations are at the same level in scale of tested bedding moisture and there is no reason to worry about using of separated manure into cubicle beddings due to emissions of ammonia and GHG.

CONCLUSIONS

The experiment was carried out in laboratory conditions and was aimed to obtain knowledges about the effects of climatic factors on production of harmful gases released from sample of recycled manure solids. Measurements of pollutant production showed that the concentrations of monitored gases increased with increasing air temperature, and using wet litter with dry matter content of 28%, the production of pollutants was higher and continually increased from the air temperature to 10° C.

ACKNOWLEDGEMENTS

This work was done in the frame of a research project funded from the European Union under the title: ITEPAg „Application of information technologies to increase the environmental and economic efficiency of production agro-system TMS 26220220014.

REFERENCES

[1] Brestenský, V. & Botto, Ľ. (2010). *Odstraňovanie a skladovanie hospodárskych hnojív.* 1st ed. Nitra: Agrokomplex Nitra (in Slovak).

[2] Harper, L. A., Flesch, T. K., Powell, J. M., Coblentz, W. K., Jokela, W. E. & Martin, N. P. (2009). *Ammonia emissions from dairy production in Wisconsin*. J. Dairy Sci. 92, 2326, 2009.

[3] Harrison, E., Bonhotal, J. & Schwarz, M. (2006). *Using Manure Solids for Dairy Barn Bedding.* Retrieved 2018-10-12 from http://cwmi.css.cornell.edu/beddingsummary.pdf

[4] Jelínek, A., Kraus, R. & Dědina, M. (2006). Nová technologie zpracování kejdy z chovu skotu jako plastického steliva pro zlepšení vztahu k životnímu prostředí a welfare chovaných zvířat. In *Separovaná kejda jako plastické stelivo v chovech skotu.* VUZT: Praha, 8-13 (in Czech).

[5] Lendelová, J., Čičo, P., Žitňák, M., Mihina, Š. & Pogran, Š. (2015). *Podlaha ležísk pre dojnice z recyklovaného biologického odpadu*: úžitkový vzor č. 7222: dátum zápisu a sprístupnenia úžitkového vzoru verejnosti 24.06.2015. Banská Bystrica: Úrad priemyselného vlastníctva, 5 s. (in Slovak).

[6] Misselbrook, T. H. & Powell, J. M. (2005). Influence of Bedding Material on Ammonia Emissions from Cattle Excreta. *Journal of Dairy Science*, Volume 88, Issue 12, December 2005, 4304-4312.

[7] Moreira, V. R. & Satter, L. D. (2006). Effect of Scraping Frequency in a Freestall Barn on Volatile Nitrogen Loss from Dairy Manure. *Journal of dairy science*, 89(7), 2579-2587.

[8] Redwine, J. S., Lacey, R. E., Mukhtar, S. & Carey, J. B. (2002). Concentration and emissions of ammonia and particulate matter in tunnel ventilated broiler houses under summer conditions in Texas. *Transactions of the ASAE*, 45(4), 1101-1109.

[9] Wu, W., Zhang, G. & Kai, P. (2012). Ammonia and methane emissions from two naturally ventilated dairy cattle buildings and the influence of climatic factors on ammonia emissions. *Atmos. Environ*., 61, 232-243.

[10] Zhao, L. Y., Brugger, M. F., Manuzon, R. B., Arnold, G. & Imerman, E. (2007). Variations in air quality of new Ohio dairy facilities with natural ventilation systems. *Appl. Eng. Agric.*, 23(3), 339-346.

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Received: 2018-10-26
Accepted: 2018-12-02
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.23-30

Original Paper

Mathematical competencies of students entering university studies. Case study of Slovakia

Janka Drábeková*, Tomáš Pechočiak, Vladimír Matušek

Slovak University of Agriculture in Nitra, Faculty of Economics and Management, Department of Mathematics, Slovak Republic

ABSTRACT

The main objective of this paper is the analysis of students' math competences via results of the mathematical experiences of students of SAU in Nitra at the beginning of winter term before the graduation of subject "Mathematics" at the different study programs. We performed a test consisted of 5 tasks. We used the National Educational Program Mathematics ISCED 2, ISCED 3A. We have compared the results of tests of respondents and determined whether the differences between study outcomes of students are significant. Our research has examined the impact of gender, type of secondary school and the grade point in mathematics from secondary school to use mathematical tools to solve tasks according ISCED 2, ISCED 3A. The obtained indices of successfulness show a beneficial effect of male gender and graduated secondary grammar school.

KEYWORDS: university students, mathematical competencies, ISCED 3A, Slovakia

JEL CLASSIFICATION: B74, D35, K40

INTRODUCTION

The Slovak University of Agriculture in Nitra with its educational, scientific and research activities represent the significant part of a European and world educational area. It has become a modern open university which reflects current needs in agri-food sector in a local and global scale. Its mission is to prepare competitive specialists for all areas of agri-food sector as well as other fields of national economy – engineering, finance, institutions and bodies of state administrations and others. It has six faculties providing wide range of knowledge in the field of natural, economic, technical and social.

One of the biggest challenges that higher education faces is preparing today's students to meet future workforce demands [1, 3, 8, 18]. As "21st century competencies" are often allowed to creativity, problem solving, critical thinking, communication, collaboration, self-management. Gosselin, Cooper & Bonnstetter [5] said that although these competencies are considered to

* Corresponding author: Janka Drábeková, Slovak University of Agriculture in Nitra, Faculty of Economics and Management, Department of Mathematics, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic, e-mail: janka.drabekova@uniag.sk

be at the foundation of individual as well as collective success in the work place, employers report substantial deficiencies in these applied skills. As a result, business leaders and educational organizations are calling for new education policies that target the development of broad, transferable skills and knowledge.

Mathematical competencies

Országhová & Horváthová [14] said that in the contemporary society the university education is the important factor for employment opportunities on the labor market. Our society needs well educated population and broadly qualified work force, all of whom are able to activate mathematical knowledge, insights, and skills in a variety of situations and contexts [11]. The growing role of science, mathematics and technology in modern life requires that all adults, not just those who aspire to scientific careers, become mathematically, science and technology literate [7]. Nevertheless, the student's interest in educational programs with a strong mathematical component is still smaller. Niss [10] said that this is reflected in the so called "relevance paradox": Even though mathematical knowledge is highly relevant in and to society, many, if not most, people have increasing difficulty at seeing that mathematics is relevant to them, as individuals. One thing is to know how to calculate the math, the second thing is to understand it, or apply it correctly and interpret current real life situations, encountered, for example, while reading a magazine about health or agriculture. One of the main aims of mathematical education as such is preparing the students for dealing effectively with the real-life situations [17]. The current task of teachers should be – to keep the students' interest in these subjects and show the importance of applying of acquired competences from mathematics [13].

Competencies and learning outcomes are two related educational terms. Competencies are combinations of attitudes, skills and knowledge that students develop and apply for successful learning, living and working. Competencies help students draw and build upon what they know. Competencies describe the desired knowledge, skills, and behaviors of a student graduating from a program or completing a course or level of education. Learning outcomes describe exactly what a student will be able to do in some measurable way. There may be more than one measurable outcome defined for a given competency. Thus, learning outcomes are the basis for an assessment program.

Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety mathematical contexts and situations. Factual knowledge and technical skills are necessary, but certainly not sufficient, prerequisites for mathematical competence. Debrenti [4] said that mathematical competence encompasses the development and use of skills and abilities related to mathematical reasoning. Which mathematical competencies need to be developed with students at different stages of the education system?

Niss [9] identified eight competencies which form two groups. The first group of competencies is to do with the ability to ask and answer questions in and with mathematics: thinking mathematically, posing and solving mathematical problems, modelling mathematically (i.e. analysing and building models), reasoning mathematically. The other group of competencies is to do with the ability to deal with and manage mathematical language and tools: representing mathematical entities (objects and situations), handling mathematical symbols and formalisms, communicating in, with, and about mathematics, making use of aids and tools (IT included).

The OECD PISA [12] indicated the following eight components of mathematical competence: reasoning, making deductions; argument and elaborating a proof; communication; modelling; finding solutions; representation; using symbolic, formal and technical language and operations; using mathematical tools.

According the Slovak state educational program, ISCED 3A [2], studying mathematics in secondary school contributes to the development of following key competencies of students:

- competence to apply the basis of mathematical thinking and basic knowledge of science and technology,
- competence to solve problems,
- competence in the field of information and communication technologies,
- competence for lifelong learning - learning to learn,
- social communication competencies,
- social and personal competencies,
- working skills,
- competences for initiative and entrepreneurship,
- competences to perceive and understand culture and to express themselves with the tools of culture.

The educational content of the mathematics, ISCED 3A, is divided into five thematic areas [2]:

- numbers, variables and arithmetic operations,
- relationships, functions, tables and diagrams,
- geometry and metric system of measurement,
- combinatorics, probability and statistics,
- logic, reasoning, making deductions, argument and elaborating a proof.

According Pietriková, Hornyák-Gregáňová & Orlíková [16] students in lower levels of educations should adopt the required education strategy so their education was the most effective, because currently it is not only about adopting the knowledge but as well as the creation and utilization of general abilities and concrete and effective techniques used for learning which can be transferred to the outside of educational environment.

The Intersegmental Committee of Academic Senates (ICAS), representing the academic senates of the three segments of California's higher education system, produced document which includes essential areas of focus for all entering college students in their country. In this document is a summary of the mathematical subjects that are an essential part of the knowledge base and skill base for all students who enter higher education. Students are best served by deep mathematical experiences in these areas [6]:

- Variables, Equations, and Algebraic Expressions: Algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols; solutions of linear equations and inequalities; absolute value; powers and roots; solutions of quadratic equations; solving two linear equations in two unknowns, including the graphical interpretation of a simultaneous solution.
- Families of Functions and Their Graphs: Applications; linear functions; quadratic and power functions; exponential functions; roots; operations on functions and the corresponding effects on their graphs; interpretation of graphs; function notation; functions in context, as models for data.
- Geometric Concepts: Distances, areas, and volumes, and their relationship with dimension; angle measurement; similarity; congruence; lines, triangles, circles, and their

properties; symmetry; Pythagorean Theorem; coordinate geometry in the plane, including distance between points, midpoint, equation of a circle; introduction to coordinate geometry in three dimensions.
- Probability: Counting (permutations and combinations, multiplication principle); sample spaces; expected value; conditional probability; independence; area representations of probability.
- Data Analysis and Statistics: Presentation and analysis of data; measures of center such as mean and median, and measures of spread such as standard deviation and interquartile range; representative samples; using lines to fit data and make predictions.

The Slovak state educational program ISCED 3A consist the same areas. So we agree that areas listed above should be an essential part of the knowledge base for all students who enter university education.

MATERIAL AND METHODS

The examined sample consisted of 218 students from three faculties of the Slovak University of Agriculture in Nitra (SAU). We selected the Faculty of Economics and Management (FEM), the Faculty of European Studies and Regional Development (FESRD) and the Faculty of Horticulture and Landscape Engineering (FHLE). The examined sample consisted of first-year students of the university, 68 graduated from secondary grammar schools (SGS), and 150 graduated from secondary vocational colleges (SVC, technical, chemical, agricultural or business). The sample structure is shown in Table 1.

Table 1 Numbers of students participating in research

	SGS		SVC		Total
	Women	Men	Women	Men	
FEM	13	22	62	20	117
FESRD	7	6	16	9	38
FHLE	9	11	29	14	63
Total	29	39	107	43	218

We performed a test consisted of 5 tasks. We used the National Educational Program Mathematics ISCED 2, ISCED 3A. The tasks were focused on the algebraic symbols and expressions, rational numbers, solutions of linear equations and inequalities, solutions of quadratic equations, linear functions. Students solved the test in 20 minutes. The maximum number of points that a student could get was 15 points in Test A and Test B.

We utilized basic methods of descriptive statistics and hypotheses testing in the assessment of survey results: test for normal distribution, z-test, analysis of variance (single factor). Statistically verifiable existence of difference in the assessment was reviewed on the base of significance of testing characteristic (p-value), what presents the error probability which we will commit if we reject H0 tested hypothesis even in fact it is valid [16]. In case the p-value of testing characteristic is lower than 0.05, the null hypothesis about the equality of observed features is rejected and the difference in values of statistical feature is considered as statistically significant [15]. We tested the following hypotheses:

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2018(4), 1

- H0: There in no dependence between the test results of students and the type of attended secondary school.
  H1: There is dependence between the test results of students and the type of attended secondary school.
- H0: There in no dependence between the test results of respondents and respondents' gender.
  H1: There is dependence between the test results of respondents and respondents' gender.
- H0: The effects of all levels of factor grade point in mathematics from secondary school (1, 2, 3, 4) are zero, insignificant.
  H1: The effect at least one i – level of the factor is significantly different from zero.

We used the program Microsoft Excel 2013 and SAS for the realization of calculations and determination of critical values.

RESULTS AND DISCUSSION

Survey subject was to determine the mathematical experiences of students in SAU in Nitra at the beginning of winter term before the graduation of subject "Mathematics" in areas listed above. For the purpose of determination of respondents mathematical experiences level the survey was realized on the first lesson. In Table 1 the educational respondent structure is presented where 68.81% of all students graduated the specialized secondary school (SVC) like business or hotel academy where as regards the school type we assume the graduation of mathematics with planned types 1 or 2 hour weekly. Other students (31.19%) graduated gymnasium (SGS) with mathematics education 3 or 4 hour weekly. Both types of secondary schools are managed by the Slovak state educational program ISCED 3A.

The results of basic methods of descriptive statistics are presented in Table 2 and an accurate representation of the distribution of numerical data on figure 1.

Table 2 Descriptive statistics

Mean	Standard Error	Median	Mode	Standard Deviation	Kurtosis	Skewness	Range	Count	Confidence Level (95%)
5.23	0.22	5	5	3.30	-0.39	0.43	15	218	0.44

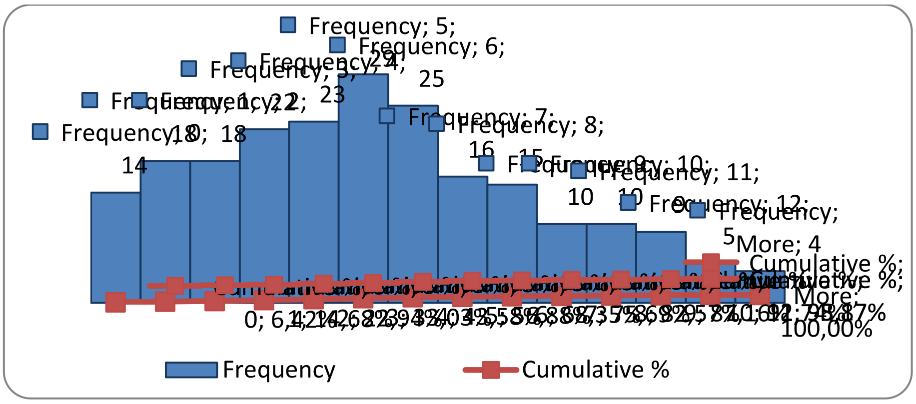


Figure 1 Histogram

Normality test was used to determine if a data set is well-modeled by a normal distribution. We detected a goodness of fit of a normal model to the data.

If we compare the test results of students in accordance with the type of attended secondary schools we find out that the students of special secondary school (SVC) obtained lower mean of points with lower variance. Based on the results of test we can assume the existence of statistically verifiable difference in mathematical knowledge of students of various types of secondary schools. We verified the validity of described hypothesis by z-test. The results are presented in Table 3. The students of gymnasium obtained significantly higher score.

Table 3 z-Test two sample for means of type secondary school

	SGS	*SVC*
Mean	6.73	4.51
Known Variance	13.41	8.24
Observations	68	150
z	4.41	
P(Z<=z) two-tail	0.00001	
Z Critical two-tail	1.96	

When testing two sexes for equality of points mean, the null hypothesis under test is that the results both gender are equal. After random samples are taken from each of the two sexes, were calculated the difference between them and this difference was used to accept or reject the null hypothesis. We detected large difference, so the null hypothesis is rejected. The results are presented in Table 4. The women obtained significantly lower score.

Table 4 z-Test two sample for means of gender

	Female	*Male*
Mean	4.85	5.78
Known Variance	9.75	12.29
Observations	136	82
z	-1.97	
P(Z<=z) two-tail	0.048	
Z Critical two-tail	1.96	

Analysis of variance was used to analyze the differences among group means in a sample. We studied the effect of different levels of grade in math from secondary school (1, 2, 3, 4) on result test. The null hypothesis assumed the same effect, insignificant difference.

Table 5 Summary

Groups	*Count*	*Sum*	*Average*	*Variance*
1	20	154	7.70	10.01
2	65	373	5.74	11.05
3	99	501.5	5.07	9.40
4	34	105.5	3.10	7.63

Table 6 One-way ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*p-value*	*F crit*
Between Groups	295.15	3	98.38	10.17	0.000003	2.65
Within Groups	2070.47	214	9.68			
Total	2365.62	217				

Table 6 summarizes the calculations needed to test the equality of several grade in mathematics means using analysis of variance. As shown in Table 6, the *between* method of estimating sigma produces a value of 98.38, whereas the *within* method estimate is 9.68. The F ratio indicates that the *between* method estimate is 10.17 times the *within* method value. The calculated *F value* of 10.17 is larger than the critical value (2.65), which means there is enough sample evidence to reject the null hypothesis. The *p-value* indicates that the probability of obtaining an *F value* greater than 10.17 by chance alone is 0.000003. We detected significantly differences in results of respondents according grade in mathematics from secondary school.

CONCLUSIONS

The level of mathematical education depends on many factors and its results are among other things the outcome of the subjective approach of each student to study the subject of mathematics. On the basis of our results, we can state that our respondents had a low ability to use mathematical apparatus, which is an important prerequisite for their professional growth and application. The research results pointed out the weaknesses that could be caused by the preference of the studied thematic areas on different type of secondary schools. Both types of researched secondary schools are managed by the Slovak state educational program ISCED 3A but with different number of lessons per week. We detected significant higher score of students graduated secondary grammar school. The women obtained significantly lower score than men. We detected significantly differences in results of respondents according grade in mathematics from secondary school. The effect at least one i – level (1, 2, 3, 4) of the factor grade point in mathematics from secondary school was significantly different from zero.

ACKNOWLEDGEMENTS

This work was supported by the Ministry of Education, Science, Research and Sport of the Slovak Republic, under contract no. KEGA 029SPU-4/2018.

REFERENCES

[1] Arum, R. & Roksa, J. (2011). *Academically adrift: limited learning on college campuses*. University of Chicago Press, Chicago.

[2] Bálint, Ľ., Janisková, J., Kubáček, Z., Oravcová, E. & Reiterová, M. (2009). *Štátny vzdelávací program*. Matematika (Vzdelávacia oblasť: Matematika a práca s informáciami). Príloha ISCED 3a. ŠPÚ, Bratislava (in Slovak). Retrieved 2014-04-07 from http://www.statpedu.sk/files/documents/svp/gymnazia/ vzdelavacie_oblasti/matematika_isced3a.pdf

[3] Bellanca, J. & Brandt, R. (2010). *21st Century skills: rethinking how students learn*. Solution Tree Press, Bloomington.

[4] Debrenti, E. (2015). Visual representations in mathematics teaching: An experiment with students. *Acta Didactica Napocensia*, 8(1), 21-26.

[5] Gosselin, D., Cooper, S., Bonnstetter, R. J. & Bonnstetter, B. J. (2013). Exploring the assessment of twenty-first century professional competencies of undergraduate students in environmental studies through a business—academic partnership. *Journal of Environmental Studies and Sciences*, 3(3), 359-368. doi: https://doi.org/10.1007/s13412-013-0140-1

[6] Intersegmental Committee of Academic Senates (2013). *Statement on Competencies in Mathematics Expected of Entering College Students*. Retrieved 2018-09-20 from http://icas-ca.org/Websites/icasca/ images/ICAS-Statement-Math-Competencies-2013.pdf

[7] Kubáček, Z., Kosper, F., Tomachová, A. & Koršňáková, P. (2004). *PISA SK 2003, Matematická gramotnosť, Správa*. ŠPÚ, Bratislava (in Slovak).

[8] National Research Council (2012). *Education for life and work: developing transferable knowledge and skills in the 21st century*. The National Academies Press, Washington.

[9] Niss, M. (1999). Kompetencer og uddannelsesbeskrivelse. *Uddannelse,* 9, 21-29.

[10] Niss, M. (2003). *Mathematical competencies and the learning of mathematics: the danish KOM project*. Retrieved 2018-10-21 from http://www.math.chalmers.se/ Math/Grundutb/ .../ KOMkompetenser.pdf

[11] Niss, M. & Jensen, T. H. (2002). *Kompetencer og matematiklæring*. Uddannelsesstyrelsens temahæfteserie, 18, 1-334, Undervisningsministeriet (Ministry of Education).

[12] Organization for Economic Co-operation and Development (1999). *Measuring Student Knowledge and Skills. A New Framework for Assessment* (PISA). OECD publications 2, Paris, France. Retrieved 2014-04-07 from http://www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/ 33693997.pdf

[13] Országhová, D. & Horváthová, J. (2015). Pedagogical and statistical analysis of students' mathematical and language competences. *Mathematics in Education, Research and Applications*, 1(1), 28-35. doi: https://doi.org/10.15414/meraa.2015.01.01.28-35

[14] Országhová, D. & Horváthová, J. (2017). Assessment of competences acquired via CLIL in selected subjects at the Slovak University of Agriculture in Nitra. *Mathematics in Education, Research and Applications*, 3(1), 24-31. https://doi.org/10.15414/meraa.2017.03.01.24-31

[15] Pacákova, V. a kol. (2009). *Štatistické metódy pre ekonómov*. Iura Edition, Bratislava (in Slovak).

[16] Pietriková, M., Hornyák Gregáňová, R. & Orlíková, M. (2016). Preferences in students' educational styles. *Mathematics in Education, Research and Applications*, 2(2), 54-60. doi: https://doi.org/10.15414/meraa.2016.02.02.54-60

[17] Švecová, V., Pavlovičová, G. & Rumanová, L. (2014). Support of Pupil's Creative Thinking in Mathematical Education. *Procedia-Social and Behavioral Sciences: 5th World Conference on Educational Sciences - WCES 2013*, 116, 1715-1719.

[18] Zemsky, R. (2009). *Making reform work*. Rutgers University Press, New Jersey.

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

Received: 2018-10-26
Accepted: 2018-12-11
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.31-38

Original Paper

Application of factor analysis to measure social capital in Slovak districts

Lukáš Varecha*

Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Department of Regional and Rural Development, Nitra, Slovakia

ABSTRACT

Factor analysis is a technique particularly suitable for analysing the patterns of complex, multidimensional relationships. Given the multidimensional nature of social capital, a plethora of indicators is often utilized for measurement purposes. In this paper, we utilize factor analysis to explore patterns of social capital and to identify spatial differentiation in the level of social capital at the level of Slovak districts. Three factors have been extracted. The first factor represents low level of bridging social capital, the second factor represents activities of associations and the third factor represents low level of bonding social capital. Factor scores of extracted factors have been visualized in order to explore spatial differentiation of social capital. Visualisation shows significant differences in the levels of individual forms of social capital in Slovak districts.

KEYWORDS: social capital, factor analysis, social network, districts

JEL CLASSIFICATION: O43, C10

INTRODUCTION

Social capital is considered to be a multidimensional concept involving relationships and contacts between actors of social groups and networks, as well as trust in other members of the group or network, the entire community or the whole society and its institutions [4, 7]. Since the early 1990s, the concept of social capital has become a subject of broad attention. It became popular mainly thanks to the research of Burdieu [3], Coleman [4], and Putnam, Leonardi, and Nanetti [15]. The first two mentioned authors conducted analysis on a micro level (individuals, small groups). The latter authors analysed social capital on a macro level (Italian municipalities) and they considered it not as an individual asset but as an attribute of the community itself. They defined it as "features of social life – networks, norms, and trust". This concept is also perceived as a tool for explaining the differences in the economic development of the regions [6].

Measuring social capital is relatively complex. This is due to the amount of its definitions and also to a large number of its forms (especially individual and collective, bonding, bridging

* Corresponding author: Lukáš Varecha, Slovak University of Agriculture, Faculty of European Studies and Regional Development, 949 76 Nitra, Slovakia, e-mail: xvarecha@uniag.sk

and linking, structural and cognitive) [12]. Bonding social capital represents trust between individuals in close, homogeneous groups. Bridging social capital connects different groups of actors, who are similar in terms of hierarchical status in society. Weak ties are characteristic for this form of social capital. Weak ties and social trust in the territory facilitate access to other forms of capital [6]. Linking social capital represents a network of relationships where individuals interact at different hierarchical levels and enables the acquisition of information, resources and power [18]. Enhanced trust among actors in community facilitates the transfer of information and reduction of transaction costs [14]. In the context of regional development, it is of paramount importance to link regional actors with actors at a higher hierarchical level, who are able to provide external resources [19]. In the context of rural development, the importance of informal relations in Slovak conditions was confirmed by Melichová [11].

Multidimensional nature of social capital cannot be represented by a single indicator. Many authors (for example Putnam [14]) rely on broad sets of indicators to measure social capital. Van Deth [21] states that the most commonly used indicators are measures of voluntary association activities and measures of personal and social trust. A problem is the usage of macro indicators not directly related to social capital components. It is not clear whether they measure the social capital directly or just its outcomes. Using believed outcomes of social capital as indicators may lead to situation where we falsely attribute a change of outcome as an effect of social capital [16]. The confounding number of various aspects, characteristics, indicators, or dimensions of social capital makes a common operationalization problematic [21].

Factor analysis is a technique suitable for analysing the patterns of multidimensional relationships and can be utilized to examine the underlying structure of a large number of variables and to determine whether the information can be condensed in a smaller set of factors [8]. Subsequently, we can assert that variables more associated with similar factors can also be conceptually grouped with each other [20]. Many authors [6, 16, 20] utilized factor analysis in order to assess the impact of broad set of social capital indicators more effectively. At sub-national level, this analysis has been conducted in Czech Republic [12], Poland [6] or Italy [16].

MATERIAL AND METHODS

The aim of the paper is to demonstrate the application of factor analysis in exploring and identifying patterns of social capital dimensions in Slovakia and to explore spatial differentiation of social capital at the level of Slovak districts. Due to the unavailability of part of the data, the districts of Kosice and Bratislava (missing data about voluntary associations) and remaining districts of the Bratislava region (ROP indicator not applicable), were not included in the analysis. Data of 67 districts was therefore used as an input for factor analysis. According to Hair et al. [8], an input for this analysis should have a minimum of 50 observations – we met this condition. It is also necessary that the number of observations is greater than the number of variables at least in the ratio of 5 observations to 1 variable. For 67 observations and 10 variables, this condition is met.

We have used ten indicators as an input into the analysis. The first three indicators are the number of the social and health voluntary associations, sport voluntary associations, and trade unions per 1,000 inhabitants. In order to enhance these indicators, we have used not only the

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
Math Educ Res Appl, 2018(4), 1

residences of voluntary associations, but also their individual organizational units, which helps to reflect their spatial localisation in an improved way. The role of voluntary associations is emphasized by Putnam [13, 14]. He claims that they are able to link people from different social backgrounds. Bonds and contacts emerge and social networks form common norms and values. The most beneficial are activities that are undertaken directly in order to develop the local community [13]. By selecting multiple types of associations, we analyse multiple types of social networks.

Table 1 Selected social capital indicators

Indicator	**Description**	**Year**	**Source**
SP	Number of sport voluntary associations per 1,000 inhabitants	2017	MVSR
TRA	Number of trade unions per 1,000 inhabitants	2017	MVSR
SH	Number of social and health voluntary associations per 1,000 inhabitants	2017	MVSR
EDU	Education index [10] $EDU = 1*P_{elementary} + 2*P_{lower\ secondary} + 3*P_{upper\ secondary} + 4*P_{university}$ where P_x are ratios of inhabitants in respective category.	2011	SODB
ELEC	Voter turnout in parliamentary elections	2016	DATACUBE
NFRA	National fractionalisation [2] $NFRA=1-\sum_{i=1}^{N} p_i^2$ (i = 1,2,..., N), where p_i is a ratio of nationality on population; N is the number of nationalities.	2016	DATACUBE
ROP	Funds received from Regional Operational Programme in the 2007 – 2013 programming period per inhabitant.	2007 - 2015	ITMS
SUI	Average number of suicides including attempts per 1,000 inhabitants	2014, 2015, 2016	National health information center
DIV	Average divorce index	2014, 2015, 2016	DATACUBE
UNM	Unemployment rate	2016	DATACUBE

Source: own processing

Education index is another indicator of bridging social capital. It represents the level of human capital. Human and social capitals are mutually contingent. By creating social networks, there is an accumulation of individual human capital and it is transformed into social capital. Strengthening the level of human capital can thus lead to strengthening the cooperative character of social and institutional networks [9, 12]. The indicator of electoral participation represents the institutional dimension of social capital. It reflects the level of development of civil society and can express public interest in public affairs [9]. The formation of social capital is also affected by the heterogeneity of the community [4]. Individual can avoid members of other social groups and limit bridging social capital [1].

In the context of Slovak Republic, the level of national fractionalizations appears to be an appropriate indicator. Some researchers consider the consequences of a lack of social capital as inverse measures of this concept. For example they use crime rates or low levels of economic growth, which point to the absence of social capital. In this way, we utilized two indicators, measuring the lack of bonding social capital. We use divorce index and suicide rates including attempts per 1,000 inhabitants. These phenomena are tied to economic and social uncertainty [20]. The amount of ROP funds received in the programming period 2007 - 2013 reflects the ability of different actors to receive support from this program. We consider this an indicator of linking social capital. Following Sýkora and Matoušek [19], we have chosen the unemployment rate as an indicator of the "success" of social capital in regional development. Unemployment can lead to the decline of social networks of the individual and social exclusion. Unemployed people often do not have access or resources to participation in society and have limited opportunities to interact with other people [17].

RESULTS AND DISCUSSION

Since the factor analysis assumes the existence of mutual linear dependence of the variables, we inspect the correlation matrix (Table 2). Before we utilize factor analysis it is necessary to verify the assumption that the input variables are correlated with each other. We can observe that using Pearson correlation coefficient, majority of variables are significantly correlated. This indicates suitability for factor analysis. The overall measure of the suitability of the data is measured by Kaiser-Meyer-Olkin MSA statistic [8]. In our case its value is 0.774. We also examined the anti-image correlation matrix and the values of MSA on the main diagonal are greater than 0.5. Bartlett's test of sphericity showed significance ($p < 0.05$). Based on the KMO criterion, and also based on the Bartlett's test of sphericity, we consider input data suitable for the use of factor analysis.

Table 2 Correlation matrix of input variables

	ROP	SP	TRA	SH	EDU	ELEC	SUI	DIV	UNM	NFRA
ROP	1.000									
SP	-.386**	1.000								
TRA	-.012	.459**	1.000							
SH	-.426**	.580**	.456**	1.000						
EDU	-.377**	.627**	.361*	.573**	1.000					
ELEC	-.261*	.397**	.076	.246*	.647**	1.000				
SUI	-.312**	.186	-.196	.129	.037	-.005	1.000			
DIV	-.389**	.301**	-.073	.268*	.052	-.120	.377**	1.000		
UNM	.384**	-.415**	.068	-.325**	-.608**	-.693**	-.164	-.159	1.000	
NFRA	.164	-.163	.102	-.101	-.455**	-.764**	.098	.109	.553**	1.000

* significant at 5 %, ** significant at 1 %
Source: own processing, SPSS

As a method of factor extraction, we have used the principal components analysis. We applied the rule of retention of factors with values of eigenvalues greater than 1.0. This condition was met by three factors which together explained 73.19 % of variability, a sufficiently high percentage. The choice of three factors was confirmed by the scree plot. By combining these criteria, we have come to the conclusion keep three factors. After examining communalities, we assume that the factors explain a sufficient amount of variability of each of the variables. Since there were only slight variations in different rotations, we decided to apply orthogonal rotation of varimax. According to Hair et al. [8], it is necessary to take into account that with the decreasing number of observation, the value of the factor loading considered significant increases. In the case of 70 observations they consider a factor loading greater than 0.65 as significant. The resulting rotated matrix of factor loadings contains significant values for all variables. In the case of one variable (EDU), factor loadings are high two factors. However, factor loadings have different signs. Factor loadings of variables are displayed in the Table 3. Only factor loadings greater than 0.4 are displayed.

Table 3 Factor loadings after varimax rotation

	Component		
	1	2	3
ELEC	-.912		
NFRA	.882		
UNM	.807		
EDU	-.637	.620	
TRA		.854	
SH		.775	
SP		.747	
DIV			.788
SUI			.766
ROP			-.643

Source: own processing, SPSS

The first factor explains 38.19 % of the variability. In the case of this factor, two variables have high positive factor loadings (nationality fractionalization, unemployment rate) and two have high negative factor loadings (voter turnout, education index). This factor represents the low level of bridging social capital resources. This is caused by the combination of low level of human capital, unemployment, and high national fractionalization, resulting in a lack of weak ties, meaning a low potential of information accumulation and transfer. This leads to low civic participation and an undeveloped civil society. We refer to this factor as to "Isolation". However, considering the national fractionalization in the context of Slovakia as the cause of low bridging social capital is questionable as it is given by historical context. It is stable and not caused, for example, by sudden migration. Spatial differentiation of the extracted factors is visualized using their factor scores (Figure 1). Indeed, the least developed districts in the southern and eastern Slovakia have the highest factor scores, namely districts of Revúca, Rimavská Sobota and Rožňava.

The second factor explains additional 18.89 % of variability and it is positively correlated with four variables. Three of these variables represent activities of the chosen associations – sport voluntary, associations, social and health voluntary associations and trade unions. The remaining variable is the education index, which also has a positive factor loading. This factor represents diverse social networks and in combination of high level of human capital these variables represent resources for the creation and accumulation of bridging social capital. We name this factor “Associational activity”. A possible limitation of this factor may be the fact that it is only the number of associations and does not account for the number of their members or the scope of their activity. This problem may be partly alleviated by the above-mentioned inclusion of organizational units of associations. Spatial differentiation of this factor is significantly different, with the highest factor scores in central Slovakia. In the context of regional development, low associational activity (districts of Detva and Poltár) may partially explain their development difficulties.

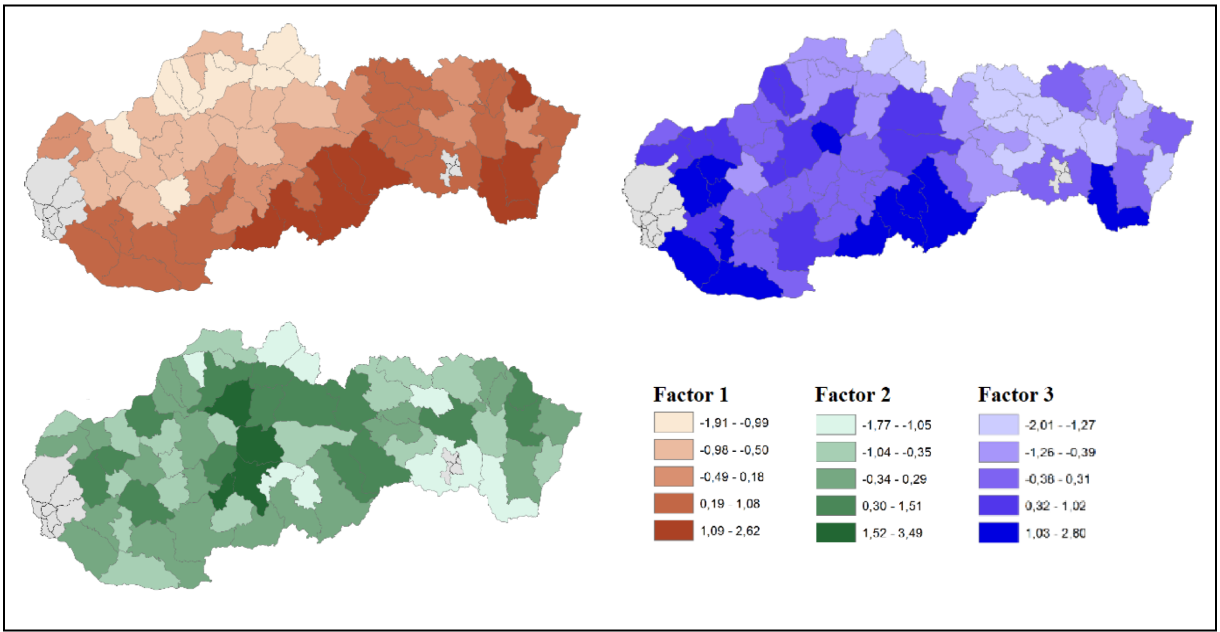


Figure 1 Factor scores of the three extracted factors
Source: own processing, ArcGis

The third extracted factor explains 16.02 % of variability. It comprises of negative societal phenomena – average divorce index and average number of suicides including attempts per 1,000 inhabitants have high positive factor loadings. Interesting is the fact that the amount of funds received from ROP per inhabitant has high although negative factor loading for this factor. We believe this factor may indicate the effects of low level of bonding social capital, meaning there is lower amount of strong bonds among the members of close, homogenous groups. We denote this factor as “Absence of trust”. Concerning factor score differentiation in the districts, we can see that the highest values are in the western Slovakia (Hlohovec, Dunajská Streda) but also in the districts in the southern Slovakia.

CONCLUSIONS

Factor analysis has been utilized in order to explore social capital patterns and to identify spatial differentiation in the level of social capital in the districts of the Slovak Republic. The inputs for factor analysis were 10 variables, commonly used as social capital indicators. The suitability of data has been proven by Kaiser-Meyer-Olkin MSA statistic and Bartlett's test of sphericity. Factor analysis led to the extraction of three interpretable factors, with the factor scores of each factor being visualized. The first factor "Isolation" represents a low level of bridging social capital; the second factor "Associational activity" represents social networks and human capital that serve as a resource for bridging social capital accumulation. The third factor, "Absence of trust" reflects the high level of negative social phenomena resulting from the lack of bonding social capital.

ACKNOWLEDGEMENTS
Authors acknowledge financial support of the Slovak Scientific Grant Agency VEGA 1/0789/18.

REFERENCES

[1] Abrams, D., Hogg, M. & Marques, J. (2005). *The social psychology of inclusion and exclusion*. New York: Psychology Press.

[2] Alesina, A., Devleeschauwer, A., Easterly W., Kurlat, S. & Wacziarg, R. (2003). Fractionalization. *Journal of Economic Growth*, 8(2), 155-194. doi: https://doi.org/10.1023/a:1024471506938

[3] Burdieu, P. (1986). The forms of capital. Riachradson, J. G. (ed.) *Handbook of Theory and Research for the Sociology of Education*. New York: Greenwood Press, 241-258.

[4] Coffé, H. (2009). Social capital and community heterogeneity. *Social Indicators Research*, 91(2), 155-170. doi: https://doi.org/10.1007/s11205-008-9275-y

[5] Coleman, J. (1988). Social Capital and the Creation of Human Capital. *American Journal of Sociology,* 94(1) 52-120. Retrieved 2018-09-21 from https://faculty.washington.edu/matsueda/courses/587/readings/Coleman%201988.pdf

[6] Działek, J. (2014). Is social capital useful for explaining economic development in Polish regions? *Geografiska Annaler, Series B: Human Geography*, 96(2), 177-193. doi: https://doi.org/10.1111/geob.12044

[7] Fukuyama, F. (1995). *Trust: the social virtues and the creation of prosperity*. New York: Free Press.

[8] Hair, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2014). *Multivariate Data Analysis*. 7 ed. London: Pearson.

[9] Jančák, V., Havlíček, T., Chromý, P. & Marada, M. (2008). Regional differentiation of selected conditions for the development of human and social capital in Czechia. *Geografie,* 113(3), 269-284 Retrieved 2018-09-04 from: https://www.researchgate.net/publication/262526400_Regional_Differentiation_of_Selected_Conditions_for_the_Development_of_Human_and_Social_Capital_in_Czechia

[10] Krejčí, T. & Ptáček, P. (2007). Educational structure of the population in the Karpatský region. *GEOGRAPHIA CASSOVIENSIS,* 1(1), 95-102. Retrieved 2018-09-04 from: http://geografia.science.upjs.sk/images/geographia_cassoviensis/articles/GC-2007-1-1/Krejci-Ptacek_tlac1.pdf (in Czech).

[11] Melichová, K. (2015). Building social capital - does it facilitate development of rural areas?. *Forward - Moving frontiers of rural and regional development, proceedings of research conference*. Nitra: Slovenská poľnohospodárska univerzita, 86-92.

[12] Pileček, J. & Jančák, V. (2010). Can Social Capital Be Measured? An analysis of territorial differences among the districts of Czechia. *Geografie,* 115(1), 78-95. Retrieved 2018-09-15 from: http://geography.cz/sbornik/wp-content/uploads/downloads/2012/11/g10-1-5pilecek-jancak1.pdf (in Czech).

[13] Putnam, R. (1993). The Prosperous Community. *The American Prospect*, 4(13), 35-42. Retrieved 2018-09-24 from: http://prospect.org/article/prosperous-community-social-capital-and-public-life

[14] Putnam, R. (2000). *Bowling alone: The collapse and revival of American community.* New York: Simon & Schuster.

[15] Putnam, R., Leonardi, R. & Nanetti, R. Y. (1993). *Making Democracy Work*. Princeton: Princeton University Press.

[16] Sabatini, F. (2008). Social Capital and the Quality of Economic Development. *KYKLOS,* 61(3), 466-499. doi: https://doi.org/10.1111/j.1467-6435.2008.00413.x

[17] Stone, W., Gray, M. & Hughes, J. (2003). Social Capital at Work: how family, friends and civic ties relate to labour market outcomes. *Australian Institute of Family Studies*, 31(1), 1-42. Retrieved 2018-09-04 from: http://www.aifs.gov.au/institute/pubs/respaper/RP31.pdf

[18] Svendsen, G. T. & Svendsen, G. L. (2010). *Handbook of social capital: The troika of sociology, political science, and economics*. Cheltenham: Edward Elgar

[19] Sýkora, L. & Matoušek, R. (2009). Social capital and social network territoriality. Poštolka, V., Lipský, Z., Popková, K., & Špída, J. (eds.) *Geodays Liberec 2008.* Liberec: Technická univerzita v Liberci, 50-56 (in Czech).

[20] Thompson M. F. & Slaper, T. F. (2016). Social capital and regional economic performance: a study across US Metropolitan Statistical Areas. Westlund, H., & Larsson, J. P. *Handbook of Social Capital and Regional Development.* Cheltenham: Edward Elgar Publishing, 296-320.

[21] van Deth, J. W. (2008). Measuring Social Capital. Castiglione, D., van Deth, J. W., & Wolleb, G. *The Handbook of Social Capital.* Oxford: Oxford University Press, 150–176.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2018 (4), 1

Received: 2018-09-28
Accepted: 2018-12-11
Online published: 2018-12-18
DOI: https://doi.org/10.15414/meraa.2018.04.01.39-48

Original Paper

Input-output modeling approach to measuring structural shift of output and macroeconomic productivity: the case of Slovakia

Martin MARIS[*]

Slovak University of Agriculture in Nitra, Faculty of European Studies and Regional Development, Nitra, Slovakia

ABSTRACT

The scope of the paper was an analysis of structural relations, sectoral output and changes in Slovak economy between the time periods of 2010 – 2014. As a main research method was used the input-output analysis, based on the Leontief input-output model. The results have shown, that in terms of the sectoral output, the highest production multipliers were found in sector of Industry (2.72), Financials (1.88) and Energetics (1.86). In terms of structural relations among the sectors highest spillover effects were recorded by the sector of Accomodation (1.44), Administrative services (1.53) and Professional services (1.35). Finally, most profounding structural changes in terms of the sectoral output were recorded in case of the Mining, Construction and Financials. Also, in these sectors, main driving forces had acted contradictory and thus countervailing.

KEYWORDS: structural changes, Leontief input-output model, input-output tables,

JEL CLASSIFICATION: E160, C1

INTRODUCTION

The process of structural changes has been attracting economists' attention for a long time and up to present it is still relevant concept. The number of authors point structural changes on changes in sectoral composition of output and employment in the national economy. During the process of economic development, employment first shifts from agriculture to manufacturing and then to services. This is a core aspect of the three-sector hypothesis, as cited in Mihnenoka and Senfelde [9]. Dietrich [3] views that structural changes are conceived in the framework provided by the three-sector hypothesis. There exists an interrelation between the two phenomena of economic growth and structural change measured either in terms of employment shares or in terms of output shares.

Marjanovic [8] specifies that the structural transformation is the accumulation of physical and human capital, but also the changes in composition of demand, production, employment and trade.

[*] Corresponding author: Martin Mariš, Slovak University of Agriculture, Faculty of European Studies and Regional Development, Slovakia, Tr. A. Hlinku 2, 949 76, Nitra, e-mail: martin.maris@uniag.sk

An income elastic demand for services is usually regarded as one of the major explanations for the observed pattern of structural change in the world economy. With rising incomes, expenditures for services are expected to increase more than proportionally. This demand-bias hypothesis, together with the hypothesis of lagging productivity growth in the service sector, was suggested as the explanation for the long-rise of the employment share of the service sector that is a common feature of all industrialized countries (as cited in [4]). However, Gundlach [4] contradicts, that both hypothesis – demand and productivity bias together do not guarantee such an outcome. He proposes the extension of the model to include socio-demographic factors such as changing age structures of the population due to declining birth rates, declining household sizes, and increasing female labor force participation, which all can be expected to have a positive impact on aggregate demand for services.

Blanchard and Johnson [1] relate structural change also to technological progress – the change in the structure of the economy induced by technological progress. Indeed, as the authors further state, technological progress has many dimensions. Among the foremost, we might mention following ones:

- It can lead to larger quantities of output for given quantities of capital and labor.
- It can lead to better products.
- It can lead to new products.
- It can lead to a larger variety of products.

Slovakian economy has been experiencing gradual structural change and their consequences since its socialism regime fall in 1989. Over 1989 in Slovakia (in that time Czechoslovakia) had become deep social and political changes, followed by the economic changes; they consisted in removing of free market barriers, the transformation of the market economy, transformation of production structure of the economy, change in political and economic orientation from the former Soviet Union on Western European countries, evolution of the private sector and others [14].

Šafr and Vltavská [12] point on neo-liberal political reforms, which had been realized in Slovakia during a period of 1998 – 2006, during the Mikulas Dzurinda, right-wing government. As a foremost important they consider the social – pension, tax code and labor code reform and others.

Kotulic et al. [5] assert, that through the period from 2000 - 2012, we can observe the enormous decline of employed persons in the primary sector in the long term (Agriculture, forestry and fishing) by 44%, similar downtrend but much moderated also recorded the sector of industry, which declined by 4%, however the chosen branches of service sector and sector of the construction marked a substantial growth in employment, like sales, transportation and accommodation rose by 29%, professional activities by 56% and construction by 41%.

Furthermore, the authors analyses the employment and output through the employment elasticity, as a change in employment given the change in output. They conclude, that during the observed period 1995 – 2012, employment elasticity indicator became $\varepsilon = 0.02$, which means increase in employment and output, together with increasing in labor productivity [6].

MATERIAL AND METHODS

In the introductory part of the paper, we have pointed to technological progress and changes in composition of demand as a one of the major factors, which likely to fuel the structural change. We will further explore this concept through the analysis of production structure of the economy, and through the analysis of the output produced in a given structure of various inputs and labor efficiency. For this purpose, we have opted as a main research method application of the input-output analysis.

The basic tool of the input-output analysis is a Leontief input-output model. However, the input-output analysis is being primarily used to quantify mutual relations and interdependencies in the production structure of the economy based on "balances" of commodities – inputs and their use – outputs.

In economics, an input-output model is a quantitative economic technique that represents the interdependencies between different branches of a national economy of different regional economies. This method builds an economic mathematic model simulating the social reproduction process that national economy sectors products inflow and outflow through establishing the input-output table and the corresponding linear algebraic equation system [2].

The basic Leontief input-output model is generally constructed from observed economic data for a specific geographic region (nation, state, country, etc.). One is concerned with the activity of a group of industries that both produce goods (outputs) and consume goods from other industries (inputs) in the process of producing each industry´s own output. In practice, the number of industries considered may vary from only few to hundreds or even thousands. The fundamental information used in input-output analysis concerns the flows of products from each industrial sector, considered as a producer, for each of the sectors, itself and others, considered as consumers [10].

The input-output analysis became an indispensable means for studying numerous views on mutual interwinements of sectors of the economy. Earlier, the input-output tables began to be used for establishing the linkages between the sectors of the economy. These linkages were studied on the side of inputs (the side of supply) to individual sectors (backward linkages) as well as on the side of outputs (the side of sales) of an individual sectors to other sectors (forward linkages) [11, 15].

The outline of each input-output model is input-output table. A table should be divided in three parts: inter-branch relation matrix, matrix of final demand, and matrix of primary inputs. Each matrix – divided by the sector or by the final demand and category of primary input, describes customer relations in economy for fixed time unit (year) [7].

Tiruneh et al. [13] opted to two different approaches in constructing of regional input-output model. They had shown on differences between the inter-regional (IRIO) and single regional models (SRIO) for all 14 regions of the Czech Republic and with 82 products according the Classification of products CZ-ZPA. According the findings in the case of the single-regional approach, using only this approach had proved a systematic undervaluation of specific products and regions contrary to other regions. For instance products K (Financials and insurance activities), J (Information and Communication), E (Water supply, sewerage, waste management and remediation activities), A (Agriculture, forestry and fishing) and B (Mining and quarrying) show the most significant differences. On average across all products, SRIO multipliers are undervalued by 14% compared to IRIO.

An essential condition for assembling the input-output model for the Slovak economy is the matrix, being recognized at the symmetric input-output table. The symmetric input-output table is being assembled by *the table of supply and use.* Rows of this table represent the commodities and columns represent the branches of the economy, which these commodities during the production process will consume. So, the dimension of this matrix is *commodity x sector*. The table of final use is being constructed in two subsets. The first subset is being recognized as a table "A" which includes the data about production consumption of domestic and also imported commodities. The second subset represents the table "B" which includes data about only domestic production relations. In our research paper we have been concerned only by the records of table "A" due to the availability in several consecutive time periods however, what might bias our results.

Input-output tables for Slovakia in its basic form consist 99 branches of the economy. However, we are able to group them into sectors according the SK-NACE in to 21 sectors representing all 99 branches of the economy. However, practically we have used only 19 sectors (T - household activities etc., U - activities of exterritorial organizations were non-available).

The data (tables of supply and use) used for the constructing of the input-output model were provided by the Statistical Office of the Slovak Republic. There were used records for 2010 and 2014 time period, adjusted on constant prices of the base year 2010.

For the purpose of the constructing the Leontief model and deriving of the multipliers we have proceeded according by [10, 13].

All kinds of economic activities (according the SK-NACE classification) we have divided into n commodities, representing goods and services. Produced commodities are being consumed for the purpose of production of the new commodities or for demand satisfaction of the final consumption. Total volume of production of the i-th commodity, we mark as x_i, intermediate consumption as of i-th commodity for the production of the j-th commodity as z_{ij} and total consumption of the i-th commodity as y_i.

Formally written as

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + y_i \tag{1.1}$$

Such system of the linear equations determines the balance of the consumption of all commodities in the economy:

$$\begin{aligned} x_1 &= z_{11} + z_{12} + \cdots + z_{1n} + y_1 \\ x_i &= z_{i1} + z_{i2} + \cdots + z_{in} + y_i \\ x_n &= z_{n1} + z_{n2} + \cdots + z_{nn} + y_n \end{aligned} \tag{1.2}$$

This system might be written in matrix form as

$$\boldsymbol{x} = \boldsymbol{Zi} + \boldsymbol{y} \tag{1.3}$$

Where x represents the volume vector of the commodity producers, $\boldsymbol{y}$ volume vector of the final consumption, $\boldsymbol{i}$ is unit vector and $\boldsymbol{Z}$ is an intermediate consumption matrix. The volumes of the intermediate consumption, or inputs, are in the Leontief input-output model directly proportional to the size of the output, what is the volume of the production of the total sector. This model uses the assumption of the so called *Leontief production function.* This means, that production of each unit of the output demands fixed units of the input. Any form of the substitution between the inputs is not possible. Thus, it exist accurate linear relation between the production volume and volume of the inputs. These relations are being determined by so called *technological coefficients* a_{ij}, being computed as a ratio between the volume of input of the i-th commodity used in production of the j-th commodity and total production volume of the j-th commodity

$$a_{ij} = \frac{z_{ij}}{x_j} \tag{1.4}$$

From the matrix notation we are able to find out the matrix of technological coefficients $\boldsymbol{A}$, by right-multiplying the intermediate consumption matrix by the diagonal matrix of inverted values of total production volumes of the commodities.

$$\boldsymbol{A} = \boldsymbol{Z}\hat{\boldsymbol{x}}^{-1} \tag{1.5}$$

Finally, linear equation system, divided the production of the commodities on intermediate and final consumption, we are able to formally write with the use of technological coefficient matrix as

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{y} \tag{1.6}$$

By the simple adjustment we can get the exciplit relation between the production and final consumption vectors

$$(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{y} \tag{1.7}$$

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{y} = \boldsymbol{L}\boldsymbol{y}$$

Where $(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{L}$ means $Leontief\ inverse\ matrix$. Using the Leontief inverse matrix, the Leontief model could be formally written as

$$\boldsymbol{x} = \boldsymbol{L}\boldsymbol{y} \tag{1.8}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} l_{11} & \cdots & l_{1n} \\ \vdots & \ddots & \vdots \\ l_{n1} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

Where vector $\boldsymbol{y}$ after left-multiplication by the matrix $\boldsymbol{L}$ gives total production in the economy of the commodity, i.e. vector $\boldsymbol{x}$. Each unit l_{ij} in the matrix $\boldsymbol{L}$ determines, what volume of the commodity i is necessary to produce for providing one unit of the commodity j for a final use. Summation of all units in each matrix column (i.e. multiplying the matrix by the unit line vector) gives us a line vector of $production\ multipliers$ with units l_j.

$$\boldsymbol{o} = \boldsymbol{e}\boldsymbol{L} \tag{1.9}$$

$$(o_1 \ldots o_n) = (1 \ldots 1) \begin{pmatrix} l_{11} & \cdots & l_{1n} \\ \vdots & \ddots & \vdots \\ l_{m1} & \cdots & l_{mn} \end{pmatrix}$$

Finally we concern with the analysis of the structural changes in the economy. When comparing the structure of the economy between the two different time periods (2010 vs. 2014) we were able to find out the change in the output structure among the sectors of the economy. For this purpose we have used the $structural\ decomposition\ method.$ This method helps us to deploy the changes of the complex variable on element factors which are being constituting it. In our case as a complex variable, we might consider the vector of production $\boldsymbol{x}$, which depends on the production structure of direct and indirect relations – Leontief inverse matrix and on the final demand y.

The most likely factors behind the structural changes are being considered as changes in final demand or in the structure of the demand or the changes in the technology as mentioned in the theoretical part of the paper. Total production in our two respected periods (2010 = 0; 2014 = 1) can be formally written as:

$$\boldsymbol{x^0} = \boldsymbol{L^0 y^0} \tag{2.0}$$

$$\boldsymbol{x^1} = \boldsymbol{L^1 y^1}$$

Where the final demand of $\boldsymbol{y^0}$ generated standing by the production structure of $\boldsymbol{L^0}$ total production in the volume and structure of $\boldsymbol{x^0}$. Similarly, the final demand of $\boldsymbol{y^1}$ generated standing by the production structure of $\boldsymbol{L^1}$ total production in the volume and structure of $\boldsymbol{x^1}$. The change in the production can be formally written as:

$$\Delta\boldsymbol{x} = \boldsymbol{x^1} - \boldsymbol{x^0} = \boldsymbol{L^1 y^1} - \boldsymbol{L^0 y^0} \tag{2.1}$$

For deriving the effect of the considered factors, we at first express the matrix $\boldsymbol{L^0}$ as a difference between the Leontief inverse matrix in time period of 1 and change in the structure of the production $\Delta\boldsymbol{L}$**:**

$$\Delta\boldsymbol{L} = \boldsymbol{L^1} - \boldsymbol{L^0} \Rightarrow \boldsymbol{L^1} - \Delta\boldsymbol{L} \tag{2.2}$$

Vector of the final demand in time period of 1 $\boldsymbol{y^1}$ can be formally written as a sum of the final use in the time period of 0 $\boldsymbol{y^0}$ and change in final use of Δy:

$$\Delta\boldsymbol{y} = \boldsymbol{y^1} - \boldsymbol{y^0} \Rightarrow \boldsymbol{y^1} = \boldsymbol{y^0} + \Delta\boldsymbol{y} \tag{2.3}$$

The change in total production of $\Delta\boldsymbol{x}$ after substituting of (2.3) and (2.4) can be formally written as:

$$\Delta\boldsymbol{x} = \boldsymbol{L^1}(\boldsymbol{y^0} + \Delta\boldsymbol{y}) - (\boldsymbol{L^1} - \Delta\boldsymbol{L})\boldsymbol{y^0} \tag{2.4}$$

This can be adjusted on:

$$\Delta\boldsymbol{x} = (\Delta\boldsymbol{L})\boldsymbol{y^1} + \boldsymbol{L^0}(\Delta\boldsymbol{y}) \tag{2.5}$$

RESULTS AND DISCUSSION

Firstly, we introduce the basic table for identifying the economic sectors which data had been used for input-output output table construction.

Next, we had computed the production multiplicators that show us the change in production invoked by the change in final demand for 2014. Furthermore, this model helps us to reveal also indirect relations, for instance the impact of change in the demand for the products of industry on services, etc.

Table 1 General classification of the national sectors of economy according the SK-NACE

A	Agriculture, forestry and fishing
B	Mining and quarrying
C	Industrial production
D	Electricity , gas, steam and air conditioning supply
E	Water supply; cleaning and waste- water treatment , waste management and remediation activities
F	Construction
G	Wholesale and retail trade ; repair of motor vehicles and motorcycles
H	Transport and Storage
I	Accommodation and food services
L	Information and communication
K	Financial and insurance activities
L	Real estate activities
M	Professional, scientific and technical activities
N	Administrative and support services
O	Public administration and defense ; compulsory social security
P	Education
Q	Health care and social assistance
R	Arts, entertainment and recreation
S	Other activities

Source: Author

Table 2 Production multipliers by the commodity in 2014

Code	Commodity	Production multiplier	Effects on other commodities	Share of effects %
A	Agriculture, forestry and fishing	1.30	0.97	43%
B	Mining and quarrying	1.18	0.94	44%
C	Industrial production	2.72	0.86	24%
D	Electricity , gas, steam and air conditioning supply	1.86	1.36	42%
E	Water supply; cleaning and waste-water treatment, waste management and remediation activities	1.08	1.32	55%
F	Construction	1.53	1.14	43%
G	Wholesale and retail trade; repair of motor vehicles and motorcycles	1.08	1.16	52%
H	Transport and Storage	1.42	1.11	44%
I	Accommodation and food services	1.01	1.48	59%
L	Information and communication	1.28	0.94	42%
K	Financial and insurance activities	1.88	0.98	34%
L	Real estate activities	1.07	0.69	39%
M	Professional, scientific and technical activities	1.25	1.35	52%
N	Administrative and support services	1.18	1.53	56%
O	Public administration and defense; compulsory social security	1.04	0.70	40%
P	Education	1.03	0.70	40%
Q	Health care and social assistance	1.03	1.13	52%
R	Arts, entertainment and recreation	1.21	0.32	21%
S	Other activities	1.03	1.09	51%

Source: Author's calculation

Table 2 shows us production multipliers by the sector for the time period 2014. The highest production multipliers record the sector of the Industry (C), Electricity (D), Finances (K) and Construction (F). So, for instance the change in the demand for industrial products by 1 million €, would give rise to the industrial production by 2.72 million € and production in other sectors by 940 thousand €. The sectors with greatest effect in terms of the production on other sectors are Accomodation (I), Administrative services (N) and Professional services (M).

Finally, we approach to analysis of the structural changes over the examined period. For this purpose the structural decomposition method seemed to fit us well. Using this method we were able to decompose the changes in the composite variable on the tribute of factors which it creates. As a composite variable we might consider the generated production $\boldsymbol{x}$. The production depends on the structure of the production – direct and indirect relations and on the final demand $\boldsymbol{y}$. The structural decomposition had been performed using the equations (2.1 - 2.6). The change in generating production $\Delta\boldsymbol{x}$ had been decomposing on change in the structure of the production and on change in the final demand.

Table 3 Structural decomposition of the change in total output on the changes in structure of production and on the changes in the final demand, over 2010 – 2014, in million €

Sector	Δ in production	Δ in final demand	∑ of changes
A	685.88	2208.82	2894.71
B	-460.57	870.08	409.51
C	1231.48	10029.13	11260.61
D	1019.68	885.03	1904.71
E	9.39	135.56	144.95
F	-1125.82	710.18	-415.64
G	-524.82	16.03	-508.79
H	200.82	3273.91	3474.72
I	246.94	238.86	485.80
J	455.04	116.50	571.54
K	322.46	-1848.50	-1526.04
L	-271.56	1301.53	1029.97
M	-223.67	1276.96	1053.29
N	1497.18	512.41	2009.59
O	186.32	-42.28	144.04
P	37.44	558.30	595.74
Q	-6.31	392.08	385.77
R	214.72	646.63	861.35
S	-76.09	80.41	4.32

Source: Author's calculation

Table 3 shows that final change in total output is the sum of the changes in the production structure and the change in the final demand. Both examined factors in several cases, for instance Mining (B), Construction (F), Finances (K) and others, are acting contrary and countervailing. For the illustration, we can show the relative share of both effects on the value of the production output deployed on sectors in 2014.

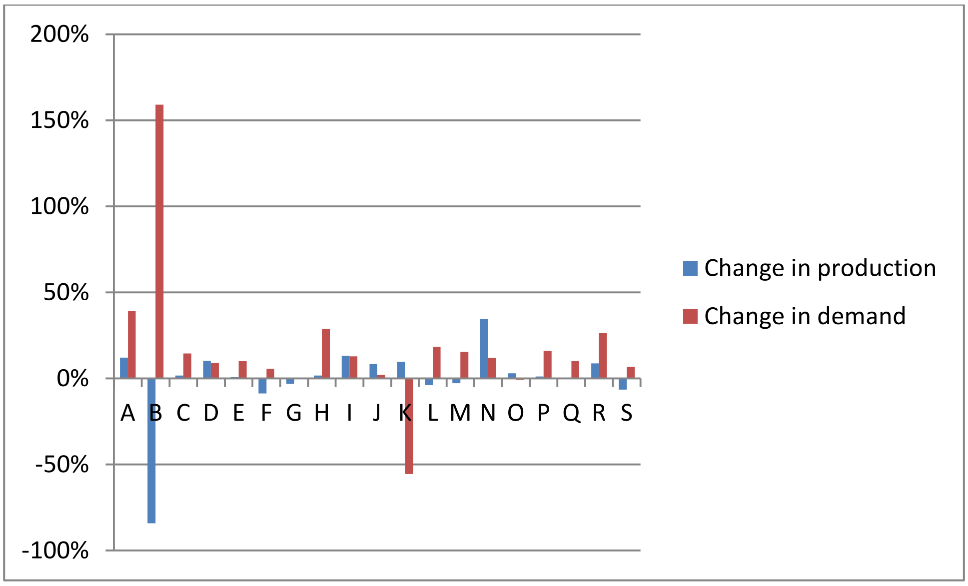


Figure 1 Relative share of effect of change in production structure and change in final demand on the value of the total production output in 2014
Source: Author

Figure 1 shows the share of the effects resulting from the change in production structure or change in demand relative to the value of the sectoral output in 2014. In relative values, the greatest change in both effects has shown the Mining sector (B), however in terms of values of the output it has marginal importance. Among the other sector, significant relative change recorded sector of Financials (K) with a drop in the demand by the 56%, followed by the sector of the Agriculture (A) with a rise in the demand by the 39%. In the case of the changes on production side, the highest drop in production recorded the sector of the Construction (F), however only a modest drop by 9% and the growth recorded the sector of Administration (N) by 35%, followed by the sector of the Accomodation (I) by 13% and others.

CONCLUSION

The paper was concerned with the analysis of the dependencies within the sectoral structure of the Slovak economy, with an emphasis on structural changes resulting from changes in the production in particular sectors and change in the demand. Basic comparing time period was a period of 2010 and 2014. As a main research method was used the IO- analysis, based on the Leontief IO-model. The main research outcomes of the paper were computed sectoral production multipliers, the coefficients of the labor intensity and structural decomposition of change in total output on changes in the production and changes in the demand.

In the case of the production multipliers the highest contribution to the total output has the sector of Industry (C), Electricity (D), Finances (K) and Construction (F). In terms of the spillover effects (e.g. effects on related sectors) the highest contributions have sectors of Accomodation (I), Administrative services (N) and Professional services (M). In the case of the structural changes, the main assessed changes were changes in the structure of the production and in the demand. The final impact of the changes on various sectors was diverse, sometimes acting contradictory, like in the sector Mining (B), Construction (F), Financials (K) and others.

The most likely causes of the structural changes, we might consider technological changes and innovations, as outlined in the introductory part of the paper, resulting in changes in labor productivity/intensity and changes in the composition of the demand, contributing to changes in the total output over the examined time period 2010 - 2014. However, we have to consider also other factors likely influencing the structural changes, like different stages of the business cycle of the economy and different volume of the import in examining time periods. Counting the value of import in input-output tables likely biased our results, what is the main shortage of the paper. However, there is no statistical evidence excluding the value of imports from national accounts over the examined periods, provided by the statistical Office the Slovak republic.

REFERENCES

[1] Blanchard, O. & Johnson, D., R. (2013). *Macroeconomics*. Sixth edition. NJ: Pearson Education, Inc.

[2] Dai, Q. & Yang, J. (2013). Input-output Analysis on the Contribution of Logistics Park Construction to Regional Economic Development. *Proceedings from CICTP '13: 13th COTA International Conference of Transportation Professionals. Procedia – Social and Behavioral Sciences*, 96, (2013) 599-608. Elsevier Ltd.

[3] Dietrich, A. (2009). *Does growth cause structural change, or is it the other way round?: a dynamic panel data analyses for seven OECD countries* (No. 2009, 034). Jena economic research papers. Retrieved 2018-09-21 from https://www.econstor.eu/bitstream/10419/31735/1/59955990X.PDF

[4] Gundlach, E. (1993). Demand bias as an explanation for structural change. *Kiel Working Paper, No. 594, Institut fur Weltwirtschaft (IFW), Kiel*. Retrieved 2018-09-21 from: http://hdl.handle.net/10419/647

[5] Kotulic, R., Huttmanova, E., Vozarova, I. & Nagy, J. (2014a). Performance of the Slovak Economy in Relation to Labor Productivity and Employment. Proceedings from *GCBEMT '15: 2nd Global Conference on Bussiness, Economics, Management and Tourism*. Prague. Procedia Economics and Finance, 23 (2015), 1590-1595. Elsevier Ltd.

[6] Kotulic R., Huttmanova, E., Vozarova, I. & Nagy, J. (2014b). The Structural Analysis of the Sectors of Slovak Economy and Employment in Times of Global Change and the Subsequent Development. Proceedings from *GCBEMT '14: 2nd Global Conference on Bussiness, Economics, Management and Tourism*. Prague. Procedia Economics and Finance, 23 (2015), 970-975.

[7] Maier, G. & Todling, F. (1997). *Regionálna a urbanistická ekonomika: Teória lokalizácie a priestorová štruktúra*. Bratislava: Elita (in Slovak).

[8] Marjanovic, V. (2015). Structural changes and structural transformation in a modern development economy. *Journal of Economic Themes*, 53 (1), 63-82. doi: https://doi.org/10.1515/ethemes-2015-0005

[9] Mihnenoka, A. & Senfelde, M. (2017). The impact of national economy structural transformation on regional employment and income: the case of Latvia. *South East European Journal of Economics and Bussiness,* 12 (2), 47-60. doi: https://doi.org/10.1515/jeb-2017-0015

[10] Miller, E. R. & Blair, D. P. (2009). *Input-output Analysis: Foudations and Extensions*. Second Edition. Cambridge University Press.

[11] Pfajfar, L. & Dolinar, A. L. (2000). Intersectoral linkages in the Slovenian economy in the years 1990, 1992, 1993 and 1995. In *Key sectors in the Slovenian economy. Papers of the 13th International Conference on Input-Output Techniques*.

[12] Šafr, K. & Vltavská, K. (2017). Ilustration of Single-regional and Inter-regional Approach in Regional Input-output Analysis. *Statistika - Statistics and Economy Journal*, 97, 18-31.

[13] Tiruneh, M. W., Lábaj, M. & Dujava D. (2011). *Štruktúra ekonomiky a ekonomický rast: Ako naplniť teóriu číslami*. Bratislava: IRIS (in Slovak).

[14] Výrostová, E. (2010). *Regionálna ekonomika a rozvoj*. Bratislava: Wolters Kluwer (Iura Edition) (in Slovak).

[15] Yay, G. G. & Keceli, S. (2009). The Intersectional Linkage Effects in Turkish Economy: An Application of Static Leontief Model. *Panoeconomicus*, 3, 301-326. doi: https://doi.org/10.2298/PAN0903301G